

LINEAR SYSTEMS AND MATRICES

Algebra 2

Chapter 3

Algebra II 3

- This Slideshow was developed to accompany the textbook
 - *Larson Algebra 2*
 - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
 - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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3.1 SOLVE LINEAR SYSTEMS BY GRAPHING

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3.1 SOLVE LINEAR SYSTEMS BY GRAPHING

- System of equations
 - More than one equation that share the same solution.
 - Often they involve more than one variable.
 - In order to solve them, you need as many equations as there are variables.

$$2x + 3y = 6$$

$$3x - 4y = 5$$

3.1 SOLVE LINEAR SYSTEMS BY GRAPHING

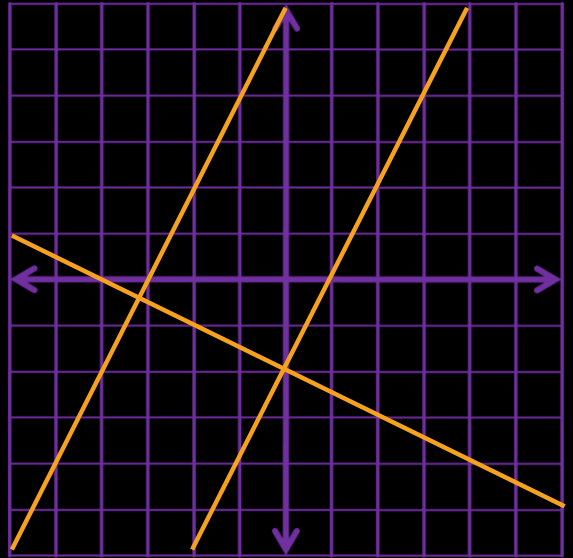
- Solutions to systems
 - An ordered pair that works in both equations.
 - If the ordered pair works in both equations, then both graphs have to go through that point.
 - Solutions are where the graphs cross.

3.1 SOLVE LINEAR SYSTEMS BY GRAPHING

- Solve by graphing
 - Graph both equations on the same graph.
 - Where they cross is the solution.

3.1 SOLVE LINEAR SYSTEMS BY GRAPHING

- **Classifying Solutions**
 - Many Solutions \rightarrow consistent (has a solution), dependant
 - One solution \rightarrow consistent, independent
 - No solution \rightarrow inconsistent



3.1 SOLVE LINEAR SYSTEMS BY GRAPHING

- Solve by graphing. Classify as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

- $3x + 2y = -4$
 $x + 3y = 1$

- Solve the equations for y .

$$3x + 2y = -4$$

$$2y = -3x - 4$$

$$y = -\frac{3}{2}x - 2$$

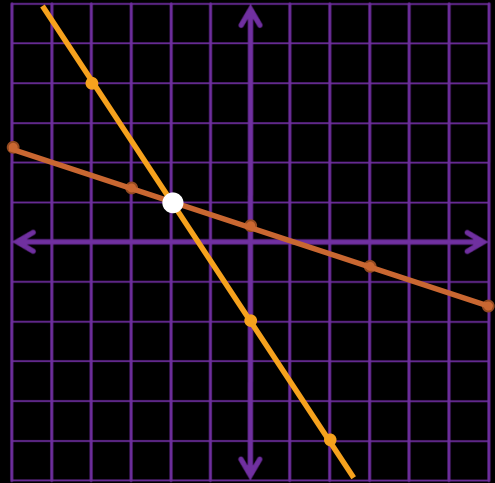
- The y -intercept is -2 and the slope is $-\frac{3}{2}$.

$$x + 3y = 1$$

$$3y = -x + 1$$

$$y = -\frac{1}{3}x + \frac{1}{3}$$

- The y -intercept is $\frac{1}{3}$ and the slope is $-\frac{1}{3}$.
- Where the lines intersect is the solution.
- $(-2, 1)$; Consistent and independent



3.1 SOLVE LINEAR SYSTEMS BY GRAPHING

- Solve by graphing. Classify as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

- $3x - 2y = 10$

- $3x - 2y = 2$

- Solve both equations for y .

$$\begin{aligned}3x - 2y &= 10 \\ -2y &= -3x + 10 \\ y &= \frac{3}{2}x - 5\end{aligned}$$

