8.1 Model Inverse and Joint Variation

Tell whether $x$ and $y$ show direct variation, inverse variation, or neither.

1. $xy = \frac{1}{5}$

2. $8y = x$

The variables $x$ and $y$ vary inversely. Use the given values to write an equation relating $x$ and $y$. Then find $y$ when $x = 3$.

3. $x = 1, y = 9$

4. $x = 7, y = 2$

5. $x = -4, y = -\frac{5}{4}$

6. $x = \frac{5}{7}, y = -7$

Determine whether $x$ and $y$ show direct variation, inverse variation, or neither.

7. $x$ | $y$
---|---
12 | 132
18 | 198
23 | 253
29 | 319
34 | 374

8. $x$ | $y$
---|---
4 | 21
6 | 14
8 | 10.5
8.4 | 10
12 | 7

Write an equation relating $x, y,$ and $z$ given that $z$ varies jointly with $x$ and $y$. Then find $z$ when $x = -4$ and $y = 5$.

9. $x = 8, y = 6, z = 12$

10. $x = 6, y = -7, z = -3$

11. $x = 5, y = -3, z = 75$

Write an equation for the given relationship.

12. $x$ varies directly with $y$ and inversely with $z$

13. $w$ varies inversely with $y$ and jointly with $x$ and $z$

Word problems

14. When you stand on snow, the average pressure $P$ (in pounds per square inch) that you exert on the snow varies inversely with the total area $A$ (in square inches) of the soles of your footwear. Suppose the pressure is 0.43 pound per square inch when you wear the snowshoes shown. Write an equation that gives $P$ as a function of $A$. Then find the pressure if you wear the boots shown.

15. The law of universal gravitation states that the gravitational force $F$ (in Newtons) between two objects varies jointly with their masses $m_1$ and $m_2$ (in kilograms) and inversely with the square of the distance $d$ (in meters) between the two objects. The constant of variation is denoted by $G$ and is called the universal gravitational constant.

**a.** Write an equation that gives $F$ in terms of $m_1, m_2, d$ and $G$.

**b.** Use the information above about Earth and the Sun to approximate the universal gravitational constant $G$. 

c. Explain what happens to the gravitational force as the masses of the two objects increase and the distance between them is held constant. Explain what happens to the gravitational force as the masses of the two objects are held constant and the distance between them increases.

Mixed Review
16. (7.7) Write an exponential function $y = ab^x$ whose graph passes through the points $(2, 24)$, $(3, 144)$.
17. (7.6) Solve $5 \ln x = 35$
18. (6.4) Find the inverse of $f(x) = -10x^6, x \leq 0$
19. (5.5) Divide using synthetic division $(x^2 + 9) \div (x - 3)$
20. (3.1) Solve by graphing $y = 5x + 2$

\[ y = 3x \]

8.2 Graph Simple Rational Functions

1. Copy and complete: The function $y = \frac{7}{x+4} + 3$ has a(n) __?__ of all real numbers except 3 and a(n) __?__ of all real numbers except -4.

Graph the function.
2. $y = \frac{3}{x}$
3. $y = \frac{0.1}{x}$

Graph the function. State the domain and range.
4. $y = \frac{4}{x} + 3$
5. $y = \frac{-5}{x} - 7$
6. $y = \frac{-4}{x+4} + 3$

7. What are the asymptotes of $y = \frac{3}{x+8} - 3$?
   (A) $x = 8, y = 3$  (B) $x = 8, y = -3$  (C) $x = -8, y = 3$  (D) $x = -8, y = -3$

Graph the function. State the domain and range.
8. $y = \frac{x+4}{x-3}$
9. $y = \frac{-5x+2}{4x+5}$

Word problems
10. The time $t$ (in seconds) it takes for sound to travel 1 kilometer can be modeled by $t = \frac{1000}{0.6T+331}$ where $T$ is the air temperature (in degrees Celsius).
   a. How long does it take for sound to travel 5 kilometers when the air temperature is 25°C? Explain.
   b. Suppose you are 1 kilometer from a lightning strike, and it takes 3 seconds to hear the thunder. Graph the given function, and use the graph to estimate the air temperature.

