This Slideshow was developed to accompany the textbook

*Larson Algebra 2*

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Some examples and diagrams are taken from the textbook.

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序列为函数，其域为整数
- 列举遵循规则的数字

2, 4, 6, 8, 10
- 有限

2, 4, 6, 8, 10, ...
- 无限

n 是 x，a_n 是 y
12.1 Define and Use Sequences and Series

- **Rule**
  - \( a_n = 2n \)

- **Domain: \( n \)**
  - Term’s location (1\(^{\text{st}}\), 2\(^{\text{nd}}\), 3\(^{\text{rd}}\)...)

- **Range: \( a_n \)**
  - Term’s value (2, 4, 6, 8...)

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"Rule
\[ a_n = 2n \]"
Writing rules for sequences
- Look for patterns
- Guess-and-check

\[ \frac{2}{5}, \frac{2}{25}, \frac{2}{125}, \frac{2}{625}, \ldots \]
- \[ a_n = \frac{2}{5^n} \]

3, 5, 7, 9, ...
- \[ 2(1) + 1, \ 2(2) + 1, \ 2(3) + 1, \ldots \]
- \[ a_n = 2n + 1 \]

\[ \frac{2}{5^1}, \frac{2}{5^2}, \frac{2}{5^3}, \frac{2}{5^4}, \ldots \rightarrow a_n = \frac{2}{5^n} \]

\[ 2(1)+1, \ 2(2)+1, \ 2(3)+1, \ldots \rightarrow a_n = 2n + 1 \]
To graph

- $n$ is like $x$; $a_n$ is like $y$
- The graph will be dots
- Do NOT connect the dots

The $n$'s are integers so there is no values between the integers.
12.1 Define and Use Sequences and Series

Series

Sum of a sequence

2, 4, 6, 8, ... \(\rightarrow\) sequence

2 + 4 + 6 + 8 + \cdots \(\rightarrow\) series
12.1 Define and Use Sequences and Series

- Sigma notation
  - Finite
    
    \[ 2 + 4 + 6 + 8 = \sum_{i=1}^{4} 2i \]

  - Infinite
    
    \[ 2 + 4 + 6 + 8 + \cdots = \sum_{i=1}^{\infty} 2i \]
12.1 Define and Use Sequences and Series

Write as a summation

\[ 4 + 8 + 12 + \cdots + 100 \]

\[ \sum_{n=1}^{25} 4n \]

\[ a_n = 4n, \text{ lower limit} = 1, \text{ upper limit} = 25 \]

\[ \sum_{n=1}^{\infty} \frac{n+1}{n^2} \]

\[ a_n = \frac{(n+1)}{n^2}, \text{ lower limit} = 1, \text{ upper limit} = \infty \]

\[ \sum_{n=1}^{\infty} \frac{n+1}{n^2} \]

Note that the index may be any letter.

\[ 2 + \frac{3}{4} + \frac{4}{9} + \frac{5}{16} + \cdots \]

\[ \sum_{n=1}^{\infty} \frac{n+1}{n^2} \]
Find the sum of the series

\[
\sum_{k=5}^{10} k^2 + 1
\]

\[
(5^2 + 1) + (6^2 + 1) + (7^2 + 1) + (8^2 + 1) + (9^2 + 1) + (10^2 + 1)
\]

\[
= 361
\]
12.1 Define and Use Sequences and Series

Some shortcut formulas

\[
\sum_{i=1}^{n} 1 = n
\]

\[
\sum_{i=1}^{n} i = \frac{n(n + 1)}{2}
\]

\[
\sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}
\]
Find the sum of the series

\[ \sum_{k=1}^{10} 3k^2 + 2 \]

\[ 3 \frac{n(n+1)(2n+1)}{6} + 2n \]

\[ 3 \frac{10(10+1)(2(10)+1)}{6} + 2(10) \]

= 1175
12.1 Homework Quiz
Arithmetic Sequences
- Common difference (d) between successive terms
  - Add the same number each time
- 3, 6, 9, 12, 15, ...
  - \(d = 3\)