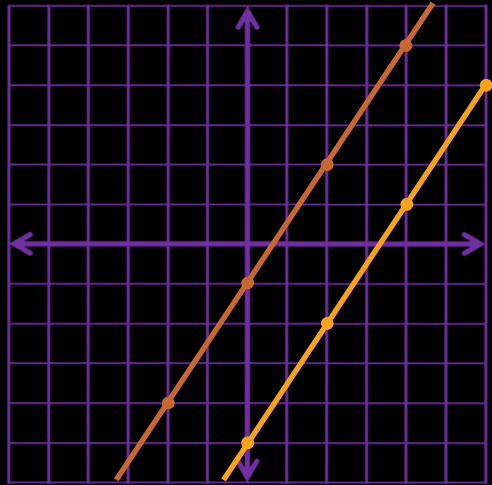
- The y -intercept is -5 and the slope is $\frac{3}{2}$.

$$\begin{aligned}3x - 2y &= 2 \\ -2y &= -3x + 2 \\ y &= \frac{3}{2}x - 1\end{aligned}$$

- The y -intercept is -1 and the slope is $\frac{3}{2}$.

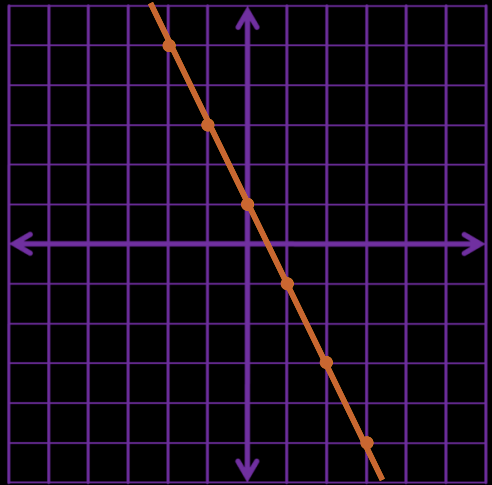
- Where the lines intersect is the solution.

- Since they are parallel there is **no solution**. Inconsistent



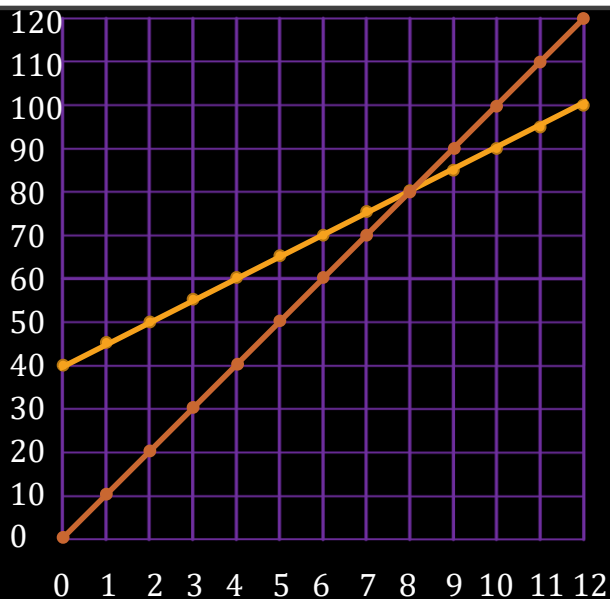
3.1 SOLVE LINEAR SYSTEMS BY GRAPHING

- Solve by graphing. Classify as *consistent and independent*, *consistent and dependent*, or *inconsistent*.
- $2x + y = 1$
- $-4x - 2y = -2$
- Solve both equations for y .
 - $2x + y = 1$
 - $y = -2x + 1$
- The y -intercept is 1 and the slope is -2 .
 - $-4x - 2y = -2$
 - $-2y = 4x - 2$
 - $y = -2x + 1$
- The y -intercept is 1 and the slope is -2 .
- Where the lines intersect is the solution.
- Since they are the same line there are **infinitely many solutions**. Consistent and dependent



3.1 SOLVE LINEAR SYSTEMS BY GRAPHING

- A soccer league offers two options for membership plans. A) \$40 initial fee and \$5 for each game played. B) \$10 for each game played. How many games must you play for both plans to be the same?
- Write an equation for each plan.
- Plan A: Cost is \$40 plus \$5 for each game. $y = 5x + 40$
- Plan B: Cost is \$10 for each game. $y = 10x$
- Where the lines intersect is the solution. (8, 80)
- You must play 8 games for both plans to be the same cost.