11. The Doppler effect occurs when the source of a sound is moving relative to a listener, so that the frequency $f_s$ (in hertz) heard by the listener is different from the frequency $f_s$ (in hertz) at the source. The frequency heard depends on whether the sound source is approaching or moving away from the listener. In both equations below, $r$ is the speed (in miles per hour) of the sound source.

   a. An ambulance siren has a frequency of 2000 hertz. Write two equations modeling the frequencies you hear when the ambulance is approaching and when the ambulance is moving away.
**Mixed Review**

12. (8.1) The variables $x$ and $y$ vary inversely. Use $x = 5$ when $y = -4$ to write an equation relating $x$ and $y$.

13. (8.1) Write an equation for $y$ varies jointly with $x$ and the square of $z$.

14. (7.6) Solve $10^{3x} + 4 = 9$

15. (7.5) Expand $\log_6 36x^2$

### 8.3 Graph General Rational Functions

**Identify the $x$-intercept(s) and vertical asymptote(s) of the graph of the function.**

1. $y = \frac{5}{x^2 - 1}$

2. $f(x) = \frac{x^2 + 9}{x^2 - 2x - 15}$

3. $y = \frac{x^3 + 27}{3x^2 + x}$

4. Describe and correct the error in finding the vertical asymptote(s) of $f(x) = \frac{x - 2}{x^2 - 8x + 7}$

**Graph the function.**

5. $y = \frac{2x}{x^2 - 1}$

6. $f(x) = \frac{x^2 - 9}{2x^2 + 1}$

7. $y = \frac{x^2 + 11x + 18}{2x + 1}$

8. $h(x) = \frac{3x^2 + 10x - 8}{x^2 + 4}$

**Word problems**

9. The mean temperature $T$ (in degrees Celsius) of the Atlantic Ocean between latitudes $40^\circ$N and $40^\circ$S can be modeled by $T = \frac{17,800d + 20,000}{3d^2 + 740d + 1000}$ where $d$ is the depth (in meters).

   a. Make a table of values showing the mean temperature for depths from 1000 meters to 1300 meters in 50 meter intervals.
   
   b. Graph the model. Use your graph to estimate the depth at which the mean temperature is $4^\circ$C.

10. The acceleration due to gravity $g$ (in meters per second squared) changes as altitude changes and is given by the function $g = \frac{3.99 \times 10^{14}}{h^2 + (1.28 \times 10^7)h + (4.07 \times 10^{12})}$ where $h$ is the altitude (in meters) above sea level.

   a. Graph the function.
   
   b. A mountaineer is climbing to a height of 8000 meters. What is the value of $g$ at this altitude?
   
   c. A spacecraft reaches an altitude of 112 kilometers above Earth. What is the value of $g$ at this altitude?
   
   d. Describe what happens to the value of $g$ as altitude increases.

**Mixed Review**

11. (8.2) To join a rock climbing gym, you must pay an initial fee of $100 and a monthly fee of $59. Write an equation that gives the average cost per month as a function of the number of months of membership.

12. (8.2) What are the asymptotes of $y = \frac{3}{x} - 2$?

13. (8.1) Tell whether $x$ and $y$ show **direct variation**, **inverse variation**, or **neither**: $\frac{y}{x} = 8$

14. (7.4) Evaluate without a calculator: $\log_9 1$

15. (6.6) $\sqrt[3]{x} - 16 = 2$
8.4 Multiply and Divide Rational Expressions

1. Match the rational expression with its simplified form. 
   \[ \frac{x^2-9x+14}{x^2-5x-14} \]
   (A) \( \frac{x-2}{x+7} \) (B) \( \frac{x-2}{x+2} \) (C) \( \frac{x+7}{x-2} \)

Simplify the rational expression, if possible.

