LINEAR EQUATIONS AND FUNCTIONS

gebra 2 nanter 2

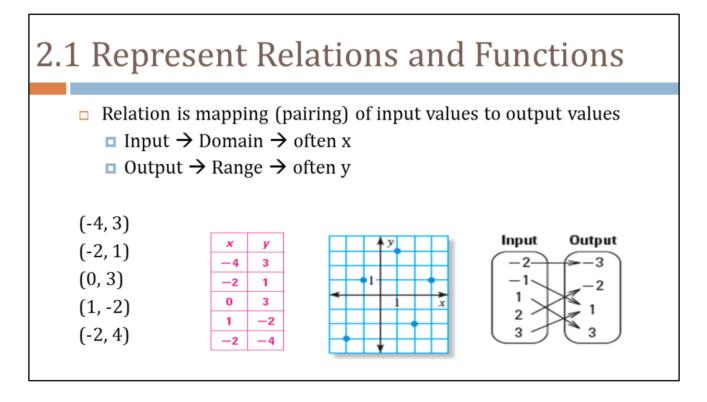
Algebra II 2

This Slideshow was developed to accompany the textbook

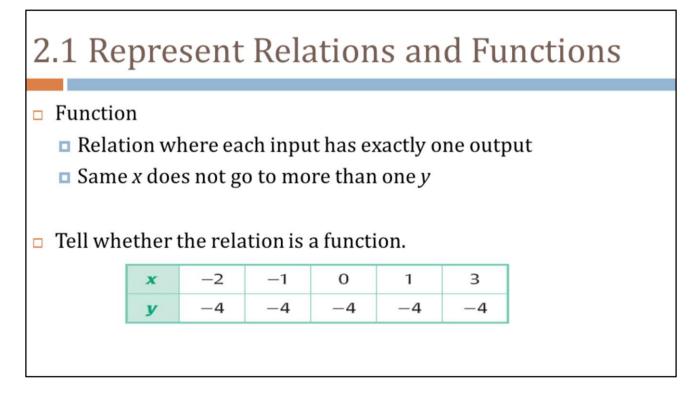
- Larson Algebra 2
- By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.
- 2011 Holt McDougal

Some examples and diagrams are taken from the textbook.

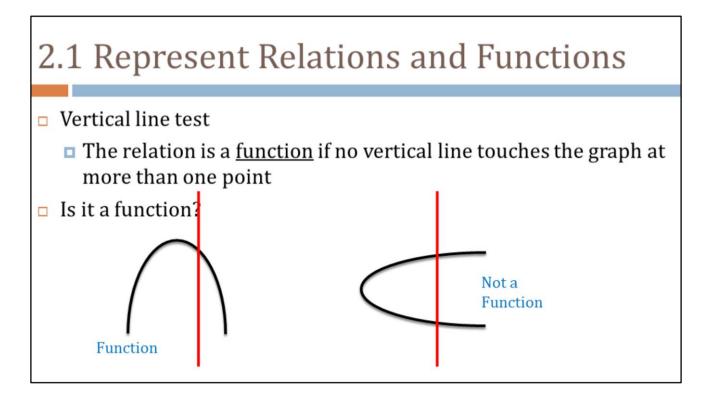
Slides created by Richard Wright, Andrews Academy <u>rwright@andrews.edu</u>



Ask domain and range questions for the relations



Yes, each x goes to only one y



First is a function Second is <u>NOT</u> a function

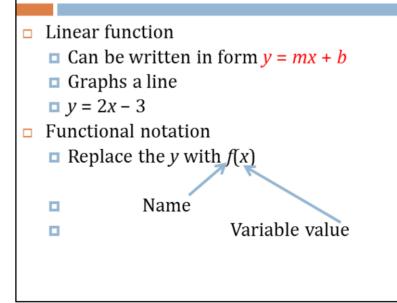
2.1 Represent Relations and Functions

- Equation in two variables
 - □ Input → usually $x \rightarrow$ independent variable
 - Output \rightarrow usually $y \rightarrow$ dependent variable
 - Solution → ordered pair (x, y) that gives a true statement
- To graph
 - Make a table of values by choosing x and calculating y
 - Plot enough points to see the pattern
 - Connect the points with a line or curve

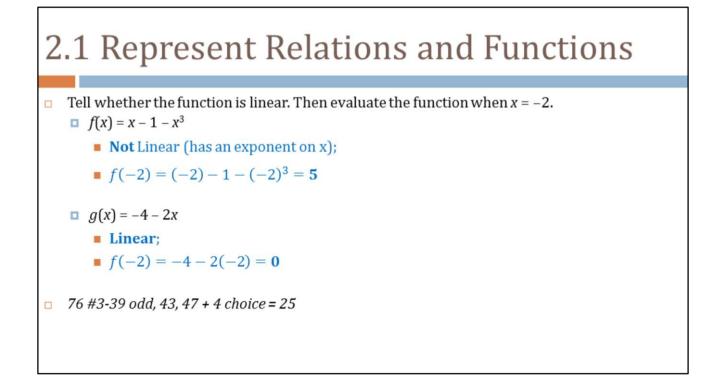
Make sure the graph actually goes through the points it should go through