HOMEWORK QUIZ

- [3.1 Homework Quiz](#)

3.2 SOLVE LINEAR SYSTEMS ALGEBRAICALLY

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3.2 SOLVE LINEAR SYSTEMS ALGEBRAICALLY

- Graphing to solve systems of equations has some problems.
- Can you guess some?
 - *Inaccurate*
 - *Sometimes hard to graph*

3.2 SOLVE LINEAR SYSTEMS ALGEBRAICALLY

- Substitution

1. Solve one equation for one variable
2. Use that expression to replace that variable in the other equation
3. Solve the new equation
4. Substitute back into the first equation
5. Solve for the second variable

3.2 SOLVE LINEAR SYSTEMS ALGEBRAICALLY

$$y = x + 2$$
$$2x + y = 8$$

y is the same thing
as $x + 2$

$$y = 2 + 2$$
$$y = 4$$

$$2x + x + 2 = 8$$

$$3x + 2 = 8$$

$$3x = 6$$

$$x = 2$$

(2,4)

3.2 SOLVE LINEAR SYSTEMS ALGEBRAICALLY

$$3x + 2y = 8$$

$$x + 4y = -4$$

$$x = -4(-2) - 4$$

$$x = 4$$

$$x = -4$$

$$3(-4y - 4) + 2y = 8$$

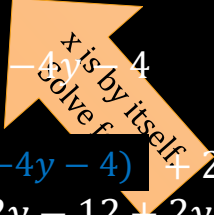
$$-12y - 12 + 2y = 8$$

$$-10y - 12 = 8$$

$$-10y = 20$$

$$y = -2$$

$(4, -2)$



3.2 SOLVE LINEAR SYSTEMS ALGEBRAICALLY

- Elimination

1. Line up the equations into columns
2. Multiply one or both equations by numbers so that one variable has the same coefficient, but opposite sign
3. **Add** the equations
4. Solve the resulting equation
5. Substitute the value into one original equation and solve

3.2 SOLVE LINEAR SYSTEMS ALGEBRAICALLY

Substitute into one of the original equations.

$$2x - 3y = -14$$

$$(-3)(3x - y) = -7(-5)$$

$$2x - 3y =$$

Add

$$\underline{-9x + 3y = 21}$$

$$-7x = 7$$

$$x = -1$$

$$2(-1) - 3y = -14$$

$$-2 - 3y = -14$$

$$-3y = -12$$

$$y = 4$$

$$(-1, 4)$$

Multiplying $-y$ by -3 will give us $3y$. We will have $-3y$ and $3y$.

3.2 SOLVE LINEAR SYSTEMS ALGEBRAICALLY

Substitute into one of the original equations.

$$(2)(3x + 11y) = 4(2)$$

$$(3)(-2x - 6y) = 0(3)$$

$$-2x - 6(2) = 0$$

$$-2x - 12 = 0$$

$$-2x = 12$$

$$x = -6$$

$$\begin{array}{r} 6x + 22y = 8 \\ \text{Add } -6x - 12y = 0 \\ \hline \end{array}$$

Multiplying the top by 2 and bottom by 3 will give $6x$ and $-6x$.

$$y = 2$$

$$(-6, 2)$$

3.2 SOLVE LINEAR SYSTEMS ALGEBRAICALLY

- Number of Solutions
 - If both variables disappear after you substitute or combine and
 - You get a true statement like $2 = 2 \rightarrow$ infinite solutions
 - You get a false statement like $2 = 5 \rightarrow$ no solution

3.2 SOLVE LINEAR SYSTEMS ALGEBRAICALLY

- Summary of Solving Techniques
 - When to graph?
 - To get general picture and estimate
 - When to use substitution?
 - When one of the coefficients is 1
 - When to use elimination?
 - When none of the coefficients is 1

HOMEWORK QUIZ

- [3.2 Homework Quiz](#)

3.3 GRAPH SYSTEMS OF LINEAR INEQUALITIES

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3.3 GRAPH SYSTEMS OF LINEAR INEQUALITIES

- To solve systems of inequalities, graph them all on one graph.
- Solution is where all graphs overlap

- Graphing linear inequalities was taught in lesson 2.8

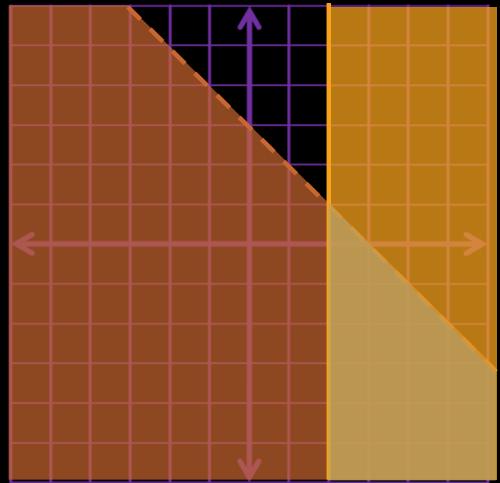
3.3 GRAPH SYSTEMS OF LINEAR INEQUALITIES

- Solve the system of inequalities

$$x \geq 2$$

$$x + y < 3$$

- Graph both inequalities.
- Where the shaded areas overlap is the solution.



