Rational Equations and Functions
Algebra II
Chapter 8

Algebra II 8
This Slideshow was developed to accompany the textbook

- *Larson Algebra 2*
- *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
- *2011 Holt McDougal*

Some examples and diagrams are taken from the textbook.
8.1 Model Inverse and Joint Variation

- Direct Variation: \( y = ax \)
  \( \star x \uparrow, y \uparrow \)

- Inverse Variation: \( y = \frac{a}{x} \)
  \( \star x \uparrow, y \downarrow \)

- Joint Variation: \( y = axz \)
  \( \star y \) depends on both \( x \) and \( z \)
What type of variation is each of the following?

- \( xy = 48 \) → inverse
- \( 2y = x \) → direct
- \( y = 2x + 3 \) → +3 means neither
Solving Variations

- Plug in x and y to find k
- Plug in k and the other value and solve

y varies inversely as x. When x = 2, y = 6. Write an equation relating x and y. Then find y when x = 4.

\[ y = \frac{k}{x} \rightarrow 6 = \frac{k}{2} \rightarrow 12 = k \]
\[ y = \frac{12}{4} \rightarrow y = 3 \]
8.1 Model Inverse and Joint Variation

- Checking data for variation
  - Plug each of the data points in one of the variation equations to find $a$
  - If the $a$ stays the same, the data has that type of variation
- What type of variation?

<table>
<thead>
<tr>
<th>X</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>8</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>
Writing variations from sentences

- $y$ varies directly with $x$ and inversely with $z^2$
- $z$ varies jointly with $x^2$ and $y$
- $y$ varies inversely with $x$ and $z$

555 #3-33 odd, 39, 41 + 2 = 20 total

Varies means “=a”

$y = ax/z^2$
$z = ax^2y$
$y = axz$
8.1 Homework Quiz
8.2 Graph Simple Rational Functions

- Rational Functions
  - Functions written as a fraction with $x$ in the denominator
  - $y = \frac{1}{x}$
- Shape called hyperbola
- Asymptotes
  - Horizontal: $x$-axis
  - Vertical: $y$-axis
8.2 Graph Simple Rational Functions

- General form
  \[ y = \frac{a}{x-h} + k \]
  - \(a\) \(\rightarrow\) stretches vertically (multiplies y-values)
  - \(h\) \(\rightarrow\) moves right
  - \(k\) \(\rightarrow\) moves up

- How is \(y = \frac{2}{x+3} + 4\) transformed from \(y = \frac{1}{x}\)?

  Stretches vertically by factor of 2
  Moves left 3
  Moves up 4
8.2 Graph Simple Rational Functions

- How to find asymptotes
  - Vertical
    - Make the denominator = 0 and solve for x

Vertical: \(3x - 6 = 0 \Rightarrow 3x = 6 \Rightarrow x = 2\)
Horizontal: \(y = \frac{2*1000000}{3*1000000 - 6} \Rightarrow y = 2/3\)
8.2 Graph Simple Rational Functions

- Horizontal
  - Substitute a very large number for x and estimate y

- Or
  - Find the degree of numerator (N)
  - Find the degree of denominator (D)
  - If N < D, then y = 0
  - If N = D, then y = leading coefficients
  - If N > D, then no horizontal asymptote

- Find the asymptotes for \( y = \frac{2x}{3x-6} \)
8.2 Graph Simple Rational Functions

- **Domain**
  - All x’s except for the vertical asymptotes

- **Range**
  - All the y’s covered in the graph
  - Usually all y’s except for horizontal asymptote
Asymptotes
Vertical: \( x + 3 = 0 \rightarrow x = -3 \)
Horizontal: \( y = \frac{2}{1000000 + 3} + 4 \rightarrow y = 4 \)

Graph by finding asymptotes and making a table
Graph \( y = \frac{2}{x+3} + 4 \)

561 #1, 3-31 every other odd, 39, 41 + 4 = 15 total
8.2 Homework Quiz
Find the asymptotes

- Simplify first
  - Factor and cancel entire factors
- Vertical
  - take the denominator = 0 and solve for x

Vertical: \( x^2 - 1 = 0 \rightarrow x^2 = 1 \rightarrow x = \pm 1 \)

Horizontal: \( y = \frac{2(1000000)^2 + 1000000}{(1000000)^2 - 1} \rightarrow y = 2 \)
8.3 Graph General Rational Functions

- Horizontal
  - Substitute a very large number for \( x \) and estimate \( y \)

