EXPONENTIAL AND LOGARITHMIC FUNCTIONS
This Slideshow was developed to accompany the textbook

- *Larson Algebra 2*
- *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
- *2011 Holt McDougal*

Some examples and diagrams are taken from the textbook.

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7.1 Graph Exponential Growth Functions
How much work will be done the last week of school?
Formula is $2^{n-1}$
Plug in 36: $2^{36-1} = 3.436 \times 10^{10}$ seconds $\rightarrow$ 9544371.769 hours $\rightarrow$ 397682.157 days $\rightarrow$ 1088.8 years
I'm a little worried about answering this one wrong.

Is that Mr. Wright he's evil?
7.1 Graph Exponential Growth Functions

- Exponential Function
  - $y = b^x$
  - Base ($b$) is a positive number other than 1
Graph
\[ y = 2^x \]
\[ y = \frac{1}{3} \cdot 2^x \]
\[ y = 3 \cdot 2^x \]
\[ y = -3 \cdot 2^x \]
7.1 Graph Exponential Growth Functions

- $y = a \cdot 2^x$
  - y-intercept = $a$
  - x-axis is the asymptote of graph
7.1 Graph Exponential Growth Functions

- Exponential Growth Function
  - $y = a \cdot b^{x-h} + k$

- To graph
  - Start with $y = b^x$
  - Multiply $y$-coordinates by $a$
  - Move up $k$ and right $h$
  - (or make table of values)

- Properties of the graph
  - $y$-intercept $= a$ (if $h$ and $k=0$)
  - $y = k$ is asymptote
  - Domain is all real numbers
  - Range
    - $y > k$ if $a > 0$
    - $y < k$ if $a < 0$
7.1 Graph Exponential Growth Functions

- Graph
- \( y = 3 \cdot 2^{x-3} - 2 \)
7.1 Graph Exponential Growth Functions

- Exponential Growth Model (word problems)
  - \( y = a(1 + r)^t \)
    - \( y \) = current amount
    - \( a \) = initial amount
    - \( r \) = growth percent
    - \( 1 + r \) = growth factor
    - \( t \) = time
7.1 Graph Exponential Growth Functions

- Compound Interest

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

- \( A \) = current amount
- \( P \) = principle (initial amount)
- \( r \) = percentage rate
- \( n \) = number of times compounded per year
- \( t \) = time in years
7.1 Graph Exponential Growth Functions

- If you put $200 into a CD (Certificate of Deposit) that earns 4% interest, how much money will you have after 2 years if you compound the interest monthly? daily?

- 482 #1, 5, 7, 9, 13, 17, 19, 21, 27, 29, 35, 37 + 3 = 15 total

Monthly: $200(1+.04/12)^{2*2} = $216.63
Daily: $200(1+.04/365)^{365*2} = $216.66
Quiz

- 7.1 Homework Quiz
7.2 Graph Exponential Decay Functions

- **Exponential Decay**
  - \( y = a \cdot b^x \)
  - \( a > 0 \)
  - \( 0 < b < 1 \)

- **Follows same rules as growth**
  - \( y \)-intercept = \( a \)
  - \( y = k \) is asymptote
  - \( y = a \cdot b^{x-h} + k \)

\[ y = \left(\frac{1}{2}\right)^x \]
7.2 Graph Exponential Decay Functions

- Graph
- \( y = 2 \cdot \left(\frac{1}{2}\right)^x + 3 - 2 \)
7.2 Graph Exponential Decay Functions

- Exponential Decay Model (word problems)
  - \( y = a(1 - r)^t \)
    - \( y \) = current amount
    - \( a \) = initial amount
    - \( r \) = decay percent
    - \( 1 - r \) = decay factor
    - \( t \) = time
A new car cost $23000. The value decreases by 15% each year. Write a model of this decay. How much will the car be worth in 5 years? 10 years?

\[ y = 23000(1-0.15)^t \rightarrow y = 23000(0.85)^t \]

5 years: \[ y = 23000(0.85)^5 = 10205.22 \]
10 years: \[ y = 23000(0.85)^{10} = 4528.11 \]
Quiz

- 7.2 Homework Quiz
7.3 Use Functions Involving $e$

- In math, there are some special numbers like $\pi$ or $i$.
- Today we will learn about $e$. 
7.3 Use Functions Involving $e$

- $e$
  - Called the natural base
  - Named after Leonard Euler who discovered it
    - (Pronounced “oil-er”)
  - Found by putting really big numbers into $\left(1 + \frac{1}{n}\right)^n = 2.718281828459...$
  - Irrational number like $\pi$
7.3 Use Functions Involving $e$

- Simplifying natural base expressions
  - Just treat $e$ like a regular variable