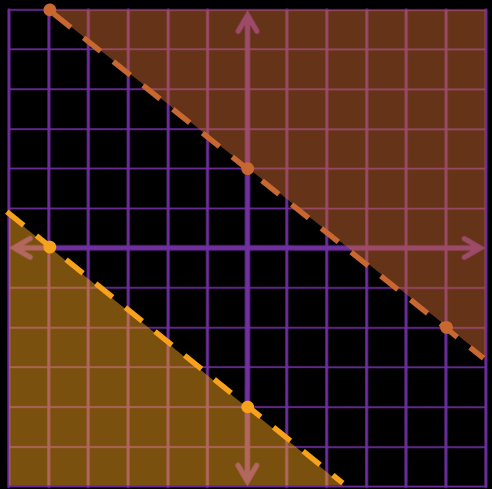
3.3 GRAPH SYSTEMS OF LINEAR INEQUALITIES

- Solve the system of inequalities

$$y < -\frac{4x}{5} - 4$$

$$y > -\frac{4x}{5} + 2$$

- Graph both inequalities.
- Where the shaded areas overlap is the solution.
- No overlap means **No Solution**.



3.3 GRAPH SYSTEMS OF LINEAR INEQUALITIES

- Solve the system of

$$y \leq 3$$

$$0 \leq x \leq 5$$

$$x > -y$$

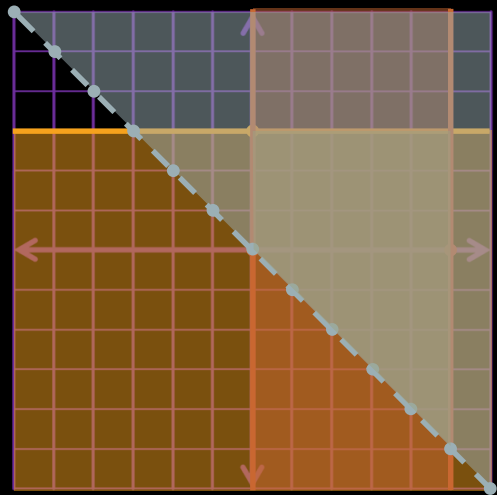
- Graph all inequalities.

$$x > -y$$

$$-x < y$$

$$y > -x$$

- Where the shaded areas overlap is the solution.



HOMEWORK QUIZ

- [3.3 Homework Quiz](#)

3.4 SOLVE SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES

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3.4 SOLVE SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES

- We have now worked with 2 variables and 2 dimensions, but sometimes there are more

dimensions.

- Linear equation in 3 variables graphs a plane

3.4 SOLVE SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES

- Solution to system in 3 variables

- Ordered triple (x, y, z)

- Example: Is $(2, -4, 1)$ a solution of

$$x + 3y - z = -11$$

- $2x + y + z = 1$

$$5x - 2y + 3z = 21$$

- Plug it in.

