7.1 Graph Exponential Growth Functions

1. In the exponential growth model \( y = 2.4(1.5)^x \), identify the initial amount, the growth factor, and the percent increase.

Graph the function.
2. \( y = 2 \cdot 3^x \)
3. \( y = -2^x \)
4. \( y = 5^x \)
5. \( y = \frac{1}{2} \cdot 3^x \)

Graph the function. State the domain and range.
6. \( y = 2^{x+1} + 3 \)
7. \( y = 2 \cdot 3^{x-2} - 1 \)
8. \( f(x) = 6 \cdot 2^{x-3} + 3 \)

Describe and correct the error in graphing the function.
9. \( y = 2^{x-3} + 3 \)

Word problems
10. From 1997 to 2002, the number \( n \) (in millions) of DVD players sold in the United States can be modeled by \( n = 0.42(2.47)^t \) where \( t \) is the number of years since 1997.
   a. Identify the initial amount, the growth factor, and the annual percent increase.
   b. Graph the function. Estimate the number of DVD players sold in 2001.

11. You deposit $2200 in a bank account. Find the balance after 4 years for each of the situations described below.
   a. The account pays 3% annual interest compounded quarterly.
   b. The account pays 2.25% annual interest compounded monthly.
   c. The account pays 2% annual interest compounded daily.

Mixed Review
12. (6.6) Solve \(-4\sqrt{x} - 6 = -20\)
13. (6.4) Find the inverse of \( f(x) = \frac{3}{4} x^5 + 5 \)
14. (6.2) Simplify \( x^{0.5} \cdot x^2 \)
15. (5.4) Factor \(-4b^4 - 500b\)
7.2 Graph Exponential Decay Functions

1. In the exponential decay model \( y = 1250(0.85)^t \), identify the initial amount, the decay factor, and the percent decrease.

Tell whether the function represents exponential growth or exponential decay.

2. \( f(x) = 3 \left( \frac{3}{4} \right)^x \)

3. \( f(x) = \left( \frac{2}{7} \right) \cdot 4^x \)

Graph the function.

4. \( y = \left( \frac{1}{4} \right)^x \)

5. \( y = -4 \left( \frac{1}{3} \right)^x \)

6. The graph of which function is shown?
   (A) \( y = 2 \left( -\frac{3}{5} \right)^x \)
   (B) \( y = -2 \left( \frac{3}{5} \right)^x \)
   (C) \( y = -2 \left( \frac{2}{5} \right)^x \)
   (D) \( y = 2 \left( -\frac{2}{5} \right)^x \)

Graph the function. State the domain and range.

7. \( y = -\left( \frac{1}{2} \right)^{x-1} \)

8. \( y = \left( \frac{2}{3} \right)^{x-4} - 1 \)

9. What is the asymptote of the graph of \( y = \left( \frac{1}{2} \right)^{x-2} + 3 \)?

Word problems

10. You buy a new mountain bike for $200. The value of the bike decreases by 25% each year.
    a. Write a model giving the mountain bike’s value \( y \) (in dollars) after \( t \) years. Use the model to estimate the value of the bike after 3 years.
    b. Graph the model.
    c. Estimate when the value of the bike will be $100.

11. The value of a car can be modeled by the equation \( y = 24,000(0.845)^t \) where \( t \) is the number of years since the car was purchased.
    a. Graph the model. Estimate when the value of the car will be $10,000.
    b. Use the model to predict the value of the car after 50 years. Is this a reasonable value? Explain.

Mixed Review

12. (7.1) The graph of which function is shown?
   (A) \( f(x) = 2(1.5)^x - 1 \)
   (B) \( f(x) = 2(1.5)^x + 1 \)
   (C) \( f(x) = 3(1.5)^x - 1 \)
   (D) \( f(x) = 3(1.5)^x + 1 \)

13. (5.2) Use synthetic substitution to evaluate the polynomial function for the given value of \( x \):
    \( f(x) = 8x^4 + 12x^3 + 6x^2 - 5x + 9; x = -2 \)

14. (4.5) Solve \( 7r^2 - 10 = 25 \)

15. (3.4) Solve the system using elimination.
    \[
    \begin{align*}
    2x - y + 2z &= -7 \\
    -x + 2y - 4z &= 5 \\
    x + 4y - 6z &= -1
    \end{align*}
    \]
7.3 Use Functions Involving $e$

1. Copy and complete: The number __?__ is an irrational number approximately equal to 2.71828.

Simplify the expression.