2. \( \frac{x^2-x-20}{x^2+2x-15} \)
3. \( \frac{x^2-11x+24}{x^2-3x-40} \)
4. \( \frac{2x^2+2x-4}{x^2-5x-14} \)

Multiply the expressions. Simplify the result.

5. \( \frac{x^2-36}{x^2+12x+36} \)
6. \( \frac{8x^2+10x-3}{6x^2+13x+6} \)
7. \( \frac{x^3-5x^2-3x+15}{x^2-8x+15} \)

Divide the expressions. Simplify the result.

8. \( \frac{4(x+5)}{x^2} \cdot \frac{x(x+1)}{2(x+5)} \)
9. \( \frac{x+5}{4x-16} \cdot \frac{2x^2-32}{x^2-25} \)

10. \( \frac{x^2-3x-10}{x^2-2x-15} \cdot \frac{x^2 + 10x + 21}{x^2 + 8x + 16} \)
11. \( \frac{4x^2+20x}{x^3+4x^2} \cdot \frac{x^2 + 8x + 16}{x^2 - 8x + 15} \)

Mixed Review

16. (8.3) Identify the x-intercepts and vertical asymptotes of the graph of \( y = \frac{x+1}{x^2+5} \).

17. (8.3) Graph \( y = \frac{8}{x^2-x-6} \).

18. (8.2) Graph \( y = \frac{x-1}{x+5} \).

19. (8.1) The variables \( x \) and \( y \) vary inversely. Use the given values to write an equation relating \( x \) and \( y \). Then find \( y \) when \( x = 3 \).
   \[ x = -3, y = 8 \]

20. (3.1) Solve the system by graphing \( \begin{cases} 2x - y = 4 \\ x - 2y = -1 \end{cases} \).
8.5 Add and Subtract Rational Expressions

Perform the indicated operation and simplify.
1. \( \frac{15}{4x} + \frac{5}{4x} \)
2. \( \frac{5x}{x+3} + \frac{15}{x+3} \)

Find the least common multiple of the polynomials.
3. \( 2x \) and \( 2x(x - 5) \)
4. \( x^2 - 25, x, \) and \( x - 5 \)

Perform the indicated operation and simplify.
5. \( \frac{3x^2 - 9x}{x^2 - 9} + \frac{6x}{x + 9} \)
6. \( \frac{8}{3x^2} - \frac{5}{4x} \)
7. \( \frac{12}{x^2 + 5x - 24} + \frac{3}{x - 3} \)
8. \( \frac{9}{x - 3} + \frac{2x}{x + 1} \)

Simplify the complex fraction.
9. \( \frac{-15x}{x^2 - 8x + 16} + \frac{12}{x - 4} \)
10. \( \frac{x}{x^2 - 9} + \frac{x + 1}{x^2 + 6x + 9} \)
11. \( \frac{x + 2}{x - 4} + \frac{2}{x} + \frac{5x}{3x - 1} \)
12. \( \frac{\frac{x}{3} - 6}{10 + \frac{4}{x}} \)
13. \( \frac{\frac{x - 2}{4}}{\frac{x + 1}{x} + \frac{6}{x}} \)

Word problem
15. The total time \( T \) (in hours) needed to fly from New York to Los Angeles and back (ignoring layovers) can be modeled by the equation \( T = \frac{d}{a-j} + \frac{d}{a+j} \), where \( d \) is the distance each way (in miles), \( a \) is the average airplane speed (in miles per hour), and \( j \) is the average speed of the jet stream (in miles per hour). Rewrite the equation so that the right side is simplified. Then find the total time if \( d = 2468 \) miles, \( a = 510 \) mi/h, and \( j = 115 \) mi/h.