$$x + 3y - z = -11$$

$$2 + 3(-4) - 1 = -11$$

$$-11 = -11 \checkmark$$

$$2x + y + z = 1$$

$$2(2) + (-4) + 1 = 1$$

$$1 = 1 \checkmark$$

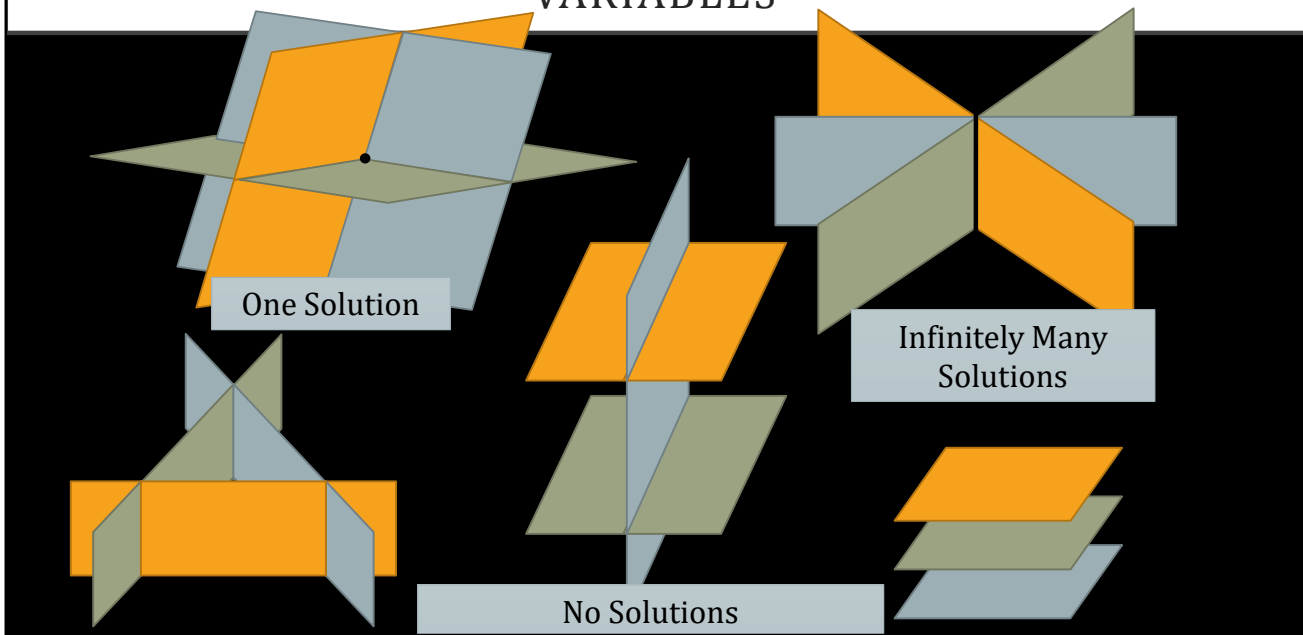
$$5x - 2y + 3z = 21$$

$$5(2) - 2(-4) + 3(1) = 21$$

$$21 = 21 \checkmark$$

- Yes

3.4 SOLVE SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES



3.4 SOLVE SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES

Elimination Method

- Like two variables, you just do it more than once.
 1. Combine first and second to eliminate a variable
 2. Combine second and third to eliminate the same variable as before
 3. Combine these new equations to find the two variables
 4. Substitute those two variables into one of the original equations to get the third variable
- If you get a false statement like $8=0 \rightarrow$ no solution
- If you get an identity like $0=0 \rightarrow$ infinitely many solutions

3.4 SOLVE SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES

$$\begin{cases} 2x + 3y + 7z = -3 \\ -2(x - 6y + z) = -4x - 2 \\ -x - 3y + 8z = 1 \end{cases}$$

- Combine first two equations (multiply second by -2 to eliminate x):

$$\begin{array}{r} 2x + 3y + 7z = -3 \\ -2x + 12y - 2z = 8 \\ \hline 15y + 5z = 5 \end{array}$$

Divide by 5

$$3y + z = 1$$

- Combine last two equations (just add to also eliminate x):

$$\begin{array}{r} x - 6y + z = -4 \\ -x - 3y + 8z = 1 \\ \hline -9y + 9z = -3 \end{array}$$

Divide by 3

$$-3y + 3z = -1$$

- Combine the combinations (just add to eliminate y):

$$\begin{array}{r} 3y + z = 1 \\ -3y + 3z = -1 \\ \hline 4z = 0 \\ z = 0 \end{array}$$

- Substitute into one of the combinations

$$\begin{array}{r} 3y + z = 1 \\ 3y + 0 = 1 \\ \hline y = \frac{1}{3} \end{array}$$

- Substitute into one of the original equations

$$\begin{array}{r} x - 6y + z = -4 \\ x - 6\left(\frac{1}{3}\right) + 0 = -4 \\ x - 2 = -4 \\ x = -2 \end{array}$$

- $\left(-2, \frac{1}{3}, 0\right)$

3.4 SOLVE SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES

$$2(-x + 2y + z) = 3 \times 2$$

$$-2(2x + 2y + z) = 5 \times -2$$

$$4x + 4y + 2z = 6$$

- Combine first two equations (multiply first by 2 to eliminate x):

$$-2x + 4y + 2z = 6$$

$$\underline{2x + 2y + z = 5}$$

$$6y + 3z = 11$$

- Combine last two equations (multiply second by -2 to also eliminate x):

$$-4x - 4y - 2z = -10$$

$$\underline{4x + 4y + 2z = 6}$$

$$0 = -4$$

- $0 = -4$ is a false statement, so there is **no solution**.