Is it arithmetic?
- -10, -6, -2, 0, 2, 6, 10, ...
  - No
- 5, 11, 17, 23, 29, ...
  - Yes, \(d = 6\)
12.2 Analyze Arithmetic Sequences and Series

Formula for $n^{th}$ term

\[ a_n = a_1 + (n - 1)d \]

Write a rule for the $n^{th}$ term

Given sequence: 32, 47, 62, 77, ...

\[ d = 15 \]
\[ a_n = 32 + (n - 1)15 \]
\[ a_n = 32 + 15n - 15 \]
\[ a_n = 17 + 15n \]

$d = 15$

\[ a_n = 32 + (n - 1)15 = 32 + 15n - 15 \rightarrow a_n = 17 + 15n \]
One term of an arithmetic sequence is \( a_8 = 50 \). The common difference is 0.25. Write the rule for the \( n^{th} \) term.

\[
a_n = a_1 + (n - 1)d
\]

\[
50 = a_1 + (8 - 1)0.25 \quad \rightarrow \quad 50 = a_1 + 1.75 \quad \rightarrow \quad 48.25 = a_1
\]

\[
a_n = 48.25 + (n - 1)0.25
\]

\[
a_n = 48.25 + 0.25n - 0.25 \quad \rightarrow \quad a_n = 48 + 0.25n
\]
Two terms of an arithmetic sequence are \( a_5 = 10 \) and \( a_{30} = 110 \). Write a rule for the \( n^{th} \) term.

\[
a_n = a_1 + (n - 1)d
\]

\[
10 = a_1 + (5 - 1)d
\quad \rightarrow \quad 10 = a_1 + 4d
\]

\[
110 = a_1 + (30 - 1)d
\quad \rightarrow \quad 110 = a_1 + 29d
\]

Linear combination

\[
-10 = -a_1 - 4d
\]

\[
110 = a_1 + 29d
\]

\[
100 = 25d
\]

\[
d = 4
\]

Substitute

\[
10 = a_1 + 4d \quad \rightarrow \quad 10 = a_1 + 4(4) \quad \rightarrow \quad a_1 = -6
\]

Rule

\[
a_n = -6 + (n - 1)4
\]

\[
a_n = -6 + 4n - 4
\]

\[
a_n = 4n - 10
\]
12.2 Analyze Arithmetic Sequences and Series

- Sum of a finite arithmetic series
  - $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$
  - Rewrite
    - $1 + 2 + 3 + 4 + 5$
    - $10 + 9 + 8 + 7 + 6$
    - $11 + 11 + 11 + 11 + 11 = 5(11) = 55$

- Formula
  - $S_n = n \left( \frac{a_1 + a_n}{2} \right)$

From example:
First and last $(a_1 + a_n) = 11$
10 numbers but only half as many pairs $(n/2)$
Consider the arithmetic series
20 + 18 + 16 + 14 + ···
Find the sum of the first 25 terms.

\[ a_{25} = 20 + (25 - 1)(-2) \]
\[ a_{25} = -28 \]

\[ S_{25} = \frac{n(a_1 + a_{25})}{2} \]
\[ S_{25} = 25 \left( \frac{20 + (-28)}{2} \right) = -100 \]

\[ a_{25} = 20 + (25-1)(-2) = -28 \]
\[ S_{25} = 25((20+(-28))/2) = -100 \]
Consider the arithmetic series

20 + 18 + 16 + 14 + …

Find n such that \( S_n = -760 \)

\[
S_n = n \left( \frac{a_1 + a_n}{2} \right)
\]

\[
a_n = 20 + (n - 1)(-2)
\]

\[
a_n = 22 - 2n
\]

\[
-760 = n \left( \frac{20 + (22 - 2n)}{2} \right)
\]

\[
-1520 = n(42 - 2n)
\]

\[
-1520 = 42n - 2n^2
\]

\[
2n^2 - 42n - 1520 = 0
\]

\[
n^2 - 21n - 760 = 0
\]

\[
(n + 19)(n - 40) = 0
\]

\[
n = 40, -19
\]
12.2 Homework Quiz
12.3 Analyze Geometric Sequences and Series

- Created by multiplying by a common ratio \((r)\)

- Are these geometric sequences?
  - 1, 2, 6, 24, 120, ...
    - No
  - 81, 27, 9, 3, 1, ...
    - Yes, \(r = \frac{1}{3}\)

No
Yes \(r = 1/3\)
Write a rule for the $n^{th}$ term and find $a_8$.