Mixed Review
16. (8.4) Simplify \( \frac{x(x - 3)}{x - 2} \cdot \frac{(x + 3)(x - 2)}{x} \)
17. (8.4) Simplify \( \frac{(x + 3)(x - 2)}{x(x + 1)} \div \frac{x + 3}{x} \)
18. (8.3) Graph \( y = \frac{x - 4}{x^2 - 3x} \)
19. (7.6) Solve \( 73^x = 18 \)
20. (7.4) Evaluate without using a calculator \( \log_3 \frac{1}{27} \)
8.6 Solve Rational Equations

Solve the equation. Check for extraneous solutions.

1. \[ \frac{9}{3x} = \frac{4}{x+2} \]
2. \[ \frac{8}{3x-2} = \frac{2}{x-1} \]
3. \[ \frac{x^3}{x+5} = \frac{x}{x+2} \]
4. \[ \frac{4(x-4)}{x^2+2x-8} = \frac{4}{x+4} \]
5. \[ \frac{3}{x+2} = \frac{6}{x-1} \]
6. \[ \frac{2}{3x} + \frac{1}{6} = \frac{4}{3x} \]
7. \[ \frac{1}{2x} + \frac{3}{x+7} = \frac{-1}{x} \]
8. \[ \frac{5}{x^2+x-6} = 2 + \frac{x-3}{x-2} \]
9. \[ \frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3} \]
10. \[ \frac{10}{x} + 3 = \frac{x+9}{x-4} \]
11. \[ \frac{x+3}{x-5} + \frac{x}{x-5} = \frac{x+5}{x-5} \]

12. Describe and correct the error in the first step of solving the equation.

\[
\frac{5}{x} + \frac{23}{6} = \frac{45}{x}
\]
\[
\frac{28}{x} = \frac{45}{x}
\]
\[
\frac{x}{x} + 6 = \frac{x}{x}
\]

In the following exercise, \( a \) is a nonzero real number. Tell whether the algebraic statement is always true, sometimes true, or never true. Explain your answer.

13. The equation \( \frac{3}{x-a} = \frac{x}{x-a} \) has exactly one solution.

Word problem

14. So far in your volleyball match, you have put into play 37 of the 44 serves you have attempted. Solve the equation \( \frac{90}{100} = \frac{37+x}{44+x} \) to find the number of consecutive serves you need to put into play in order to raise your service percentage to 90%.

15. Golden rectangles are rectangles for which the ratio of the width \( w \) to the length \( l \) is equal to the ratio of \( l \) to \( l + w \). The ratio of the length to the width for these rectangles is called the golden ratio. Find the value of the golden ratio using a rectangle with a width of 1 unit.

Mixed Review

16. (8.5) Simplify \( \frac{12}{5x} + \frac{7}{6x} \)
17. (8.5) Simplify \( \frac{15-2}{x} \)
18. (8.1) Simplify \( \frac{4x^2}{20x^2-12x} \)
19. (8.2) What are the asymptotes of \( y = \frac{x+6}{4x-8} \)
20. (8.1) Tell whether \( x \) and \( y \) show direct variation, inverse variation, or neither. \( 4x = y \)
Chapter 8 Review

Classify the following variations as direct, inverse, or neither.
1. \( xy = 16 \)
2. \( x = \frac{y}{3} \)

The variables \( x \) and \( y \) vary inversely. Use the given values to write an equation relating \( x \) and \( y \). The find \( y \) when \( x = 10 \).
3. \( x = 2, y = 9 \)
4. \( x = 15, y = -5 \)

Graph the function.
5. \( y = \frac{1}{x+1} + 2 \)
6. \( y = \frac{2x}{x^2 - 1} \)
7. \( y = \frac{10}{x^2 + 9} \)

Find the least common multiple of the polynomials.
8. \( 10x(x + 2)(x - 1) \) and \( 15x(x + 3)(x - 1) \)
9. \( x^2 + x - 2 \) and \( x^2 - x - 6 \)