2.1 Represent Relations and Functions				
□ Graph the equation $y = 3x - 2$				
x -3 -2 -1 0 1 2 3	y -11 -8 -5 -2 1 4 7			

2.1 Represent Relations and Functions



Point out that functions can be named more than just f



Not Linear (has an exponent on x);

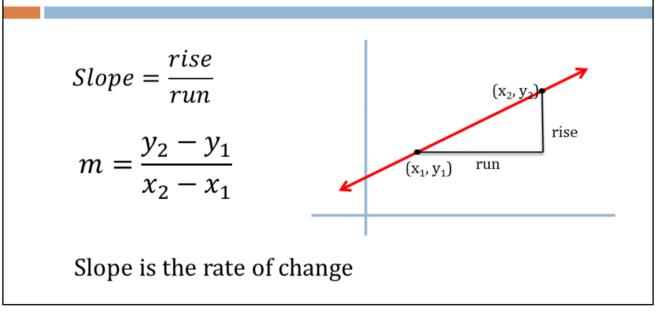
$$f(-2) = (-2) - 1 - (-2)^3 = 5$$

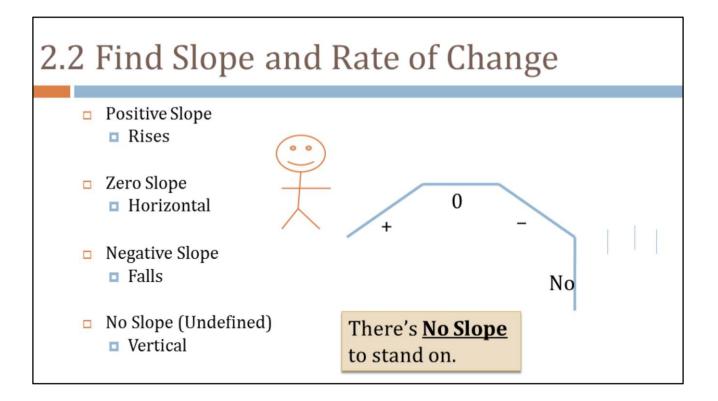
Linear;

$$f(-2) = -4 - 2(-2) = 0$$

Homework Quiz

<u>2.1 Homework Quiz</u>





□ Find the slope of the line passing through the given points. Classify as *rises, falls, horizontal,* or *vertical*.

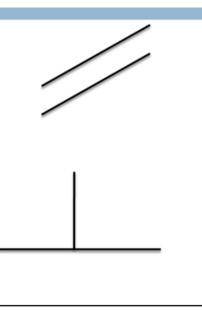
• (0, 3), (4, 8)
•
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 3}{4 - 0} = \frac{5}{4}$$
; rises
• (7, 3), (-1, 7)
• $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 3}{-1 - 7} = \frac{4}{-8} = -\frac{1}{2}$; falls
• (7, 1), (7, -1)
• $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 1}{7 - 7} = -\frac{2}{0}$ = undefined; vertical
• $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 1}{7 - 7} = -\frac{2}{0}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 3}{4 - 0} = \frac{5}{4}; \text{ rises}$$
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 3}{-1 - 7} = \frac{4}{-8} = -\frac{1}{2}; \text{ falls}$$
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 1}{7 - 7} = -\frac{2}{0} = \text{ undefined}; \text{ vertical}$$



- In the same plane and do not intersect
- Go the same direction
- Slopes are the same
- Perpendicular Lines
 - Intersect to form a right angle
 - Slopes are negative reciprocals
 - OR Product of slopes is -1

$$\square \frac{2}{3}$$
 and $-\frac{3}{2}$



Tell whether the lines are *parallel*, *perpendicular*, or *neither*.

- □ Line 1: through (-2, 8) and (2, -4)
- □ Line 2: through (–5, 1) and (–2, 2)

Line 1:
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 8}{2 - (-2)} = -\frac{12}{4} = -3$$

Line 2: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{-2 - (-5)} = \frac{1}{3}$
Perpendicular

Line 1:
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 8}{2 - (-2)} = -\frac{12}{4} = -3$$

Line 2: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{-2 - (-5)} = \frac{1}{3}$
Perpendicular

Tell whether the lines are *parallel*, *perpendicular*, or *neither*.

- □ Line 1: through (-4, -2) and (1, 7)
- □ Line 2: through (−1, −4) and (3, 5)

Line 1:
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-2)}{1 - (-4)} = \frac{9}{5}$$

Line 2: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-4)}{3 - (-1)} = \frac{9}{4}$
Neither

Line 1:
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-2)}{1 - (-4)} = \frac{9}{5}$$

Line 2: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-4)}{3 - (-1)} = \frac{9}{4}$
Neither

Set #3-23 every other odd, 25-35 odd, 39, 43, 47 + 5 choice = 20 In 1983, 87% of New Hampshire was forested. By 2001, that percent had fallen to 81.1%. What is the average rate of change of forested land? Then predict what percentage will be forested in 2005. x = time in years y = percent Points are (1983, 87) and (2001, 81.1) m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{81.1 - 87}{2001 - 1983} = -\frac{5.9}{18} = -0.3278 Start + 4(-0.3278) = 79.8%

x = time in years y = percent Points are (1983, 87) and (2001, 81.1) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{81.1 - 87}{2001 - 1983} = -\frac{5.9}{18} = -0.3278$

To get the percent for 2005, take the amount from 2001 and add 4 times the slope to get four more years.