- $5, 2, 0.8, 0.32, ...$
- $r = \frac{2}{5}$
- $a_n = 5 \left(\frac{2}{5}\right)^{n-1}$
- $a_8 = 5 \left(\frac{2}{5}\right)^7 = 0.008192$

$r = 2/5$

$a_n = 5(2/5)^{n-1}$

$a_8 = 5(2/5)^7 = 0.008192$
One term of a geometric sequence is $a_4 = 3$ and $r = 3$. Write the rule for the $n^{th}$ term.

$a_n = a_1 r^{n-1}$

$3 = a_1 3^{4-1}$

$3 = a_1 27$

$a_1 = \frac{1}{9}$

$a_n = \left(\frac{1}{9}\right) 3^{n-1}$
If two terms of a geometric sequence are $a_2 = -4$ and $a_6 = -1024$, write rule for the $n^{th}$ term.

- $a_n = a_1 r^{n-1}$
- $-4 = a_1 r^{2-1}$
- $-4 = a_1 r$
- $-1024 = a_1 r^{6-1}$
- $-1024 = a_1 r^5$

Solve first for $a_1$: $a_1 = \frac{-4}{r}$

Plug into second:

- $-1024 = \left(\frac{-4}{r}\right) r^5$
- $-1024 = \frac{-4r^5}{r}$
- $-1024 = -4r^4$
- $256 = r^4$
- $r = 4$

Plug back into first: $a_1 = \frac{-4}{4} = -1$

Write rule: $a_n = -1 \cdot 4^{n-1}$
12.3 Analyze Geometric Sequences and Series

- **Sum of geometric series**
  \[ S_n = a_1 \left( \frac{1-r^n}{1-r} \right) \]

- Find the sum of the first 10 terms of
  \[ 4 + 2 + 1 + \frac{1}{2} + \cdots \]
  \[ r = \frac{1}{2}, a_1 = 4 \]

  \[ S_{10} = 4 \left( \frac{\left(1 - \left(\frac{1}{2}\right)^{10}\right)}{1 - \frac{1}{2}} \right) \]
  \[ = 4 \left( \frac{0.99902}{0.5} \right) \]
  \[ = \frac{1023}{128} = 7.992 \]

\[ r = \frac{1}{2}, a_1 = 4 \]

\[ S_{10} = 4 \left( \frac{1 - \left(\frac{1}{2}\right)^{10}}{1 - \frac{1}{2}} \right) \]

\[ = 4 \left( \frac{0.99902}{0.5} \right) \]

\[ = \frac{1023}{128} = 7.992 \]
12.3 Analyze Geometric Sequences and Series

Find \( n \) such that \( S_n = \frac{31}{4} \)

\[ 4 + 2 + 1 + \frac{1}{2} + \ldots \]

\[ S_n = a_1 \left( \frac{1-r^n}{1-r} \right) \]

\[ \frac{31}{4} = 4 \left( \frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}} \right) \]

\[ \frac{31}{16} = \frac{1-\left(\frac{1}{2}\right)^n}{\frac{1}{2}} \]

\[
\begin{align*}
\frac{31}{32} &= 1 - \left(\frac{1}{2}\right)^n \\
-\frac{1}{32} &= - \left(\frac{1}{2}\right)^n \\
\frac{1}{32} &= \left(\frac{1}{2}\right)^n \\
\log_{1/2} \left(\frac{1}{32}\right) &= n \log_{1/2} \left(\frac{1}{2}\right) \\
5 &= n
\end{align*}
\]