- Or
  - Find the degree of numerator (\( N \))
  - Find the degree of denominator (\( D \))
  - If \( N < D \), then \( y = 0 \)
  - If \( N = D \), then \( y \) = leading coefficients
  - If \( N > D \), then no horizontal asymptote

- Find the asymptotes for
  \[ y = \frac{2x^2 + x}{x^2 - 1} \]
8.3 Graph General Rational Functions

- How to find x-intercepts
  - Let $y = 0$
    - If $y = \frac{\text{numerator}}{\text{denominator}} = 0$
      - Only happens if numerator = 0
- How to find y-intercepts
  - Let $x = 0$ and simplify
8.3 Graph General Rational Functions

- To graph rational functions
  - Find the asymptotes
  - Make a table of values around the vertical asymptotes
  - Graph the asymptotes and points
  - Start near an asymptote, go through the points and end near another asymptote
    - Each graph will have several sections
    - NEVER cross a vertical asymptote
Graph $y = \frac{2x^2 + x}{x^2 - 1}$

568 #3-15 odd, 19, 23, 33, 35 + 4 = 15 total

Vertical: $x^2 - 1 = 0 \rightarrow x^2 = 1 \rightarrow x = \pm 1$
Horizontal: $y = (2(1000000)^2 + 1000000)/(1000000^2 - 1) \rightarrow y = 2$
8.3 Homework Quiz
8.4 Multiply and Divide Rational Expressions

- Simplified form → numerator and denominator can have no common factors

- Steps to simplify
  - Factor numerator and denominator
  - Cancel any common factors
Simplify

\[
\frac{x^2-5x-6}{x^2-1} \quad \frac{x^3+5x^2+6x}{x^3+2x^2}
\]

\[
\frac{(x-6)(x+1)}{(x-1)(x+1)} \rightarrow \frac{x-6}{x-1}
\]

\[
\frac{x(x+3)(x+2)}{x^2(x+2)} \rightarrow \frac{x+3}{x}
\]
8.4 Multiply and Divide Rational Expressions

- Multiplying Rational Expressions
  - Factor numerators and denominators
  - Multiply across top and bottom
  - Cancel factors
8.4 Multiply and Divide Rational Expressions

\[
\frac{3x-27x^3}{3x^2-2x-1} \cdot \frac{3x^2-4x+1}{3x} \quad \cdot \quad \frac{x+2}{27x^3+8} \cdot (9x^2 - 6x + 4)
\]

\[
\Rightarrow \frac{-3x(9x^2-1)}{(3x+1)(x-1)} \cdot (3x-1)(x-1) \Rightarrow \frac{-3x(3x-1)(3x+1)(3x-1)(x-1))(x-1)}{(3x+1)(x-1))
\Rightarrow -(3x-1)^2
\]

\[
\frac{x+2}{(3x+2)(9x^2-6x+4))} \cdot (9x^2-6x+4)/1 \Rightarrow (x+2)/(3x+2)
\]
Dividing Rational Expressions

- Take reciprocal of divisor
- Multiply

\[
\frac{3}{4x-8} \div \frac{x^2+3x}{x^2+x-6}
\]

\[
\Rightarrow \frac{3}{4(x-2)} \times \frac{(x-2)(x+3)}{x(x+3)} \Rightarrow \frac{3}{4x}
\]
8.4 Multiply and Divide Rational Expressions

- Combined Operations
  - Do the first two operations
  - Use that result with the next operation

- 577 #3, 7-17 odd, 25-43 odd, 49 + 2 = 20
Quiz

- 8.4 Homework Quiz
8.5 Add and Subtract Rational Expressions

- Adding and Subtracting
  - Need least common denominator (LCD)
    - If LCD already present, add or subtract numerators only
  - To get fractions with LCD
    - Factor all denominators
    - LCD is the common factors times the unique factors
    - Whatever you multiply the denominator by, multiply the numerator also
-4/(2x) → -2/x

(3x+6)/(x-4)
8.5 Add and Subtract Rational Expressions

\[
\frac{4}{3x^2} + \frac{x}{6x^3 + 3x^2} \quad \text{and} \quad \frac{x+1}{x^2 + 6x + 9} - \frac{1}{x^2 - 9}
\]

\[
\frac{4}{3x^2} + \frac{x}{3x^2 (2x + 1)} \rightarrow \frac{4(2x + 1)}{3x^2 (2x + 1)} + \frac{x}{3x^2 (2x + 1)} \rightarrow \frac{9x + 4}{3x^2 (2x + 1)}
\]