2. $(2e^{3x})^3$
3. $\sqrt{9e^6}$
4. $\frac{1}{3}e^{9x}$

Describe and correct the error in simplifying the expression.

\[(3e^{5x})^2 = 3e^{(5x)(2)} = 3e^{10x}\] 

5. Use a calculator to evaluate the expression.

6. $e^{2.2}$
7. $e^7$
8. $-6e^{2.4}$

Tell whether the function is an example of exponential growth or exponential decay.

9. $f(x) = e^{-4x}$
10. $f(x) = 2e^{4x}$

Graph the function. State the domain and range.

11. $y = 2e^{-3x} - 1$
12. $g(x) = \frac{4}{3}e^{x-1} + 1$

Word problems

13. The number of camera phones shipped globally can be modeled by the function $y = 1.28e^{1.31x}$ where $x$ is the number of years since 1997 and $y$ is the number of camera phones shipped (in millions). How many camera phones were shipped in 2002?

14. You deposit $2000 in an account that pays 4% annual interest compounded continuously. What is the balance after 5 years?

15. The area of a wound decreases exponentially with time. The area $A$ of a wound after $t$ days can be modeled by $A = A_0e^{-0.05t}$ where $A_0$ is the initial wound area. If the initial wound area is 4 square centimeters, what is the area after 14 days?

Mixed Review

16. (7.2) Tell whether the function represents exponential growth or exponential decay. $f(x) = 4 \left(\frac{5}{2}\right)^x$
17. (7.1) Graph the function. State the domain and range. $y = 5 \cdot 4^x + 2$
18. (6.4) Find the inverse of $f(x) = x^7$
19. (6.2) Simplify $2\sqrt{3} + 7\sqrt{3}$
20. (2.5) The variables $x$ and $y$ vary directly and $x = -3$ when $y = -5$. Write an equation that relates $x$ and $y$. Then find $y$ when $x = 12$. 
7.4 Evaluate Logarithms and Graph Logarithmic Functions

Rewrite the equation in exponential form.
1. \( \log_{4} 16 = 2 \)
2. \( \log_{6} \frac{1}{36} = -2 \)

Evaluate the logarithm without using a calculator.
3. \( \log_{7} 49 \)
4. \( \log_{\frac{1}{2}} 8 \)
5. \( \log_{8} 512 \)

Use a calculator to evaluate the logarithm.
6. \( \ln 6 \)
7. \( \ln 5.38 \)

Simplify the expression.
8. \( \log_{5} 5^x \)
9. \( \log_{3} 81^x \)

Find the inverse of the function.
10. \( y = \log_{9} x \)
11. \( y = e^{x+2} \)

Graph the function. State the domain and range.
12. \( y = \log_{4} x \)
13. \( y = \log_{2} (x - 3) \)

Word problems
14. The pH value for a substance measures how acidic or alkaline the substance is. It is given by the formula \( pH = -\log[H^+] \) where \( H^+ \) is the hydrogen ion concentration (in moles per liter). Lemon juice has a hydrogen ion concentration of \( 10^{-2.3} \) moles per liter. What is its pH value?
15. The energy magnitude \( M \) of an earthquake can be modeled by \( M = 0.29(\ln E) - 9.9 \) where \( E \) is the amount of energy released (in ergs).
   a. In 2001, a powerful earthquake in Peru, caused by the slippage of two tectonic plates along a fault, released \( 2.5 \times 10^{24} \) ergs. What was the energy magnitude of the earthquake?
   b. Find the inverse of the given function. Describe what it represents.