Perform the indicated operation and simplify.
10. \( \frac{2x^2 + 12x + 10}{8x^2 + 16x - 120} \)
11. \( \frac{x^2 + 8x + 15}{x^2 - x - 12} \cdot \frac{x - 4}{x^2 + 4x - 5} \)
12. \( \frac{x^2 - 4x - 12}{x^2 - 9} \div \frac{x + 2}{x^2 - 9x + 18} \)
13. \( \frac{x - 3}{x^3} \cdot \frac{5x + 4}{x^2 + 3x} \)
14. \( \frac{3x}{6(x + 1)} + \frac{9}{18(x + 1)} \)

Solve the equation. Check for extraneous solutions.
15. \( \frac{2x}{x^2 - 4} = \frac{5}{x - 2} \)
16. \( \frac{2}{x + 10} = \frac{5}{x + 11} \)
17. \( \frac{3}{x} + \frac{4}{x + 10} = \frac{5}{x + 10} \)
18. \( \frac{2x}{x + 1} + \frac{3}{x + 2} = \frac{5x}{x + 1} \)
19. Simplify the complex fraction.
\[
\frac{4}{\frac{5}{x + 1} + \frac{3}{x^2 + x}}
\]
20. A factory will begin making chairs. The startup costs are $20,000 for the machines to make the chairs. The materials and labor cost $15 for each chair. Write an equation that gives the average cost per chair as a function of the number of chairs made. How many chairs will have to be made to have an average cost of $30?
8.1

1. inverse variation
2. direct variation
3. \( y = \frac{9}{x} \cdot \frac{3}{14} \)
4. \( y = \frac{14}{x} \cdot \frac{14}{3} \)
5. \( y = \frac{5}{x} \cdot \frac{5}{7} \)
6. \( y = -\frac{35}{3x} \cdot -\frac{35}{9} \)
7. Direct variation
8. Inverse variation
9. \( z = \frac{1}{4} xy; -5 \)
10. \( z = \frac{3}{14} xy; -\frac{10}{7} \)
11. \( z = -5xy; 100 \)
12. \( x = ay \)
13. \( w = \frac{ax}{y} \)
14. \( P = \frac{172}{x}; \) about 2.87 lb/in²
15. \( F = \frac{g m_1 m_2}{d^2}; 6.7 \times 10^{-11}; \) increases, decreases
16. \( y = \frac{2}{3} \cdot 6^x \)
17. 1096.633
18. \( f^{-1}(x) = -\sqrt{\frac{x}{-10}} \)
19. \( x + 3 + \frac{18}{x^3} \)
20. (-1, -3)

8.2

1. Range, domain
   - graph with domain: \( x \neq 0, \) range: \( y \neq 3 \)
2. 
   - graph with domain: \( x \neq -4, \) range: \( y \neq 3 \)
3. 
   - graph with domain: \( x \neq 0, \) range: \( y \neq 3 \)
4. 
   - graph with domain: \( x \neq 0, \) range: \( y \neq -7 \)
5. 
   - graph with domain: \( x \neq 0, \) range: \( y \neq -7 \)
6. 
   - graph with domain: \( x \neq -4, \) range: \( y \neq 3 \)
7. D
8. 
   - graph with domain: \( x \neq 3, \) range: \( y \neq 1 \)
9. 
   - graph with domain: \( x \neq -\frac{3}{4}, \) range: \( y \neq -\frac{5}{4} \)
10. About 14.5 sec;

   - graph with domain: \( x \neq 0, \) range: \( y \neq -7 \)

11. approaching: \( f_1 = \frac{1480,000}{740-r}, \) moving away: \( f_1 = \frac{1480,000}{740+r} \)

   - frequency of a sound that is approaching is greater than that of a sound moving away.
12. \( y = \frac{20}{x} \)
13. \( y = axz^2 \)
14. 0.233
15. \( \log_6 36 + 2 \log_6 x \)
8.3

