11.7 Use Geometric Probability

Goal • Use lengths and areas to find geometric probabilities.

Your Notes

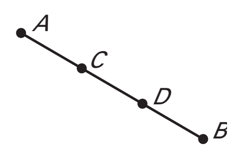
VOCABULARY

Probability The probability of an event is a measure of the likelihood that the event will occur.

Geometric probability A geometric probability is a ratio that involves a geometric measure such as length or area.

PROBABILITY AND LENGTH

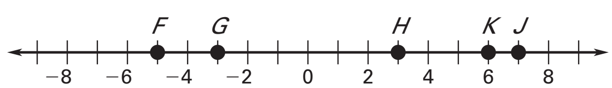
Let $\overline{AB}$ be a segment that contains the segment $\overline{CD}$. If a point K on $\overline{AB}$ is chosen at random, then the probability that it is on $\overline{CD}$ is the ratio of the length of $\overline{CD}$ to the length of $\overline{AB}$.



$$P(K \text{ is on } \overline{CD}) = \frac{\text{Length of } \overline{CD}}{\text{Length of } \overline{AB}}$$

Example 1 ***Use lengths to find a geometric probability***

Find the probability that a point chosen at random on $\overline{FJ}$ is on $\overline{GK}$.



To apply the geometric probability formulas on this page and the next, you need to know that every point on the segment or in the region is *equally likely* to be chosen.

Solution

$$P(\text{Point is on } \overline{GK}) = \frac{\text{Length of } \overline{GK}}{\text{Length of } \overline{FJ}}$$

$$= \frac{|6 - (-3)|}{|7 - (-5)|}$$

$$= \frac{9}{12} = \frac{3}{4}, \ 0.75, \text{ or } 75\%$$

Your Notes

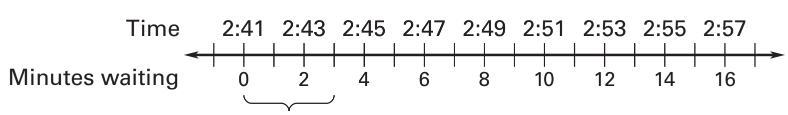
Example 2 ***Use a segment to model a real-world probability***

Shuttle A shuttle to town runs every 10 minutes. The ride from your boarding location to town takes 13 minutes. One afternoon, you arrive at the boarding location at 2:41. You want to get to town by 2:57. What is the probability you will get there by 2:57?

Solution

Step 1 **Find** the longest you can wait for the shuttle and still get to town by 2:57. The ride takes 13 minutes, so you need to catch the shuttle no later than 13 minutes before 2:57, or 2:44. The longest you can wait is 3 minutes (2:44 − 2:41 = 3 min).

Step 2 **Model** the situation. The shuttle runs every 10 minutes, so it will arrive in 10 minutes or less. You need it to arrive within 3 minutes.



The shuttle needs to arrive within the first 3 minutes.

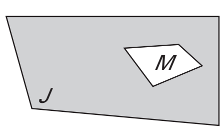
Step 3 **Find** the probability.

$$P(\text{Get to town by 2:57}) = \frac{\text{Favorable waiting time}}{\text{Maximum waiting time}} = \frac{3}{10}$$

The probability that you get to town by 2:57 is $\frac{3}{10}$, or 30%.

PROBABILITY AND AREA

Let *J* be a region that contains region *M*. If a point *K* in *J* is chosen at random, then the probability that it is in region *M* is the ratio of the area of *M* to the area of *J*.

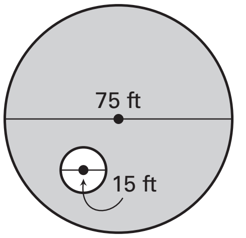


$$P(K \text{ is in region } M) = \frac{\text{Area of } M}{\text{Area of } J}$$

Your Notes

Example 3 ***Use areas to find a geometric probability***

Golf A golf ball is hit and stops on the green. A prize is won if it stops in the painted circle. The diameters of the green and circle are shown at the right. If the ball is equally likely to stop on any point on the green, what is the probability that a prize is won?



Solution

Find the ratio of the circle's area to the green's area.

$$P(\text{prize is won}) = \frac{\text{Area of painted circle}}{\text{Area of green}}$$

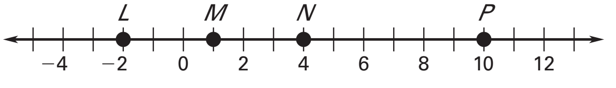
$$= \frac{\pi(\boxed{7.5})^2}{\pi(\boxed{37.5})^2} = \frac{\boxed{56.25}\,\pi}{\boxed{1406.25}\,\pi} = \frac{1}{25}$$

All circles are similar and the Area of Similar Polygons Theorem also applies to circles. The ratio of radii is 7.5 : 37.5, or 1 : 5, so the ratio of areas is $1^2 : 5^2$, or 1 : 25.

The probability that a prize is won is $\frac{1}{25}$, or 4 %.

Checkpoint **Complete the following exercises.**

1. Find the probability that a point chosen at random on $\overline{LP}$ is on $\overline{MN}$.



$\frac{1}{4}$, 0.25, or 25%

2. In Example 2, suppose you arrive at the pickup location at 2:38. What is the probability that you will get to town by 2:57?

60%

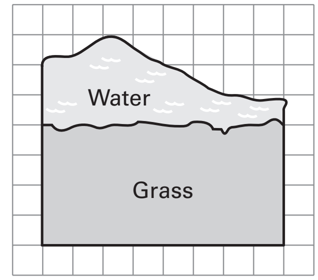
3. On the green in Example 3, the hole is 4.25 inches in diameter. Find the probability that your ball stops in the hole.

about 0.000022

Your Notes

Example 4 ***Estimate area on a grid to find a probability***

Property A homeowner's property is shown in the scale drawing. If a deer is equally likely to be anywhere on the property, estimate the probability that it is on grass.



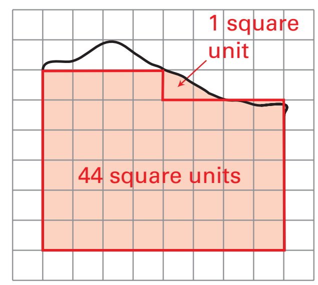
Solution

Step 1 **Find** the grass area. The shape is a rectangle, so the area is bh = 8 • 4 = 32 square units.

Step 2 **Find** the total area of the property.

Count the squares that are fully covered. There are 32 squares in the grass and 12 in the water. So, there are 44 full squares.

Make groups of partial squares so the area of each is about 1 square unit. The total area of partial squares is about 3 square units. So use 44 + 3 = 47 square units for the total area.



The deer must be in the grass or in the water, so check that the probabilities in Example 4 and Checkpoint Exercise 4 add up to 100%.

Step 3 **Write** a ratio of the areas to find the probability.

$$P(\text{deer on grass}) = \frac{\text{Area of grass}}{\text{Total area of property}} \approx \frac{32}{47}$$

The probability that the deer is on grass is about $\frac{32}{47}$, or about 68.1%.

Checkpoint **Complete the following exercise.**

4. In Example 4, estimate the probability that the deer is in the water.

about 31.9%

Homework