3.4 SOLVE SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES

- Substitution

1. Solve one of the equations for one variable
2. Substitute that into both of the other equations
3. Solve the resulting system of two variables

3.4 SOLVE SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES

$$\begin{aligned} x + y + z &= 6 \\ x - y + z &= 6 \\ 4x + y + 4z &= 24 \end{aligned}$$

- Solve 1st for $y \rightarrow y = -x - z + 6$

- Substitute this into 2nd

$$\begin{aligned} x - (-x - z + 6) + z &= 6 \\ 2x + 2z - 6 &= 6 \\ 2x + 2z &= 12 \\ x + z &= 6 \end{aligned}$$

- Substitute the solved equation into the 3rd

$$\begin{aligned} 4x + (-x - z + 6) + 4z &= 24 \\ 3x + 3z + 6 &= 24 \\ 3x + 3z &= 18 \\ x + z &= 6 \end{aligned}$$

- Write the new system

$$\begin{aligned} x + z &= 6 \\ x + z &= 6 \end{aligned}$$

- Solve the 1st for $x \rightarrow x = 6 - z$
- Substitute into 2nd and solve

$$\begin{aligned} (6 - z) + z &= 6 \\ 6 &= 6 \end{aligned}$$

- This is true, so **many solutions**.

- Write answer by letting

$$z = a$$

- Substitute into $x + z = 6$ and find x :

$$x = 6 - a$$

- Substitute back into first and find y :

$$\begin{aligned} y &= -x - z + 6 \\ y &= -(6 - a) - a + 6 \\ y &= 0 \end{aligned}$$

- Solution: $(6 - a, 0, a)$

3.4 SOLVE SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES

- If there are infinitely many solutions
 - Let $x = a$
 - Solve for y in terms of a
 - Substitute those to find z in terms of a
 - Sample answer $(a, a + 4, 2a)$

HOMEWORK QUIZ

- [3.4 Homework Quiz](#)

3.5 PERFORM BASIC MATRIX OPERATIONS

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3.5 PERFORM BASIC MATRIX OPERATIONS

- Matrices are simply a way to organize data.
- For example, a computer desktop wallpaper (bitmap) is a matrix. Each element tells what color pixel goes in that spot.

3.5 PERFORM BASIC MATRIX OPERATIONS

- A matrix is a rectangular arrangement of things (variables or numbers in math)

- $$\begin{bmatrix} 2 & -1 & 5 & a \\ 2 & y & 6 & b \\ 3 & 14 & x & c \end{bmatrix}$$

- Dimensions
 - Rows by columns
 - 3×4 for the above matrix

3.5 PERFORM BASIC MATRIX OPERATIONS

- In order for two matrices to be equal, they must be the same dimensions and corresponding elements must be the same

- $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

- Find the variables

$$\begin{bmatrix} 2 & y + 1 \\ x/3 & 4 \end{bmatrix} = \begin{bmatrix} w & -4 \\ 5 & z - 4 \end{bmatrix}$$

- $2 = w$
- $y + 1 = -4 \rightarrow y = -5$
- $\frac{x}{3} = 5 \rightarrow x = 15$
- $4 = z - 4 \rightarrow z = 8$

3.5 PERFORM BASIC MATRIX OPERATIONS

- Adding and Subtracting

- You can only add and subtract matrices that are the same dimensions
- When you add or subtract, add the corresponding elements.

- $\begin{bmatrix} 1 & 2 \\ -5 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 5 \\ 4 & -3 \end{bmatrix}$

- $= \begin{bmatrix} 1 + (-2) & 2 + 5 \\ -5 + 4 & 4 + (-3) \end{bmatrix}$

- $= \begin{bmatrix} -1 & 7 \\ -1 & 1 \end{bmatrix}$

3.5 PERFORM BASIC MATRIX OPERATIONS

- $[2 \quad -3] - [3 \quad 4] + [1 \quad 0]$

- $= [2 - 3 + 1 \quad -3 - 4 + 0]$

- $= [0 \quad -7]$

- $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 3 & 1 \\ 2 & 5 & 2 \end{bmatrix}$

- Can't add because the matrices are different dimensions

3.5 PERFORM BASIC MATRIX OPERATIONS

- Scalar Multiplication
 - Multiply each element by the scalar
 - Distribute

$$\bullet 3 \begin{bmatrix} 5 & -2 & 7 \\ -3 & 8 & 4 \end{bmatrix}$$

$$\bullet = \begin{bmatrix} 3(5) & 3(-2) & 3(7) \\ 3(-3) & 3(8) & 3(4) \end{bmatrix}$$

$$\bullet = \begin{bmatrix} 15 & -6 & 21 \\ -9 & 24 & 12 \end{bmatrix}$$

3.5 PERFORM BASIC MATRIX OPERATIONS

- The National Weather Service keeps track of weather.