81.1 + 4(-0.3278) = 79.8%

Homework Quiz

<u>2.2 Homework Quiz</u>

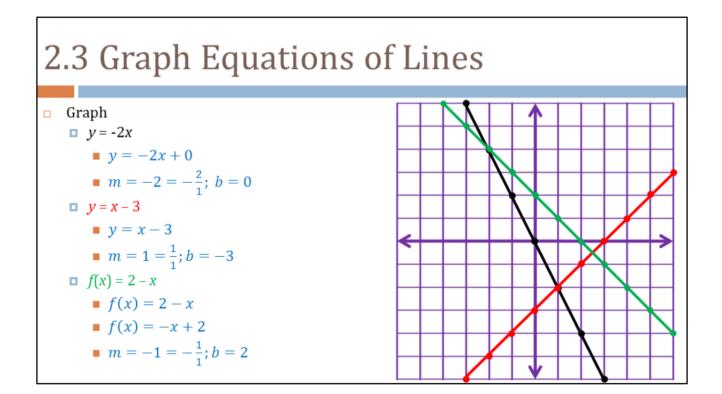
Slope-intercept form

 $\Box y = mx + b$

- m is slope
- b is y-intercept

- To graph
 - Solve equation for *y*
 - Plot the y-intercept
 - From there move up and over the slope to find another couple of points
 - Draw a line neatly through the points

y-intercept is where the line crosses the y-axis



$$y = -2x + 0$$

$$m = -2 = -\frac{2}{1}; b = 0$$

$$y = x - 3$$

$$m = 1 = \frac{1}{1}; b = -3$$

$$f(x) = 2 - x$$

$$f(x) = -x + 2$$

$$m = -1 = -\frac{1}{1}; b = 2$$

- Standard Form
 - Ax + By = C
 - A, B, and C are integers
- To graph
 - Find the x- and y-intercepts by letting the other variable = 0
 - Plot the two points
 - Draw a line through the two points

- *x*-intercept:
- $\Box Ax + B(0) = C$

$$\Box$$
 Ax = C

$$\Box \quad x = \frac{C}{A}$$

- □ *y*-intercept:
- $\Box A(0) + By = C$
- \square By = C

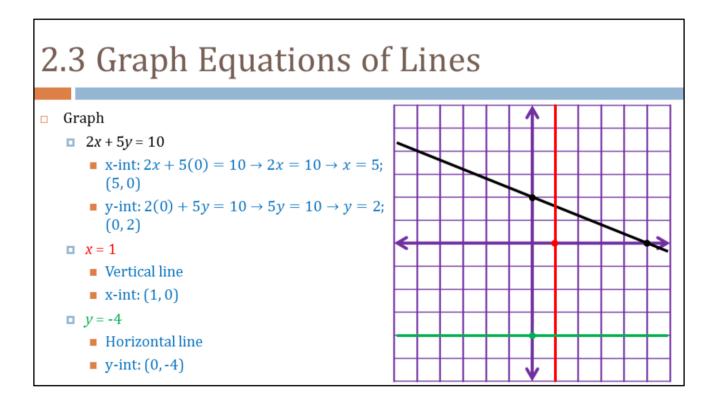
$$\Box \quad y = \frac{C}{B}$$

Horizontal Lines

 $\Box y = c$

Vertical Lines

 $\square x = c$



x-int: $2x + 5(0) = 10 \rightarrow 2x = 10 \rightarrow x = 5$; (5, 0) y-int: $2(0) + 5y = 10 \rightarrow 5y = 10 \rightarrow y = 2$; (0, 2)

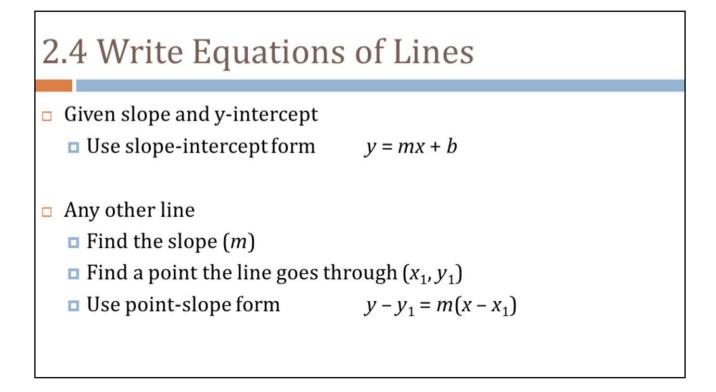
Vertical line x-int: (1, 0)

Horizontal line y-int: (0, -4)