Mixed Review
16. (7.3) Simplify \( \frac{4e^x}{e^{4x}} \)
17. (7.3) Tell whether the function is exponential growth or exponential decay. \( f(x) = \frac{1}{3}e^{4x} \)
18. (7.2) State the domain and range of \( y = 2 \left( \frac{1}{3} \right)^{x+1} - 3 \)
19. (7.1) Write an exponential growth model. You purchase an antique table for $450. The value of the table increases by 6% per year.
20. (6.6) Solve \( \frac{1}{2}x^2 = 16 \)
7.5 Apply Properties of Logarithms

Use $\log 4 \approx 0.602$ and $\log 12 \approx 1.079$ to evaluate the logarithm.

1. $\log 3$
2. $\log 144$

Expand the expression.

3. $\log_3 4x$
4. $\log_2 \frac{x}{5}$
5. $\log_7 5x^3yz^2$
6. $\log_2 \sqrt{x}$

Describe and correct the error in expanding the logarithmic expression.

7. $\log_2 5x = (\log_2 5)(\log_2 x)$

Condense the expression.

8. $\log_4 7 - \log_4 10$
9. $2 \log x + \log 11$
10. $5 \log x - 4 \log y$
11. $\ln 40 + 2 \ln \frac{1}{2} + \ln x$
12. $6 \ln 2 - 4 \ln y$
13. Which of the following is equivalent to $3 \log_4 6$?
   - (A) $\log_4 18$
   - (B) $\log_4 72$
   - (C) $\log_4 216$
   - (D) $\log_4 256$

Use the change-of-base formula to evaluate the logarithm.

14. $\log_3 15$
15. $\log_4 17$
16. $\log_9 27$
17. $\log_3 \frac{9}{40}$

Word problems

18. The intensity of the sound of a trumpet is $10^3$ watts per square meter. Use the model $L(I) = 10 \log \frac{I}{10^{-12}}$

where $L$ is the loudness in decibels and $I$ is the intensity to find the decibel level of a trumpet.

19. The intensity of the sound TV ads make is ten times as great as the intensity for an average TV show. How many decibels louder is a TV ad? Use the model $L = 10 \log \frac{I}{I_0}$ where $I$ and $I_0$ are the intensities of two sounds.

20. The f-stops on a 35 millimeter camera control the amount of light that enters the camera. Let $s$ be a measure of the amount of light that strikes the sensor and let $f$ be the f-stop. Then $s$ and $f$ are related by the equation: $s = \log_2 f^2$

a. Expand the expression for $s$.

b. The table shows the first eight f-stops on a 35 millimeter camera. Copy and complete the table. Describe the pattern you observe.

<table>
<thead>
<tr>
<th>$f$</th>
<th>1.414</th>
<th>2.000</th>
<th>2.828</th>
<th>4.000</th>
<th>5.657</th>
<th>8.000</th>
<th>11.314</th>
<th>16.000</th>
</tr>
</thead>
</table>

c. Many 35 millimeter cameras have nine f-stops. What do you think the ninth f-stop is? Explain your reasoning.

Mixed Review

21. (7.4) Graph $y = \log_3 x + 4$
22. (7.4) Rewrite in exponential form $\log_7 343 = 3$
23. (7.3) Simplify $e^x \cdot 5e^{x+3}$
24. (7.2) What is the asymptote of $y = \left(\frac{1}{3}\right)^x + 1$?
25. (7.1) Write an exponential model for the situation. In 1992, 1219 monk parakeets were observed in the United States. For the next 11 years, about 12% more parakeets were observed each year.
7.6 Solve Exponential and Logarithmic Equations

Solve the equation.
1. \(5^{x-4} = 25^{x-6}\)
2. \(4^{2x-5} = 64^{3x}\)
3. \(25^{10x+8} = \left(\frac{1}{125}\right)^{4-2x}\)
4. \(11^{5x} = 33\)
5. \(-3e^{2x} + 16 = 5\)
6. \(\frac{3}{4}e^{2x} + \frac{7}{2} = 4\)