\[
\frac{x + 1}{(x + 3)(x + 3)} - \frac{1}{(x + 3)(x - 3)} \rightarrow \frac{(x + 1)(x - 3)}{(x + 3)(x - 3)} + \frac{x + 3}{x + 3} \rightarrow \frac{(x + 3)^2(x - 3)}{(x + 3)^2(x - 3)} - \frac{(x + 3)^2(x - 3)}{(x + 3)^2(x - 3)} \rightarrow \frac{(x + 3)^2(x - 3)}{(x + 3)^2(x - 3)}
\]

\[
\frac{4}{3x^2} + \frac{x}{6x^3 + 3x^2} \quad \text{and} \quad \frac{x+1}{x^2 + 6x + 9} - \frac{1}{x^2 - 9}
\]
8.5 Add and Subtract Rational Expressions

- Simplifying Complex Fractions
  - Fractions within fractions
  - Follow order of operations (groups first)
  - Divide
8.5 Add and Subtract Rational Expressions

\[
\frac{3}{x-4} + \frac{1}{x-4} = \frac{3}{x-4} + \frac{3}{x+1} \\
\rightarrow \frac{3x + 12}{(x-4)(x+1)} = \frac{3x-11}{4x-11}
\]

586 #3, 7, 11, 13, 15, 17, 19, 21, 23, 27, 29, 31, 33, 35, 39, 41 + 4 = 20
Quiz

8.5 Homework Quiz
8.6 Solve Rational Equations

- Only when the = sign is present!!!

- Method 1: simplify both sides and cross multiply

- Method 2:
  - Multiply both sides by LCD to remove fractions
  - Solve
  - Check answers
8.6 Solve Rational Equations

\[
\frac{3}{x} - \frac{1}{2} = \frac{12}{x}
\]

\[
\frac{5x}{x+1} = 4 - \frac{5}{x+1}
\]

\[
\frac{(3(2x))/x - 2x/2 = (12(2x))/x}{6 - x = 24 \rightarrow -x = 18 \rightarrow x = -18}
\]

\[
5x = 4(x+1) - 5 \rightarrow 5x = 4x + 4 - 5 \rightarrow x = -1
\]

Check answer: can’t divide by -1 so NO SOLUTION
(3x-2)/(x-2) = 6/((x-2)(x+2)) + 1 \rightarrow (3x-2)(x+2) = 6 + (x-2)(x+2) \rightarrow 3x^2 + 4x - 4 = 6 + x^2 - 4 \rightarrow 2x^2 + 4x - 6 = 0 \rightarrow x^2 + 2x - 3 = 0 \rightarrow (x - 1)(x + 3) = 0 \rightarrow x = 1, -3

3/(x(x+4)) = 1/(x+4) \rightarrow 3 = x
8.6 Solve Rational Equations

592 #5-27 odd, 31, 35, 37 + 5 = 20
Quiz

8.6 Homework Quiz
The variables $x$ and $y$ vary inversely. Use the given values to write an equation relating $x$ and $y$. Then find $y$ when $x = 4$.

1. $x = 5, y = 2$
2. $x = -2, y = 8$
3. $x = \frac{3}{2}, y = 10$
4. $x = 3, y = 6$
5. $x = -4, y = \frac{7}{2}$
6. $x = \frac{3}{4}, y = \frac{5}{8}$

Graph the function. State the domain and range.

7. $y = \frac{2}{x + 5} - 3$
8. $y = -\frac{1}{x - 4} - 1$
9. $f(x) = \frac{6 - x}{2x + 1}$

Graph the function.

10. $y = \frac{4}{x^2 + 2}$
11. $y = \frac{x^2 - 4}{x^2 + 8x + 15}$
12. $g(x) = \frac{x^2 + 3}{2x - 1}$

Find the least common multiple of the polynomials.

13. $(x - 3)(x + 5)$ and $x(x + 5)$
14. $4x^2(x - 2)$ and $8x(x + 2)$
15. $x^2 - 4x$ and $x^2 - 2x - 8$
16. $2x + 6$ and $x^3 + 10x^2 + 21x$

Perform the indicated operation and simplify.

17. $\frac{3xy^2}{4x^2y} - \frac{6y^3}{2xy^3}$
18. $\frac{x^3 - 3x - 4}{x^2 - 3x - 10} \cdot \frac{x - 3}{x + 1}$
19. $\frac{x^2 - 8x + 15}{x^2 + 12x + 32} \cdot \frac{x + 4}{x^2 - 25}$
20. $\frac{x^2 - 11x + 28}{x^2 + 5x + 4} = (x^2 - 16)$