\[
\begin{align*}
31/4 &= 4(1 - (1/2)^n)/(1-1/2) \rightarrow 31/16 = (1 - (1/2)^n)/(1/2) \rightarrow 31/32 = 1 - (1/2)^n \rightarrow -1/32 = -(1/2)^n \rightarrow 1/32 = (1/2)^n \rightarrow \log (1/32) = n \log (1/2) \rightarrow n = (\log (1/32)/\log (1/2)) = 5
\end{align*}
\]
12.3 Homework Quiz
Think of the box a 1 whole piece
12.4 Find the Sums of Infinite Geometric Series

Cut in half
12.4 Find the Sums of Infinite Geometric Series

Cut in half
12.4 Find the Sums of Infinite Geometric Series

\[
\frac{1}{2} + \frac{1}{4} = \frac{3}{4}
\]

Cut in half
12.4 Find the Sums of Infinite Geometric Series

\[
\frac{1}{2} + \frac{1}{4} + \frac{1}{8}
\]

Cut in half
12.4 Find the Sums of Infinite Geometric Series

Cut in half
What is the sum of the pieces if we keep cutting forever?

What is the sum of the pieces if we keep going? 1 piece
12.4 Find the Sums of Infinite Geometric Series

◊ Sum of an infinite geometric series

◊ \( S = \frac{a_1}{1-r} \)

◊ \(|r| < 1\)

◊ If \(|r| > 1\), then no sum (\(\infty\))
12.4 Find the Sums of Infinite Geometric Series

Find the sum

\[
\sum_{i=1}^{\infty} 2(0.1)^{i-1}
\]

\(a_1 = 2(0.1)^{1-1} = 2,\)
\(r = 0.1\)
\(S = \frac{a_1}{1-r}\)
\(S = \frac{2}{1-0.1} = \frac{20}{0.9} = \frac{20}{9}\)

\[a_1 = 2(0.1)^{1-1} = 2,\ r = 0.1\]
\[S=2/(1-0.1) = 2/.9 = 20/9\]

\(a_1 = 12,\ r = \frac{1}{3}\)
\(S = \frac{a_1}{1-r}\)
\(S = \frac{12}{1-\frac{1}{3}} = \frac{12}{2/3} = \frac{12}{\frac{2}{3}} = 18\)

\[a_1 = 12,\ r = \frac{1}{3}\]
\[S=12/(1-\frac{1}{3}) = 12/(2/3) = 36/2 = 18\]
An infinite geometric series has $a_1 = 5$ has sum of $27/5$. Find the common ratio.

$S = \frac{a_1}{1-r}$

$\frac{27}{5} = \frac{5}{1-r}$

$27(1 - r) = 25$

$\frac{27}{5} = \frac{5}{1-r}$

$27(1 - r) = 25$

$1 - r = \frac{25}{27}$

$-r = -\frac{2}{27}$

$r = \frac{2}{27}$

$27/5 = 5/(1-r) \Rightarrow 27(1-r) = 5*5 \Rightarrow 1-r = 25/27 \Rightarrow -r = -2/27 \Rightarrow r = 2/27$
Write 0.27272727... as a fraction.

Write the repeating unit as a sum of fractions

$$\frac{27}{100} + \frac{27}{10000} + \frac{27}{1000000} + \cdots$$

$$a_1 = \frac{27}{100}, r = \frac{1}{100}$$

$$S = \frac{\frac{27}{100}}{1 - \frac{1}{100}} = \frac{27/100}{99/100} = \frac{27}{99} = \frac{3}{11}$$
Write 0.416666666... as a fraction.

\[
\frac{41}{100} + \frac{6}{1000} + \frac{6}{10000} + \frac{6}{100000} + \ldots
\]

Ignore the \( \frac{41}{100} \) for now.

\[
a_1 = \frac{6}{1000}, \quad r = \frac{1}{10}
\]

\[
S = \frac{\frac{6}{1000}}{1 - \frac{1}{10}} = \frac{6}{1000} \times \frac{10}{9} = \frac{60}{9000} = \frac{1}{150}
\]

Now add the \( \frac{41}{100} \)

\[
\frac{41}{100} + \frac{1}{150} + \frac{41 \times 3}{300} + \frac{1 \times 2}{300} + \frac{123}{300} + \frac{2}{300} + \frac{125}{300} = \frac{5}{12}
\]

41/100 + 6/1000 + 6/10000 + 6/100000 + ...