- $(2e^{-5x})^{-2}$

- $\frac{24e^8}{8e^5}$
7.3 Use Functions Involving $e$

- Evaluate the natural base expressions using your calculator
- $e^3$
- $e^{-0.12}$

$e^3 = 20.085537$
$e^{-0.12} = 0.88692044$
7.3 Use Functions Involving $e$

- To graph make a table of values
- $f(x) = a \cdot e^{rx}$
  - $a > 0$
  - If $r > 0 \rightarrow$ growth
  - If $r < 0 \rightarrow$ decay
- Graph $y = 2e^{0.5x}$
7.3 Use Functions Involving $e$

- **Compound Interest**
  $$A = P \left(1 + \frac{r}{n}\right)^{nt}$$
  - $A =$ current amount
  - $P =$ principle (initial amount)
  - $r =$ percentage rate
  - $n =$ number of times compounded per year
  - $t =$ time in years

- **Compounded continuously**
  - $A = Pe^{rt}$
7.3 Use Functions Involving $e$

- 495 #1-49 every other odd, 55, 57, 61 + 4 = 20 total
Quiz

- 7.3 Homework Quiz
7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Definition of Logarithm with Base \( b \)

- \( \log_b y = x \iff b^x = y \)

- Read as “\( \log \) base \( b \) of \( y \) equals \( x \)”

- Rewriting logarithmic equations
  - \( \log_3 9 = 2 \)
  - \( \log_8 1 = 0 \)
  - \( \log_5(1/25) = -2 \)

\[ 3^2 = 9 \]
\[ 8^0 = 1 \]
\[ 5^{-2} = 1/25 \]
7.4 Evaluate Logarithms and Graph Logarithmic Functions

- **Special Logs**
  - \( \log_b 1 = 0 \)
  - \( \log_b b = 1 \)

- **Evaluate**
  - \( \log_4 64 \)
  - \( \log_2 \frac{1}{8} \)
  - \( \log_{1/4} 256 \)

Rewrite \( \log_b 1 = 0 \rightarrow b^0 = 1 \)
Rewrite \( \log_b b = 1 \rightarrow b^1 = b \)

Rewrite \( \log_4 64 = x \rightarrow 4^x = 64 \rightarrow x = 3 \)
Rewrite \( \log_2 0.125 = x \rightarrow 2^x = 1/8 \rightarrow x = -3 \)
Rewrite \( \log_{1/4} 256 = x \rightarrow (\frac{1}{4})^x = 256 \rightarrow 4^{-x} = 256 \rightarrow 4^{-x} = 4^4 \rightarrow -x = 4 \rightarrow x = -4 \)
7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Using a calculator
- Common Log (base 10)
  - $\log_{10} x = \log x$
  - Find $\log 12$

- Natural Log (base $e$)
  - $\log_e x = \ln x$
  - Find $\ln 2$

1.0792
0.6931
7.4 Evaluate Logarithms and Graph Logarithmic Functions

- When the bases are the same, the base and the log cancel
- \(5^{\log_5 7} = 7\)

- \(\log_3 81^x\)
- \[= \log_3 3^{4x}\]
- \[= 4x\]
7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Finding Inverses of Logs
- $y = \log_b x$
- $x = \log_b y$ Switch $x$ and $y$
- $y = 8^x$ Rewrite to solve for $y$

- To graph logs
  - Find the inverse
  - Make a table of values for the inverse
  - Graph the log by switching the $x$ and $y$ coordinates of the inverse.
7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Properties of graphs of logs

- \( y = \log_b(x - h) + k \)
  - \( x = h \) is vert. asymptote
  - Domain is \( x > h \)
  - Range is all real numbers
  - If \( b > 1 \), graph rises
  - If \( 0 < b < 1 \), graph falls
Graph the points with x and y switched

(1/8, -3)
(1/4, -2)
(1/2, -1)
(1, 0)
(2, 1)
(4, 2)
(8, 3)
7.4 Evaluate Logarithms and Graph Logarithmic Functions

- 503 #3, 5-49 every other odd, 59, 61 + 5 = 20 total
Quiz

- 7.4 Homework Quiz
7.5 Apply Properties of Logarithms

- **Product Property**
  - \( \log_b uv = \log_b u + \log_b v \)

- **Quotient Property**
  - \( \log_b \frac{u}{v} = \log_b u - \log_b v \)

- **Power Property**
  - \( \log_b u^n = n \log_b u \)
7.5 Apply Properties of Logarithms

- Use \( \log_9 5 = 0.732 \) and \( \log_9 11 = 1.091 \) to find
  
  - \( \log_9 \frac{5}{11} \)
  
  - \( \log_9 55 \)
  