Solve the equation. Check for extraneous solutions.
7. \(\log_5(2x - 7) = \log_5(3x - 9)\)
8. \(\log_6(5 - 12x) = \log_8(6x - 1)\)
9. \(5.2 \log_4 2x = 16\)
10. \(\ln(x + 3) + \ln x = 1\)
11. \(\log_3(x - 9) + \log_3(x - 3) = 2\)

Solve the equation.
12. \(10^{3x-8} = 2^{5-x}\)
13. \(5^{2x} + 20 \cdot 5^x - 125 = 0\)

Word problems
14. As you are hanging an outdoor thermometer, its reading drops from the indoor temperature of 75°F to 37°F in one minute. If the cooling rate is \(r = 1.37\), what is the outdoor temperature? Use Newton's Law of Cooling \(T = (T_0 - T_R)e^{-rt} + T_R\) where \(T_0\) is the initial temperature, \(T_R\) is the surrounding temperature, \(r\) is the cooling rate, and \(t\) is time in minutes.

15. One hundred grams of radium are stored in a container. The amount \(R\) (in grams) of radium present after \(t\) years can be modeled by \(R = 100e^{-0.00043t}\). After how many years will only 5 grams of radium be present?

Mixed Review
16. (7.5) Expand \(\ln 4x^2y\)
17. (7.5) Condense \(2(\log_3 20 - \log_3 4) + 0.5 \log_3 4\)
18. (7.5) Use the change-of-base formula to evaluate \(\log_3 13\)
19. (7.4) Graph \(g(x) = \log_6(x - 4) + 2\)
20. (7.3) Graph \(y = 2.5e^{-0.5x} + 2\)
7.7 Write and Apply Exponential and Power Functions

Write an exponential function \( y = ab^x \) whose graph passes through the given points.
1. \((1, 3), (2, 12)\)
2. \((1, 2), (3, 50)\)

Use the points \((x, y)\) to draw a scatter plot of the points \((x, \ln y)\). Then find an exponential model for the data.
3. \((1, 18), (2, 36), (3, 72), (4, 144), (5, 288)\)
4. \((1, 9.8), (2, 12.2), (3, 15.2), (4, 19), (5, 23.8)\)

Write a power function \( y = ax^b \) whose graph passes through the given points.
5. \((4, 3), (8, 15)\)
6. \((4, 8), (8, 30)\)

Use the given points \((x, y)\) to draw a scatter plot of the points \((\ln x, \ln y)\). Then find a power model for the data.
7. \((1, 0.6), (2, 4.1), (3, 12.4), (4, 27), (5, 49.5)\)
8. Which equation is equivalent to \(\log y = 2x + 1\)?
   (A) \(y = 10(100)^x\)    (B) \(y = 10^x\)    (C) \(y = e^{2x+1}\)    (D) \(y = e^2\)

Word problems
9. Scientists use the circumference of an animal’s femur to estimate the animal’s weight. The table shows the femur circumference \(C\) (in millimeters) and the weight \(W\) (in kilograms) for several animals.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Giraffe</th>
<th>Polar bear</th>
<th>Lion</th>
<th>Squirrel</th>
<th>Otter</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C) (mm)</td>
<td>173</td>
<td>135</td>
<td>93.5</td>
<td>13</td>
<td>28</td>
</tr>
<tr>
<td>(W) (kg)</td>
<td>710</td>
<td>448</td>
<td>143</td>
<td>0.399</td>
<td>9.68</td>
</tr>
</tbody>
</table>

   a. Draw a scatter plot of the data pairs \((\ln C, \ln W)\).
   b. Find a power model for the original data.
   c. Predict the weight of a cheetah if the circumference of its femur is 68.7 millimeters.

10. Your visual near point is the closest point at which your eyes can see an object distinctly. Your near point moves farther away from you as you grow older. The diagram shows the near point \(y\) (in centimeters) at age \(x\) (in years).
   a. Draw a scatter plot of the data pairs \((x, \ln y)\).
   b. Draw a scatter plot of the data pairs \((\ln x, \ln y)\).
   c. Based on your scatter plots, does an exponential function or a power function best fit the original data? Explain your reasoning.
   d. Based on your answer for part (c), write a model for the original data. Use your model to predict the near point for an 80-year-old person.