93 #1-69 every other odd + 2 choice = 20

Homework Quiz

<u>2.3 Homework Quiz</u>



2.4 Write Equations of Lines Write the equation of the line given... m = -2, b = -4Given slope and yintercept y = mx + b y = -2x - 4 y = 4x + 10y = 4x + 10

Given slope and y-intercept

$$y = mx + b$$
$$y = -2x - 4$$

Given slope and point

$$y - y_1 = m(x - x_1)$$

y - 6 = 4(x - (-1))
y - 6 = 4x + 4
y = 4x + 10

Write the equation of the line given...
it passes through (-1, 2) and (10, 0)

$$m = \frac{y_2 - y_2}{x_2 - x_1} = \frac{0 - 2}{10 - (-1)} = -\frac{2}{11}$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{2}{11}(x - 10)$$

$$y = -\frac{2}{11}x + \frac{20}{11}$$

$$m = \frac{y_2 - y_2}{x_2 - x_1} = \frac{0 - 2}{10 - (-1)} = -\frac{2}{11}$$
$$y - y_1 = m(x - x_1)$$
$$y - 0 = -\frac{2}{11}(x - 10)$$
$$y = -\frac{2}{11}x + \frac{20}{11}$$

Write an equation of the line that passes through (4, -2) and is (a) parallel to, and (b) perpendicular to, the line y = 3x - 1.

a) m = 3; Parallel lines have same slope

 $y - y_1 = m(x - x_1)$ y - (-2) = 3(x - 4) y + 2 = 3x - 12y = 3x - 14

b) $m = -\frac{1}{2}$; Perpendicular lines have negative reciprocal slopes

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{1}{3}(x - 4)$$
$$y + 2 = -\frac{1}{3}x + \frac{4}{3}$$
$$y = -\frac{1}{3}x - \frac{2}{3}$$

a) m = 3; Parallel lines have same slope $y - y_1 = m(x - x_1)$ y - (-2) = 3(x - 4) y + 2 = 3x - 12y = 3x - 14

b) $m = -\frac{1}{3}$; Perpendicular lines have negative reciprocal slopes

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -\frac{1}{3}(x - 4)$$

$$y + 2 = -\frac{1}{3}x + \frac{4}{3}$$

$$y = -\frac{1}{3}x - \frac{2}{3}$$

- A certain farmer can harvest 44000 bushels of crops in a season. Corn averages 155 bushels per acre and soybeans average 44 bushels per acre in Michigan in 2013. Write an equation that models this situation.
- The "per" means this rate problem.
 Rate × amount = total
- \Box 155*x* + 44*y* = 44000



The "per" means this rate problem. Rate \times amount = total 155x + 44y = 44000

- □ In a chemistry experiment, you record the temperature to be -5 °F one minute after you begin. Six minutes after you begin the temperature is 20 °F. Write a linear equation to model this.
 - Two points: (1, -5), (6, 20)
 - $m = \frac{y_2 y_1}{x_2 x_1} = \frac{20 (-5)}{6 1} = \frac{25}{5} = 5$
 - $y y_1 = m(x x_1)$
 - y (-5) = 5(x 1)
 - y + 5 = 5x 5
 - y = 5x 10
- 101 #1-57 every other odd + 5 choice = 20

Two points: (1, -5), (6, 20)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - (-5)}{6 - 1} = \frac{25}{5} = 5$$
$$y - y_1 = m(x - x_1)$$
$$y - (-5) = 5(x - 1)$$
$$y + 5 = 5x - 5$$
$$y = 5x - 10$$

Homework Quiz

<u>2.4 Homework Quiz</u>

2.5 Model Direct Variation

- Direct Variation
 - y = ax can be used to model the situation
 - a = constant of variation (slope)
- Write and graph a direct variation equation that has the given ordered pair as a solution.
 (6,-2)
 - y = ax-2 = a(6)a = $-\frac{1}{3}$ y = $-\frac{1}{3}x$

Slope is $\left(-\frac{2}{6}\right) = -\frac{1}{3}$ $y = -\frac{1}{3}x$

2.5 Model Direct Variation

- Hooke's Law states that the distance d a spring stretches varies directly with the force f that is
 applied to it.
 - Suppose a spring stretches 15 in. when a force of 9 lbs. is applied. Write an equation to relate *d* to *f*.
 - Hooke's law: d = af

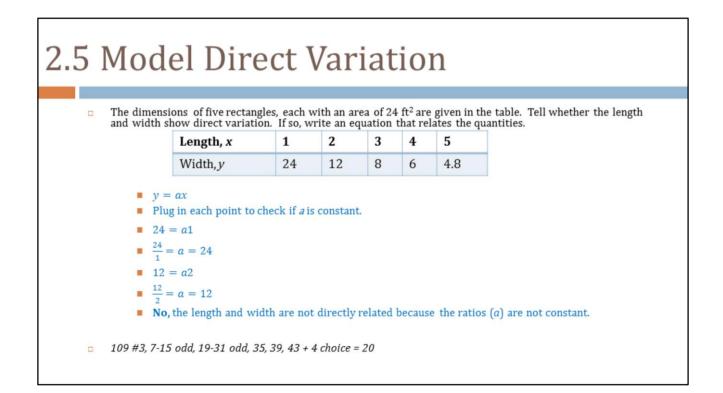
■
$$15 = a(9) \rightarrow a = \frac{15}{9} = \frac{5}{3}$$

- $d = \frac{5}{3}f$
- Predict the distance that the spring will stretch when a force of 6 lbs. is applied.

a $d = \left(\frac{5}{3}\right)6 = 10$ in.