  - \( \log_9 25 \)

\[
\begin{align*}
\log_9 5/11 & \rightarrow \log_9 5 - \log_9 11 \rightarrow 0.732 - 1.091 \rightarrow -0.359 \\
\log_9 55 & \rightarrow \log_9 (5\cdot11) \rightarrow \log_9 5 + \log_9 11 \rightarrow 0.732 + 1.091 \rightarrow 1.823 \\
\log_9 25 & \rightarrow \log_9 5^2 \rightarrow 2 \log_9 5 \rightarrow 2(0.732) \rightarrow 1.464
\end{align*}
\]
7.5 Apply Properties of Logarithms

- Expand: $\log_5 2x^6$
- Condense: $2 \log_3 7 - 5 \log_3 x$

\[
\log_5 2 + \log_5 x^6 \rightarrow \log_5 2 + 6 \log_5 x
\]
\[
\log_3 7^2 - \log_3 x^5 \rightarrow \log_3 (49/x^5)
\]
7.5 Apply Properties of Logarithms

- Change-of-Base Formula
  \[ \log_c u = \frac{\log_b u}{\log_b c} \]

- Evaluate \( \log_4 8 \)

- 510 #3-31 every other odd, 33-43 odd, 47, 51, 55, 59, 63, 71, 73 + 4 = 25 total

\[ \log_4 8 = (\log 8)/(\log 4) = 1.5 \]
Quiz

- 7.5 Homework Quiz
7.6 Solve Exponential and Logarithmic Equations

- Solving Exponential Equations
  - Method 1) if the bases are equal, then exponents are equal

- \(2^{4x} = 32^{x-1}\)

\[
2^{4x} = 2^{5(x-1)} \rightarrow 4x = 5(x-1) \rightarrow 4x = 5x - 5 \rightarrow -x = -5 \rightarrow x = 5
\]
7.6 Solve Exponential and Logarithmic Equations

- Solving Exponential Equations
  - Method 2) take log of both sides
  - $4^x = 15$

- $5^{x+2} + 3 = 25$

$log 4^x = log 15 \rightarrow x \log 4 = log 15 \rightarrow x = log 15 / log 4 \rightarrow x = 1.95$

$5^{x+2} = 22 \rightarrow log 5^{x+2} = log 22 \rightarrow (x+2) \log 5 = log 22 \rightarrow x+2 = log 22 / log 5 \rightarrow x = -0.079$
7.6 Solve Exponential and Logarithmic Equations

- **Solving Logarithmic Equations**
  - Method 1) if the bases are equal, then logs are equal
    - \( \log_3 (5x - 1) = \log_3 (x + 7) \)

\[ 5x - 1 = x + 7 \rightarrow 4x = 8 \rightarrow x = 2 \]
7.6 Solve Exponential and Logarithmic Equations

- Solving Logarithmic Equations
  - Method 2) exponenitiation both sides
    - Make both sides exponents with the base of the log
  - $\log_4 (x + 3) = 2$

$$4^{\log_4 (x + 3)} = 4^2 \Rightarrow x + 3 = 16 \Rightarrow x = 13$$
7.6 Solve Exponential and Logarithmic Equations

- \( \log_2 2x + \log_2 (x - 3) = 3 \)

- \( 519 \#3-43 \) every other odd, 49, 53, 55, 57 + 5 = 20 total

\[
\log_2 2x + \log_2 (x - 3) = 3 \\
\log_2 (2x \cdot (x - 3)) = 3 \\
2x(x - 3) = 2^3 \\
2x^2 - 6x - 8 = 0 \\
x^2 - 3x - 4 = 0 \\
(x - 4)(x + 1) = 0 \\
x - 4 = 0, x + 1 = 0 \\
x = 4, -1 \\
-1 extraneous
\]

Solution x=4
Quiz

7.6 Homework Quiz
7.7 Write and Apply Exponential and Power Functions

- Just as 2 points determine a line, so 2 points will determine an exponential equation.
7.7 Write and Apply Exponential and Power Functions

- Exponential Function
  - $y = a b^x$

- If given 2 points
  - Fill in both points to get two equations
  - Solve for $a$ and $b$ by substitution
7.7 Write and Apply Exponential and Power Functions

- Find the exponential function that goes through (-1, 0.0625) and (2, 32)

\[
0.0625 = ab^{-1} \rightarrow 0.0625 = a/b \rightarrow a = 0.0625b \\
32 = ab^2
\]

Substitute

\[
32 = (0.0625b)b^2 \rightarrow 32 = 0.0625b^3 \rightarrow 512 = b^3 \rightarrow b = 8 \\
a = 0.0625b \rightarrow a = 0.0625(8) = 0.5 \\
y = 0.5 \times 8^x
\]
7.7 Write and Apply Exponential and Power Functions