Mixed Review
11. \((7.6)\) Solve \(0.5^x - 0.25 = 4\)
12. \((7.6)\) Solve \(\log_2(x - 4) = 6\)
13. \((7.5)\) Condense \(5 \log_4 2 + 7 \log_4 x + 4 \log_4 y\)
14. \((7.4)\) Find the inverse of \(y = 7^x\)
15. \((7.2)\) Graph \(y = 3(0.25)^x + 3\)
Chapter 7 Review

Graph and state the domain and range.
1. \( y = 2^x - 1 \)
2. \( y = -e^{-x} \)
3. \( y = \log_2 x + 1 \)
4. \( y = 2 \ln(x - 1) \)
5. Rewrite \( 10^2 = 100 \) as a logarithm.

Evaluate.
6. \( \log_4 256 \)
7. \( \log_2 \frac{1}{1024} \)
8. Find the inverse of \( y = 2e^x \).

Condense the expression.
9. \( 10 \log_6 a - 8 \log_6 b \)
10. \( \ln 12 + 3 \ln x - \ln x^2 \)

Expand the logarithm.
11. \( \log_3 a^6 b^{23} \)
12. \( \ln \frac{2x^7}{y^2} \)

Use the change-of-base formula to evaluate the logarithm. (Round to three decimal places.)
13. \( \log_4 150 \)
14. \( \log_{17} 1321 \)

Solve. (Round to three decimal places.)
15. \( 4^{2x+1} = 32^{x-1} \)
16. \( 7^{x+4} + 3 = 51 \)
17. \( \log_{21}(2x + 17) = \log_{21}(x - 1) \)
18. \( \log_5(2x + 7) = 15 \)

Solve the word problems. Round to two decimal places.
19. You charge $1200 on a credit card that charges 20% interest compound daily. If you do not make a payment, how much will you owe after 1 year?
20. A rabbit population starts with 20 individuals. If the population increases 30% every year, estimate the number of rabbits in the population after 5 years.
21. Write a power function \( y = ax^b \) whose graph passes through \((1, 2)\) and \((3, 54)\).
Algebra 2 7 Exponential and Logarithmic Functions Practice Problems

Answers

7.1

1. 2.4, 1.5, 50%
2. all real numbers, range: \( y > 3 \)
3. all real numbers, range: \( y > 3 \)
4. all real numbers, range: \( y > -1 \)
5. domain:
6. domain:
7. domain:
8. all real numbers, range: \( y > 3 \)
9. The power of \( (x - 3) \) translates the parent graph 3 units to the right, not to the left.
10. 0.42 million, 2.47, 147%; about 16 million
11. $2479.38; $2406.98; $2383.23
12. 12
13. \( f^{-1}(x) = \sqrt{\frac{4x - 20}{3}} \)
14. \( x^{2.5} \)
15. \(-4(b + 5)(b^2 - 5b + 25)\)

7.2

1. 1250, 0.85, 15%
2. exponential decay
3. exponential growth
4. all real numbers, range: \( y < 0 \)
5. domain:
6. domain:
7. domain:
8. all real numbers, range: \( y > -1 \)
9. \( y = 3 \)
10. \( y = 200(0.75)^t \), about $84.38;
11. after about 2.5 yr
12. B
13. 75
14. \( \pm \sqrt{15} \)
15. \((-3, -1, -1)\)
7.3

1. \(e\)
2. \(8e^{9x}\)
3. \(3e^{3}\)
4. \(2e^{3x}\)
5. The 3 should be raised to the second power also; \((3e^{5x})^2 = 9e^{10x}\)
6. about 9.025
7. about 1096.633
8. about -66.139
9. exponential decay
10. exponential growth

11. domain: all real numbers, range: \(y > -1\)
12. domain: all real numbers, range: \(y > 1\)
13. about 895 million camera phones
14. \(2442.81\)
15. about 1.986 cm
16. exponential growth
17. domain: all real numbers, range: \(y > 2\)
18. \(f^{-1}(x) = \sqrt{x}\)
19. \(9\sqrt{3}\)
20. \(y = \frac{5}{3}x; 20\)