Hooke's law: d = af 15 = a(9) → a = 15/9 = 5/3 d = 5/3 f

d = (5/3) 6 = 10 in.



y = ax

Plug in each point to check if *a* is constant.

$$24 = a1$$

$$\frac{24}{1} = a = 24$$

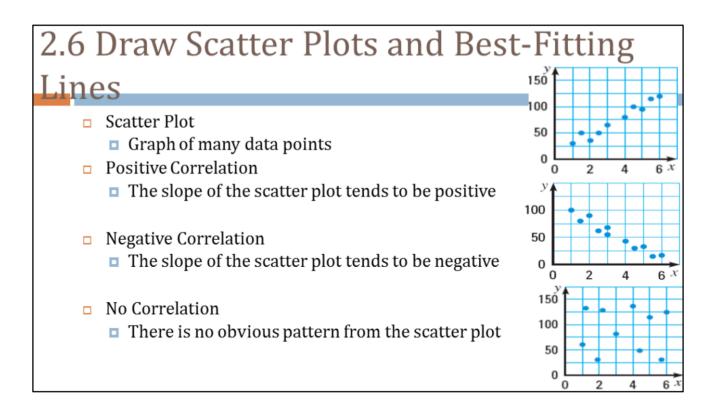
$$12 = a2$$

$$\frac{12}{2} = a = 12$$

No, the length and width are not directly related because the ratios (a) are not constant.

Homework Quiz

<u>2.5 Homework Quiz</u>

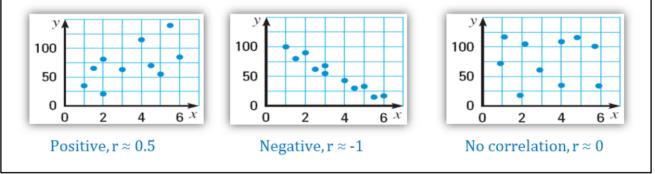


2.6 Draw Scatter Plots and Best-Fitting Lines

- □ Correlation Coefficient (*r*)
 - Number between -1 and 1 that measures how well the data fits a line.
 - Positive for positive correlation, negative for negative
 - □ *r* = 0 means there is no correlation

2.6 Draw Scatter Plots and Best-Fitting Lines

For each scatter plot, (a) tell whether the data have a *positive correlation*, a *negative correlation*, or *approximately no correlation*, and (b) tell whether the correlation coefficient is closest to -1, -0.5, 0, 0.5, or 1.



Positive, $r \approx 0.5$

Negative, $r \approx -1$

No correlation, $r \approx 0$

2.6 Draw Scatter Plots and Best-Fitting

- Best-fitting line
 - Line that most closely approximates the data
- Find the best-fitting line
 - 1. Draw a scatter plot of the data
 - 2. Sketch the line that appears to follow the data the closest
 - There should be about as many points below the line as above
 - 3. Choose two points on the line and find the equation of the line
 - These do not have to be original data points

See example 5 in the textbook to see how to do this on a TI graphing calculator

2.6 Draw Scatter Plots and I									
Lines Contraction Contraction									
x		0	1	2	3	4	5	6	7
у		16.5	11.4	12.5	4.7	9.9	7.1	2.9	1.7
		Monarch shows the by Monar spend wi years afte	e area in l [.] ch Butter nter, <i>y</i> , in	Mexico us flies to		 Approximate the best-fitting line for the data. Use your equation from part (a) to predict the area used by the butterflies in 2016. 			

Sample Answer: y = -1.89x + 14.97

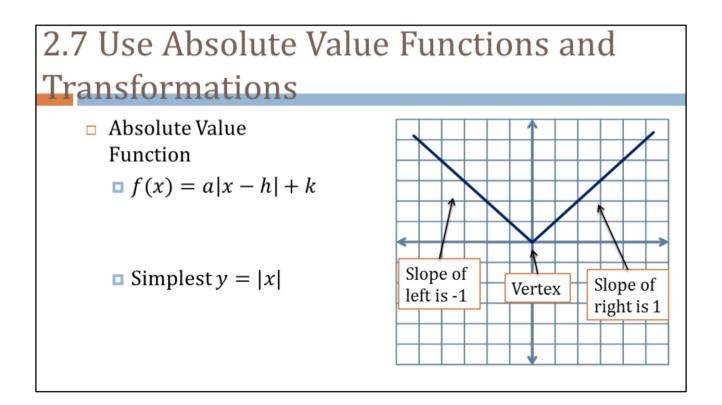
Sample Answer: y = -1.89(10) + 14.97 = -3.93 acres (they would be gone, extinct!)