June 2014	Benton Harbor	South Bend	July 2014	Benton Harbor	South Bend
Precip Days	13	18	Precip Days	14	15
Clear Days	16	13	Clear Days	18	18
Ab Norm T	12	19	Ab Norm T	2	8

- What is meaning of the first matrix + second matrix?
 - The total number of days of each type for each city.
- Use matrix operations to find the total weather stats of each city

$$\bullet = \begin{bmatrix} 13 + 14 & 18 + 15 \\ 16 + 18 & 13 + 18 \\ 12 + 2 & 19 + 8 \end{bmatrix} = \begin{bmatrix} 27 & 33 \\ 34 & 31 \\ 14 & 27 \end{bmatrix}$$

*Precip Days = Days with precipitation

Clear Days = Days with no clouds

Ab Norm T = Days with Above Normal Temperature

HOMEWORK QUIZ

- [3.5 Homework Quiz](#)

3.6 MULTIPLY MATRICES

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3.6 MULTIPLY MATRICES

- Yesterday we learned all about matrices and how to add and subtract them. But how do you multiply or divide matrices?
- Today we will multiply matrices.
- Later we will find out that you can't divide by a matrix.

3.6 MULTIPLY MATRICES

- Matrix multiplication can only happen if the number of columns of the first matrix is the same as the number of rows on the second matrix.
- You can multiply a 3×5 with a 5×2 .
 - $3 \times 5 \cdot 5 \times 2 \rightarrow 3 \times 2$ will be the dimensions of the answer
- Because of this **order does matter!**

3.6 MULTIPLY MATRICES

$$\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}$$



$$\begin{bmatrix} -2 & 1 \\ 4 & 3 \end{bmatrix}$$



- Pick another row in the 1st matrix and a column in the 2nd matrix.
- Multiply the first numbers.
- Plus.
- Multiply the second numbers.

$$\begin{bmatrix} 1 \cdot -2 + 2 \cdot 4 & 1 \cdot 1 + 2 \cdot 3 \\ 0 \cdot -2 + -3 \cdot 4 & 0 \cdot 1 + -3 \cdot 3 \end{bmatrix}$$

Simplify each element.

$$\begin{bmatrix} 6 & 7 \\ -12 & -9 \end{bmatrix}$$

The answers go in the same location as the row and column.

For example, the result of the 1st row and 2nd column will go in the 1st row 2nd column of the answer.

3.6 MULTIPLY MATRICES

$$\begin{bmatrix} 1 & 0 & 4 \\ -2 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$$

- Pick another row in the 1st matrix and a column in the 2nd matrix.
- Multiply the first numbers.
- Plus multiply the second numbers.
- Plus multiply the third numbers.

$$\begin{bmatrix} 1 \cdot -1 + 0 \cdot 3 + 4 \cdot 5 \\ -2 \cdot -1 + 3 \cdot 3 + 2 \cdot 5 \end{bmatrix}$$

Simplify each element.

$$\begin{bmatrix} 19 \\ 21 \end{bmatrix}$$

The answers go in the same location as the row and column.

For example, the result of the 1st row and 1nd column will go in the 1st row 1nd column of the answer.

HOMEWORK QUIZ

- [3.6 Homework Quiz](#)

3.7 DETERMINANTS AND CRAMER'S RULE

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3.7 DETERMINANTS AND CRAMER'S RULE

- You had to know that all this matrix stuff must have some purpose.
- Uses of matrices (that we will investigate today)
 - Solve systems of equations
 - Find the area of a triangle when we only know the coordinates of its vertices

3.7 DETERMINANTS AND CRAMER'S RULE

- Determinant
 - Number associated with square matrices
 - Symbolized by $\det A$ or $|A|$
 - Vertical lines mean determinant
 - I won't answer that question on the test for you!

3.7 DETERMINANTS AND CRAMER'S RULE

- Determinant of 2x2 matrix
 - Multiply along the down diagonal and subtract the product of the up diagonal.

$$\bullet \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$$

$$\bullet = 2(4) - 3(-1)$$

$$\bullet = 8 + 3$$

$$\bullet = 11$$

3.7 DETERMINANTS AND CRAMER'S RULE

- Determinant of 3x3 Matrix
 - Copy the first 2 columns behind the matrix and then add the products of the down diagonals and subtract the product of the up diagonals.