7.4

1. \(4^2 = 16\)
2. \(6^{-2} = \frac{1}{36}\)
3. \(2\)
4. \(-3\)
5. \(3\)
6. \(1.792\)
7. \(1.683\)
8. \(x\)
9. \(4x\)
10. \(y = 8^x\)
11. \(y = \ln x - 2\)
12. domain: \(x > 0\), range: all real numbers
13. domain: \(x > 3\), range: all real numbers
14. \(2.3\)
15. about 6.4; \(E = e^{\frac{M99}{a}}\), the inverse represents the amount of energy released, in ergs, as a function of the energy magnitude
16. \(\frac{4}{e^{3\omega}}\)
17. exponential growth
18. domain: all real numbers; range: \(y > -3\)
19. \(y = 450(1.06)^t\)
20. 4

7.5

1. \(0.477\)
2. \(2.158\)
3. \(\log_3 4 + \log_3 x\)
4. \(\log_2 2 - \log_2 5\)
5. \(\log_3 5 + 3 \log_7 x + \log_7 y + 2 \log_7 z\)
6. \(\frac{1}{x} \log_2 x\)
7. The two parts should be added, not multiplied; \(\log_2 5 + \log_2 x\)
8. \(\log_{10} 4\)
9. \(\log 11x^2\)
10. \(\log \frac{x^3}{y^4}\)
11. \(\ln 10x\)
12. \(\ln \frac{6+6}{y^2}\)
13. \(C\)
14. \(2.465\)
15. \(1.581\)
16. \(1.5\)
17. \(-1.358\)
18. 150 decibels
19. 10 decibels
20. \(s = 2 \log_2 f; 1, 2, 3, 4, 5, 6, 7, 8; 9\)
about 22.627

21. \(7^2 = 343\)
22. \(5e^{2x+3}\)
23. \(y = 1\)
24. \(5e^{2x+3}\)
25. \(y = 1219(1.12)^t\)
7.6

1. 8
2. $-\frac{5}{7}$
3. -2
4. 0.292
5. 0.650
6. -0.203
7. no solution
8. $\frac{1}{3}$
9. 35.601
10. 0.729
11. 10.243
12. 2.879
13. 1

14. about 24°F
15. about 6967 yr
16. $\ln 4 + 2 \ln x + \ln y$
17. $\log_5 50$
18. 2.335

7.7

1. $y = \frac{3}{4} \cdot 4^x$
2. $y = \frac{3}{5} \cdot 5^x$
3. $y = 9(2)^x$
4. $y = 7.83(1.25)^x$
5. $y = 0.12x^{2.32}$
6. $y = 0.569x^{1.91}$
7. $y = 0.606x^{2.74}$
8. $y = 3.4^x$

9. $y = 0.000466x^{2.89}$, about 64.8 kg

10. $y = \log_7 x$

Exponential function; the points for $(x, \ln y)$ appear more linear than the points $(\ln x, \ln y)$, so an exponential model appears to be the best fit for the data; $y = 4.98(1.05)^x$, about 247 cm

11. 2.087
12. 68
13. $\log_4 32x^2y^4$
14. $y = \log_7 x$

15. Exponential function; $y = 0.000466x^{2.89}$, about 64.8 kg
7. Review

1. \[ \log_{10} 100 = 2 \]

2. \[ y = \ln \left( \frac{x}{2} \right) \]

3. \[ \log_6 \left( \frac{a^{10}}{b^8} \right) \]

4. \[ \ln(12x) \]

5. \[ 10 \log_3 a + 23 \log_3 b \]

6. \[ \ln 2 + 7 \ln x - 2 \ln y \]

7. \[ 3.614 \]

8. \[ 2.536 \]

9. \[ 7 \]

10. \[ -2.011 \]

11. No solution (-18 is extraneous)

12. \[ 1.526 \times 10^{10} \]

13. \[ \$1465.60 \]

14. 74 rabbits

15. \[ y = 2x^3 \]