- Steps if given a table of values
  - Find $\ln y$ of all points
  - Graph $\ln y$ vs $x$
  - Draw the best fit straight line
  - Pick two points on the line and find equation of line (remember to use $\ln y$ instead of just $y$)
  - Solve for $y$

- OR use the ExpReg feature on a graphing calculator
  - Enter points in STAT $\rightarrow$ EDIT
  - Go to STAT $\rightarrow$ CALC $\rightarrow$ ExpReg $\rightarrow$ Enter $\rightarrow$ Enter
7.7 Write and Apply Exponential and Power Functions

- Writing a Power Function
  - $y = a x^b$

- Steps are the same as for exponential function
  - Fill in both points to get two equations
  - Solve for $a$ and $b$ by substitution
7.7 Write and Apply Exponential and Power Functions

Write power function through (3, 8) and (9, 12)

\[ 8 = a^{3^b} \rightarrow a = \frac{8}{3^b} \]
\[ 12 = a^{9^b} \]

Substitute
\[ 12 = (\frac{8}{3^b})^{9^b} \rightarrow 12 = 8 \cdot (\frac{9^b}{3^b}) \rightarrow 12 = 8 \cdot (\frac{9}{3})^b \rightarrow 12 = 8 \cdot 3^b \rightarrow 12/8 = 3^b \rightarrow \log 3^b = \log 3/2 = b \cdot \log 3 \rightarrow b = \log 3/2 / \log 3 \rightarrow b = 0.369 \]
\[ a = 8/3^b \rightarrow a = 8 / 3^{0.369} \rightarrow a = 16/3 \]
\[ y = 16/3 \cdot x^{0.369} \]
7.7 Write and Apply Exponential and Power Functions

- Steps if given a table of values
  - Find \( \ln y \) and \( \ln x \) of all points
  - Graph \( \ln y \) vs \( \ln x \)
  - Draw the best fit straight line
  - Pick two points on the line and find equation of line (remember to use \( \ln y \) and \( \ln x \) instead of just \( y \))
  - Solve for \( y \)

- OR use the PwrReg feature on a graphing calculator
  - Enter points in STAT \( \rightarrow \) EDIT
  - Go to STAT \( \rightarrow \) CALC \( \rightarrow \) PwrReg \( \rightarrow \) Enter \( \rightarrow \) Enter
7.7 Write and Apply Exponential and Power Functions

- 533 #3, 7, 11, 13, 15, 19, 23, 27, 33, 35 + 5 = 15 total
Quiz

- 7.7 Homework Quiz
7. Review

543 choose 20

Graph the function. State the domain and range.

1. \( y = 3^x \)  
2. \( y = 2 \cdot 4^{x-2} \)  
3. \( f(x) = -5 \cdot 2^x + 3 \)

4. \( y = 4(0.25)^x \)  
5. \( y = 2\left(\frac{1}{3}\right)^x + 2 \)

6. \( g(x) = \left\{ \begin{array}{ll} 2^x & \text{if } x \leq 1 \\ 1 & \text{if } x > 1 \end{array} \right. \)

7. \( y = \frac{1}{2}e^{-x} \)  
8. \( y = 2.5e^{-0.5x} + 1 \)  
9. \( h(x) = \frac{1}{3}e^{x-1} - 2 \)

Evaluate the logarithm without using a calculator.

10. \( \log_5 25 \)  
11. \( \log_3 \frac{1}{32} \)  
12. \( \log_6 1 \)

Graph the function. State the domain and range.

13. \( y = \log_2 x \)  
14. \( y = \ln x - 3 \)  
15. \( f(x) = \log (x + 3) + 2 \)

Condense the expression.

16. \( 2 \ln 7 - 3 \ln 4 \)  
17. \( \log_3 5 + 5 \log_3 2 \)  
18. \( \log 5 + \log x - 2 \log 3 \)

Use the change-of-base formula to evaluate the logarithm.

19. \( \log_9 50 \)  
20. \( \log_{23} 50 \)  
21. \( \log_{45} 5 \)

Solve the equation. Check for extraneous solutions.

22. \( 2^x = 30 \)  
23. \( 3 \log (x - 4) = 6 \)  
24. \( \log_3 x + \log_3 (x + 6) = 2 \)

25. Write an exponential function \( y = ab^x \) whose graph passes through \((-1, 48)\) and \((2, 6)\).

26. Write a power function \( y = ax^b \) whose graph passes through \((3, 8)\) and \((6, 15)\).

27. **LANDSCAPING** From 1996 to 2001, the number of households that purchased lawn and garden products at home gardening centers increased by about