1. None; \( x = 1, x = -1 \)
2. None; \( x = 5, x = -3 \)
3. \(-3; x = 0, x = -\frac{1}{3} \)
4. The vertical asymptote occurs at the zeros of the denominator not the numerator; \( x = 7, x = 1 \)

8.4

1. B
2. Already simplified
3. \( \frac{x-1}{x+5} \)
4. \( \frac{x^2-5}{x+6} \)
5. \( \frac{x^3+2}{4x-1} \)
6. \( \frac{3x+2}{x-3} \)
7. \( \frac{x^2-3}{x-2} \)
8. \( \frac{x^2+4}{2(x+7)} \)
9. \( \frac{x(x+2)}{2(x-5)} \)
10. \( (x+2)(x+7) \)
11. \( \frac{4(x+5)(x+4)}{x} \)
12. \( \frac{16x-4}{x+5} \)
13. \( \frac{x+4}{x-5} \)
14. \( \frac{(x+5)^2}{5(x+1)} \)
15. \( \frac{x-1}{6x} \)
16. -1; none
17. \( y = \frac{-24}{x} - 8 \)
18. (3, 2)
8.5
1. \( \frac{5}{x} \)
2. 5
3. \( 2x(x - 5) \)
4. \( x(x - 5)(x + 5) \)
5. \( 6x^2(x - 3) \)
6. \( \frac{(32-15x)}{(12x^7)} \)
7. \( \frac{3(x+12)}{(x+b)(x-3)} \)
8. \( \frac{2x^2+3x+9}{(x+1)(x-3)} \)
9. \( \frac{-3(x+16)}{(x-4)^2} \)

10. \( \frac{(2x+3)(x-1)}{(x-3)(x+3)^2} \)
11. \( \frac{8x^2-9x^2-28x+8}{x(x-4)(3x-1)} \)
12. \( \frac{x(x-18)}{6(5x+2)} \)
13. \( \frac{8(x+1)}{(x-2)(5x+3)} \)
14. \( \frac{3x}{4(x-1)} \)
15. \( T = \frac{2da}{(a-j)(a+j)}; \) about 10.2 h
16. \( x^2 - 9 \)
17. \( \frac{x-2}{x+1} \)

8.6
1. 6
2. 2
3. -1
4. No solution
5. -5
6. 4
7. \(-\frac{7}{3}\)
8. \(-\frac{1+\sqrt{7}}{3}\)
9. 1
10. \(-\frac{5}{2}\)
11. 0, 7
12. The student simply added numerators and denominators on the left side of the equation. Both sides of the equation should have been multiplied by the LCD, 6x; 6x \( \left( \frac{2}{x} + \frac{23}{6} \right) = 6x \left( \frac{45}{x} \right) \), 30 + 23x = 270.
13. Sometimes true; the equation will have exactly one solution except when \( a = 3 \).
14. 26 serves
15. \( \frac{3}{16} \)
16. \( \frac{2}{107} \)
17. \( \frac{5(15x-2)}{x(x+20)} \)
18. \( \frac{x}{5x-2} \)
19. \( x = 2, y = \frac{1}{4} \)
20. Direct variation

8. Review
1. Inverse
2. Direct
3. \( y = \frac{16}{x}; y = \frac{9}{2} \)
4. \( y = \frac{75}{x}; y = -\frac{15}{2} \)

9. \( (x + 2)(x - 1)(x - 3) \)
10. \( \frac{x+1}{4(x-3)} \)
11. \( \frac{x-1}{(x-6)^2} \)
12. \( \frac{x^2+5x-4}{x(x+3)} \)
13. \( \frac{1}{7} \)
14. \( -\frac{10}{3} \)
15. \( -\frac{5}{7} \)
16. \( -\frac{20}{3} \)
17. -15
18. \( -1+\sqrt{3} \)
19. \( \frac{4x}{5x+3} \)
20. \( C = \frac{15x+20000}{x}; 1333 \) chairs