Answer key to review 3-Math 240

1. Answer the following based on the level curves.
(a) Two ways to get $\nabla f(1,1)$,

(First method) Since $\nabla f(1,1) = < f_x(1,1), f_y(1,1) >$, estimate $f_x(1,1)$, and $f_y(1,1)$ and sketch it.

Note that your answer must be orthogonal to the level curve at the point.

(Second method) The gradient vector at the point must be normal to the level curve pointing to the direction the function is increasing and its length is determined by the rate of change.

(b) $\sqrt{8}$

(c) Find the value of $D_u f(1,1)$ using the several choices of u given below.

u	(1,0)	(0,1)	(1,1)	(2,-1)	(-1,1)	(1,-1)
$D_u f(1,1)$	-2	2	0	$-6/\sqrt{5}$	$2\sqrt{2}$	$-2\sqrt{2}$

In which direction does f increase most rapidly? –i+j Decrease most rapidly? i-j
Or change nothing? i+j

2. (a) <3,-1> (b) $\sqrt{10}$ (c) <1,3>

3. (a) Absolute max 9 at (-1,-1) and absolute min -6 at (4,-1)
(b) Maximum 1 at (0,0), Minimum e^{-25} at (4,3)

4. 1.06

5. $z = -2x - 2y + 9$

6. (a) 5 (b) 12π (c) 26 (d) 0

7. $2^\circ C/\sec$

8. $z_x = -\dfrac{2xz^3 - yze^{xyz}}{4yz^3 + 3x^2z^2 - e^{xyz}xy}$, $z_y = \dfrac{xze^{xyz} - z^4}{4yz^3 + 3x^2z^2 - e^{xyz}xy}$

9. 2

10. Absolute max is $\dfrac{2}{3\sqrt{3}}$ at $(\pm\sqrt{\dfrac{2}{3}}, \dfrac{1}{\sqrt{3}})$ and absolute min is $-\dfrac{2}{3\sqrt{3}}$ at $(\pm\sqrt{\dfrac{2}{3}}, -\dfrac{1}{\sqrt{3}})$.

11. (a) $\int_0^1 \int_{-\sqrt{1-(x-1)^2}}^{x} (x+y)\,dydx + \int_1^2 \int_{-\sqrt{1-(x-1)^2}}^{-x+2} (x+y)\,dydx$

(b) $\int_{-1}^0 \int_{-\sqrt{1-y^2}+1}^{\sqrt{1-y^2}+1} (x+y)\,dxdy + \int_0^1 \int_y^{2-y} (x+y)\,dxdy$