This is only the review and besides this, you should redo all Hw, Quiz problems with problems done in class. This exam is based upon sections 9.1 thru 9.7 and some theorems, facts, or formulas will be asked to prove, show or derive in the exam.

1. Describe and sketch the following in rectangular coordinates.

(a) The intersection of the surfaces given in cylindrical coordinates by  $\theta = \frac{\pi}{4}$  and  $\theta = \frac{2\pi}{3}$ .

(b) The intersection of the surfaces given in spherical coordinates by  $\rho = 1$  and  $\theta = \frac{\pi}{2}$ .

2. Find  $b \times c$  from the graph shown to the right side.

3. Let a = (x+y)i + 2j + yk and b = 3i + (4x+y+1)j + 4k.

(a) Find values of x and y such that  $a \perp b$  (orthogonal).

(b) Find the values of x and y such that a is parallel to b.

4. Let a, b and c be three vectors in the plane 3x-5y+6z=7. Compute  $(-a+4b-7c) \cdot (-3i+5j-6k)$ .

5. Describe and sketch the following in space defined by the following equations.

(a) y = -z + 1 (b)  $x^2 + 4y^2 + z^2 = 4$  (c)  $y^2 - \frac{1}{9}z^2 = 1$  (d)  $z = \sqrt{9 - x^2 - y^2}$ (e)  $z = x^2 + y^2$  (f)  $z = \sqrt{x^2 + y^2}$  (g)  $0 \le y \le \sqrt{1 - x^2 - z^2}$ 

6. Calculate the given quantity if a = i + j - 2k, b = 3i - 2j + k and c = j - 5k. (a) comp<sub>a</sub>b (b) proj<sub>a</sub>b (c) The angle between a and b (d)  $a \cdot (b \times c)$  (e) comp<sub>b</sub>a (f) proj<sub>b</sub>a

7. Find the equation of the following.

(a) The line thru (4,-1,2) and parallel to the line  $\frac{1}{3}(x-4) = \frac{1}{2}y = z+2$ .

(b) The plane thru (3,-1,1),(4,0,2) and (6,3,1)

(c) The line of intersection of the planes x-z=1 and y+2z=3.

8. Suppose A, B, and C are mutually orthogonal vectors. Let D=5A-6B+3C. If A,B, and C are unit vectors, find ||D||.

9. Fill in the blanks in the table to the right.

10. What points satisfy the equations r = 2,  $\theta = \pi/4$ ?

Rectangular	Cylindrical	Spherical
	$(1, \pi/2, 1)$	
		$(\sqrt{3}, 3\pi/2, \pi/3)$
$(1,\sqrt{3},-1)$		

11. The parallelogram ABCD has vertices at A(2,-1,4), B(1,0,-1), C(1,2,3) and D. Find

- (a) the coordinates of D.
- (b) the cosine of the interior angle at B.
- (c) the vector projection of  $\overrightarrow{BA}$  onto  $\overrightarrow{BC}$ .
- (d) the area of the parallelogram ABCD.
- (e) the equation of the plane of the parallelogram.



This is a brief outline of the main topics we had in class.

Chapter 9. Vectors and the geometry of spaces 9.1 Three-dimensional coordinates systems Coordinates planes and their equations Distance and midpoint between two points The equation of the sphere at (a,b,c) and a radius r Finding the center and radius of a sphere, Finding the region given by an inequality

9.2 Vectors

Vector addition, subtraction, constant multiple of a vector Vector representation  $\overrightarrow{AB}$  from two points A and B and its length Vector notation using i, j, and k, standard basis vectors and  $\langle a, b, c \rangle$ Finding the unit vector in the direction of the given vector Application: resulting force vector, example 7 in page 645

9.3 The dot product
Two methods to find the dot product of two vectors
Finding the amount of work done by the force vector
The angle between two vectors
Properties of dot product
Projections: scalar projection and vector projection and rewrite the given vector as a sum of vector projection and its orthogonal projection

9.4 The cross productTwo methods to find the cross product of two vectorsProperties of cross productThe area of parallelogram spanned by two vectorsThe volume of parallelepiped spanned by three vectorsTriple scalar product, coplanarDistance from a point to a line and distance from a point to a plane

9.5 Equations of lines and planesEquation of a line thru two points: vector equation, parametric equation, symmetric equationTwo lines: parallel, intersecting, skewEquation of a plane thru three points or having two vectorsThe angle between two planes and intersecting lineExtra: distance between planes and between skew lines

9.6 Functions and surfacesDomain and range of functions with two variablesGraphs of several functions of two variables: cylinders, planes, quadric surfacesTraces and cross sectionsMatching the functions with graphs

9.7 Cylindrical and spherical coordinates: Only Cylindrical coordinates Change from rectangular to cylindrical or cylindrical to rectangular Identify or describe the surface given by cylindrical coordinates