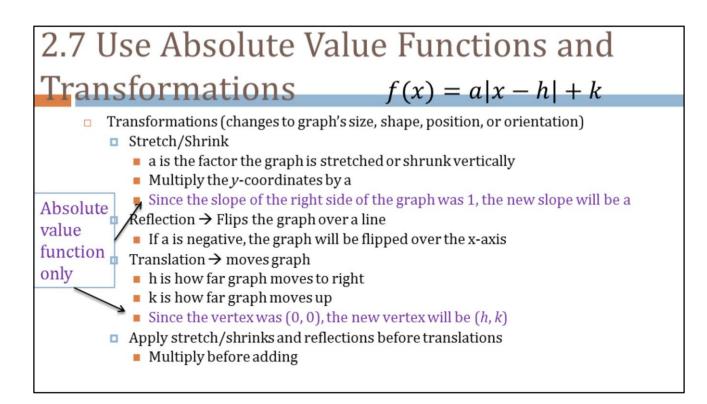
2.6 Draw Scatter Plots and Best-Fitting Lines

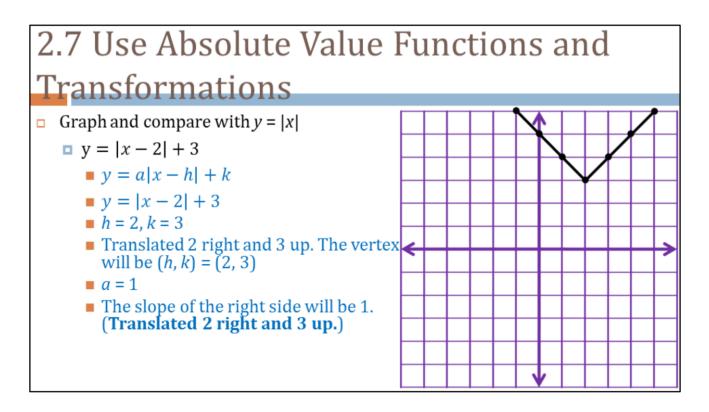
□ 117 #1-29 odd + 5 choice = 20

Homework Quiz

<u>2.6 Homework Quiz</u>







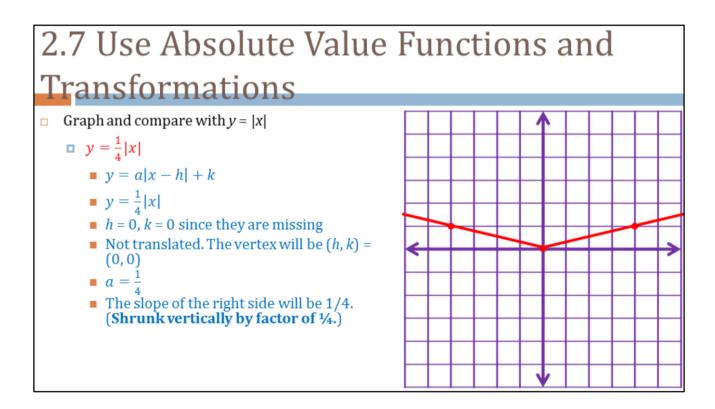
$$y = a|x - h| + k$$
$$y = |x - 2| + 3$$

h = 2, k = 3

Translated 2 right and 3 up. The vertex will be (h, k) = (2, 3)

a = 1

The slope of the right side will be 1. (Translated 2 right and 3 up.)



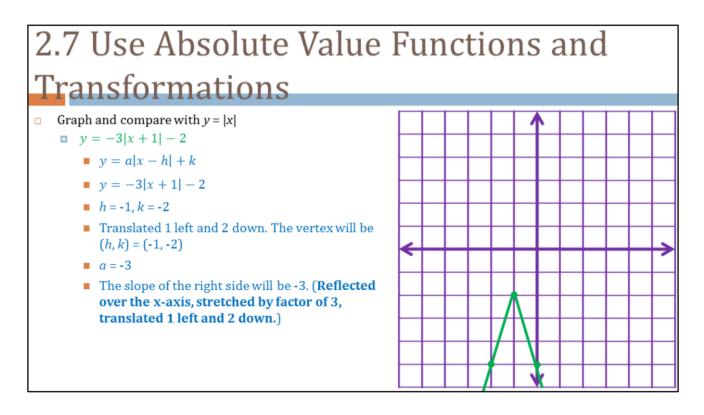
$$y = a|x - h| + k$$
$$y = \frac{1}{4}|x|$$

h = 0, k = 0 since they are missing

Not translated. The vertex will be (h, k) = (0, 0)

$$a = \frac{1}{4}$$

The slope of the right side will be 1/4. (Shrunk vertically by factor of ¼.)