Ignore the 41/100 for now.

\[
a_1 = \frac{6}{1000}, \quad r = \frac{1}{10}
\]

\[
S = \frac{\frac{6}{1000}}{1 - \frac{1}{10}} = \frac{6}{1000} \times \frac{10}{9} = \frac{60}{9000} = \frac{1}{150}
\]

Now add the 41/100

\[
41/100 + 1/150 \rightarrow (41 \times 3)/300 + (1 \times 2)/300 \rightarrow 123/300 + 2/300 \rightarrow 125/300 \rightarrow 5/12
\]
12.4 Homework Quiz
12.5 Use Recursive Rules with Sequences and Functions

Explicit Rule
- Gives the $n^{th}$ term directly
- $a_n = 2 + 4n$

Recursive Rule
- Each term is found by knowing the previous term
- $a_1 = 6$; $a_n = a_{n-1} + 4$

Both these rules give the same sequence
12.5 Use Recursive Rules with Sequences and Functions

Write the first 5 terms

\[ a_1 = 1, a_n = (a_{n-1})^2 + 1 \]

\[ a_1 = 1, \]
\[ a_2 = 1^2 + 1 = 2, \]
\[ a_3 = 2^2 + 1 = 5, \]
\[ a_4 = 5^2 + 1 = 26, \]
\[ a_5 = 26^2 + 1 = 677 \]

\[ a_1 = 2, a_2 = 2, a_n = a_{n-2} - a_{n-1} \]

\[ a_1 = 2, \]
\[ a_2 = 2, \]
\[ a_3 = 2 - 2 = 0, \]
\[ a_4 = 2 - 0 = 2, \]
\[ a_5 = 0 - 2 = -2 \]

\[ a_1 = 1, a_2 = 1^2 + 1 = 2, a_3 = 2^2 + 1 = 5, a_4 = 5^2 + 1 = 26, a_5 = 26^2 + 1 = 677 \]

\[ a_1 = 2, a_2 = 2, a_3 = 2 - 2 = 0, a_4 = 2 - 0 = 2, a_5 = 0 - 2 = -2 \]
Write the rules for the arithmetic sequence where \( a_1 = 15 \) and \( d = 5 \).

**Explicit**
- \( a_n = a_1 + (n - 1)d \)
- \( a_n = 15 + (n - 1)5 \)
- \( a_n = 5n + 10 \)

**Recursive**
- \( a_1 = 15, a_n = a_{n-1} + 5 \)

Explicit: \( a_n = 15 + (n-1)5 \) \( \rightarrow \) \( a_n = 5n + 10 \)

Recursive: \( a_1 = 15, a_n = a_{n-1} + 5 \)
Write the rule for the geometric sequence where \( a_1 = 4 \) and \( r = 0.2 \)

**Explicit**
- \( a_n = a_1 r^{n-1} \)
- \( a_n = 4(0.2)^{n-1} \)

**Recursive**
- \( a_1 = 4, a_n = 0.2a_{n-1} \)

Explicit: \( a_n = 4(0.2)^{n-1} \)

Recursive: \( a_1 = 4, a_n = 0.2a_{n-1} \)
Write a recursive rule for

1, 1, 4, 10, 28, 76, ...

\[ a_n = 2(a_{n-2} + a_{n-1}) \]
\[ a_1 = 1, a_2 = 1 \]

1, 2, 2, 4, 8, 32, ...

\[ a_n = (a_{n-2})(a_{n-1}) \]
\[ a_1 = 1, a_2 = 2 \]
12.5 Homework Quiz