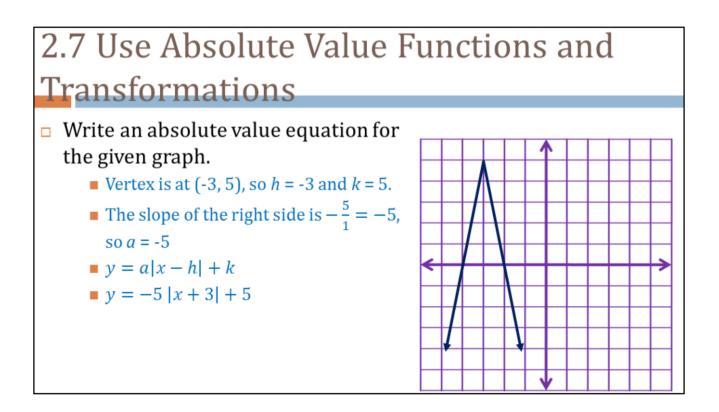
$$y = a|x - h| + k$$
$$y = -3|x + 1| - 2$$

h = -1, k = -2

Translated 1 left and 2 down. The vertex will be (h, k) = (-1, -2)

a = -3

The slope of the right side will be -3. (**Reflected over the x-axis, stretched by factor of 3, translated 1 left and 2 down.**)



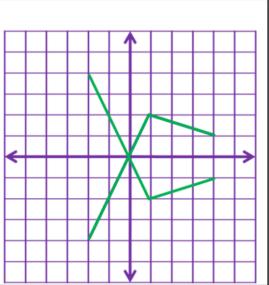
Vertex is at (-3, 5), so h = -3 and k = 5. The slope of the right side is $-\frac{5}{1} = -5$, so a = -5y = a|x - h| + ky = -5 |x + 3| + 5

2.7 Use Absolute Value Functions and Transformations

□ The graph of f(x) is given. Sketch the following functions.

$$y = -\frac{1}{2}f(x)$$

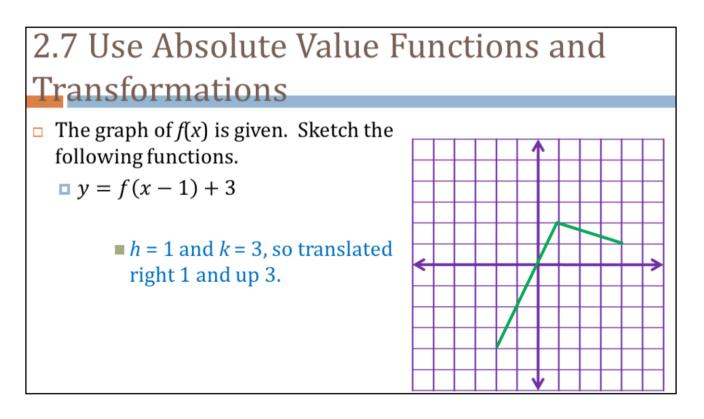
- Reflected over the x-axis because of the -, shrunk vertically by factor of ½ because of the ½.
- Reflect the graph over the *x*-axis first.
- Make the distance from each point to the *x*-axis half the distance.



Reflected over the x-axis because of the -, shrunk vertically by factor of $\frac{1}{2}$ because of the $\frac{1}{2}$.

Reflect the graph over the *x*-axis first.

Make the distance from each point to the *x*-axis half the distance.



h = 1 and k = 3, so translated right 1 and up 3.

2.7 Use Absolute Value Functions and Transformations

□ 127 #3-37 odd + 2 choice = 20

Homework Quiz

<u>2.7 Homework Quiz</u>

- Linear Inequality in two variables
 - Like linear equation, but with inequality instead of =
- Tell whether the given ordered pair is a solution of $5x 2y \le 6$
 - **(**0, -4)
 - $5x 2y \le 6$
 - $5(0) 2(-4) \le 6$
 - 8 ≤ 6
 - Not true, so **not** a solution

- (-3, 8)
 - $5x 2y \le 6$
 - $5(-3) 2(8) \le 6$
 - -15 16 ≤ 6
 - -31 ≤ 6
 - True, so it **is** a solution

 $5x - 2y \le 6$ $5(0) - 2(-4) \le 6$ $8 \le 6$

Not true, so not a solution

$$5x - 2y \le 6$$

$$5(-3) - 2(8) \le 6$$

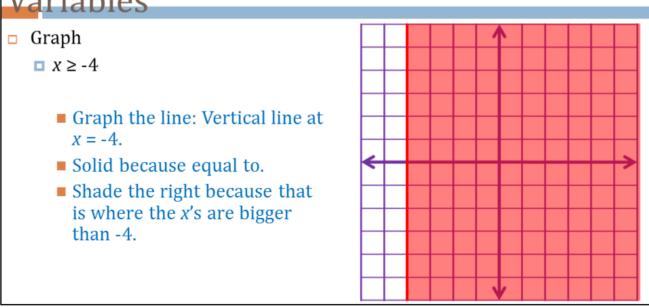
$$-15 - 16 \le 6$$

$$-31 \le 6$$

True, so it is a solution

- Graphing a linear inequality
 - Graph the line as if it was =
 - Dotted or Solid line
 - Dotted if <, >
 - Solid if \leq , =, \geq
 - Shade
 - Test a point not on the line
 - If the point is a solution, shade that side of the line
 - If the point is not a solution, shade the other side of the line

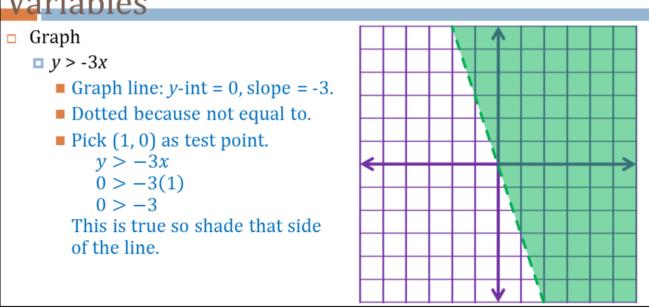




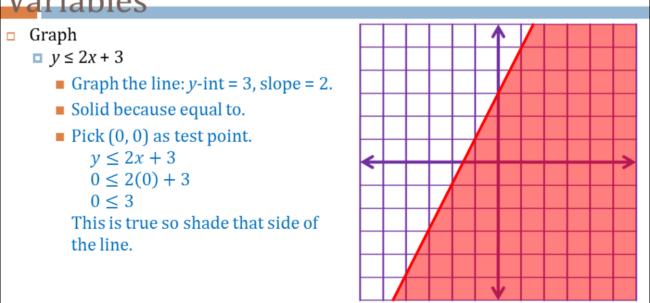
Graph the line: Vertical line at x = -4.

Solid because equal to.

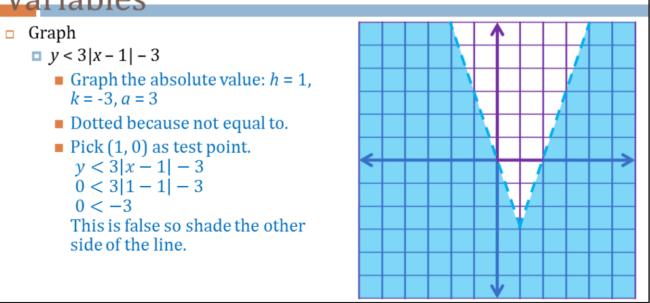
Shade the right because that is where the x's are bigger than -4.



Graph line: y-int = 0, slope = -3. Dotted because not equal to. Pick (1, 0) as test point. $y > -3x \rightarrow 0 > -3(1) \rightarrow 0 > -3$. This is true so shade that side of the line.



Graph the line: *y*-int = 3, slope = 2. Solid because equal to. Pick (0, 0) as test point. $y \le 2x + 3 \rightarrow 0 \le 2(0) + 3 \rightarrow 0 \le 3$. This is true so shade that side of the line.



Graph the absolute value: h = 1, k = -3, a = 3Dotted because not equal to.

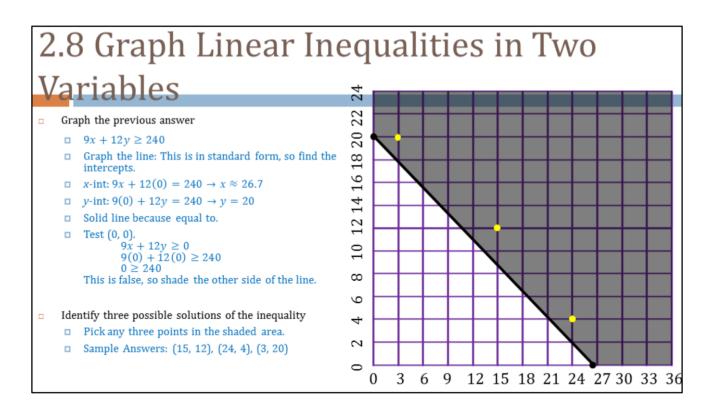
Pick (1, 0) as test point. $y < 3|x - 1| - 3 \rightarrow 0 < 3|1 - 1| - 3 \rightarrow 0 < -3$. This is false so shade the other side of the line.

- You have two part-time summer jobs, one that pays \$9 an hour and another that pays \$12 an hour. You would like to earn at least \$240 a week. Write an inequality describing the possible amounts of time you can schedule at both jobs.
 - Rate problem: rate × amount = total
 - $9x + 12y \ge 240$
 - Greater than sign because the 240 is the smallest we want, so the small side of the sign is pointed at 240.

Rate problem: rate × amount = total

$$9x + 12y \ge 240$$

Greater than sign because the 240 is the smallest we want, so the small side of the sign is pointed at 240.



$$9x + 12y \ge 240$$

Graph the line: This is in standard form, so find the intercepts.

x-int: $9x + 12(0) = 240 \rightarrow x \approx 26.7$ *y*-int: $9(0) + 12y = 240 \rightarrow y = 20$ Solid line because equal to. Test (0, 0). $9x + 12y \ge 0 \rightarrow 9(0) + 12(0) \ge 240 \rightarrow 0 \ge 240$. This is false, so shade the other side of the line.

Pick any three points in the shaded area. Sample Answers: (15, 12), (24, 4), (3, 20)

135 #5-37 odd, 41, 43 + 1 choice = 20

Homework Quiz

<u>2.8 Homework Quiz</u>

2.Review

145 #20 choice = 20