$$\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 9 & 7 & 8 \end{array}$$

$$= 1 \cdot 5 \cdot 9 + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8 - 7 \cdot 5 \cdot 3 - 8 \cdot 6 \cdot 1 - 9 \cdot 4 \cdot 2$$

$$= 45 + 84 + 96 - 105 - 48 - 72$$

$$= 0$$

3.7 DETERMINANTS AND CRAMER'S RULE

- Area of a Triangle

$$Area = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where x 's and y 's are the coordinates of the vertices

3.7 DETERMINANTS AND CRAMER'S RULE

- Find the area of a triangle with vertices of (2,4), (5,1), and (2,-2)

$$\bullet \text{ Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\bullet \text{ Area} = \pm \frac{1}{2} \begin{vmatrix} 2 & 4 & 1 & 2 & 4 \\ 5 & 1 & 1 & 5 & 1 \\ 2 & -2 & 1 & 2 & -2 \end{vmatrix}$$

$$\bullet = \frac{1}{2} (2 \cdot 1 \cdot 1 + 4 \cdot 1 \cdot 2 + 1 \cdot 5 \cdot (-2) - 2 \cdot 1 \cdot 1 - (-2) \cdot 1 \cdot 2 - 1 \cdot 5 \cdot 4)$$

$$\bullet = \frac{1}{2} (2 + 8 + (-10) - 2 - (-4) - 20)$$

$$\bullet = \frac{1}{2} (-18) = -9$$

$$\bullet \text{ Area} = 9$$

3.7 DETERMINANTS AND CRAMER'S RULE

- Cramer's Rule

- Write the equations in standard form
- Make a matrix out of the coefficients

- 2x2

- $ax + by = e$
- $cx + dy = f$

gives

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

Coefficients

x-column replaced with constants to find *x*.
y-column replaced with constants to find *y*.

Notice that the numerator and denominator are the same except for the columns containing the coefficients of the variable you are solving for is replaced with the numbers from the constants column.

3.7 DETERMINANTS AND CRAMER'S RULE

$$\bullet 2x + 1y = 1$$

$$\bullet 3x - 2y = -23$$

$$\bullet x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

$$\bullet x = \frac{\begin{vmatrix} \cancel{1} & \cancel{1} \\ -23 & -2 \end{vmatrix}}{\begin{vmatrix} \cancel{2} & \cancel{1} \\ \cancel{3} & -2 \end{vmatrix}}$$

$$\bullet x = \frac{1(-2) - (-23)1}{2(-2) - 3(1)}$$

$$\bullet x = \frac{21}{-7} = -3$$

$$\bullet y = \frac{\begin{vmatrix} \cancel{2} & \cancel{1} \\ 3 & -23 \end{vmatrix}}{\begin{vmatrix} \cancel{2} & \cancel{1} \\ 3 & -2 \end{vmatrix}}$$

$$\bullet y = \frac{2(-23) - 3(1)}{2(-2) - 3(1)}$$

$$\bullet y = \frac{-49}{-7} = 7$$

$$\bullet \text{Solution } (-3, 7)$$

3.7 DETERMINANTS AND CRAMER'S RULE

- Cramer's Rule on a 3x3 System
 - Same as 2x2 system
 - The denominator is the determinant of the coefficient matrix and the numerator is the same only with the column of the variable you are solving for replaced with the = column.

3.7 DETERMINANTS AND CRAMER'S RULE

$$2x - 1y + 6z = -4$$

$$\bullet \quad 6x + 4y - 5z = -7$$

$$-4x - 2y + 5z = 9$$

$$\bullet \quad x = \frac{\begin{vmatrix} -4 & -1 & 6 & -4 & -1 \\ -7 & 4 & -5 & -7 & 4 \\ 9 & -2 & 5 & 9 & -2 \\ 2 & -1 & 6 & 2 & -1 \\ 6 & 4 & -5 & 6 & 4 \\ -4 & -2 & 5 & -4 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 6 & 2 & -1 \\ 6 & 4 & -5 & 6 & 4 \\ -4 & -2 & 5 & -4 & -2 \end{vmatrix}}$$

$$\bullet \quad x = \frac{-4(4)(5) + (-1)(-5)(9) + 6(-7)(-2) - 9(4)(6) - (-2)(-5)(-4) - 5(-7)(-1)}{2(4)(5) + (-1)(-5)(-4) + 6(6)(-2) - (-4)(4)(6) - (-2)(-5)(2) - 5(6)(-1)}$$

$$\bullet \quad x = \frac{-80 + 45 + 84 - 216 - (-40) - 35}{40 + (-20) + (-72) - (-96) - 20 - (-30)}$$

$$\bullet \quad x = \frac{-162}{54} = -3$$

3.7 DETERMINANTS AND CRAMER'S RULE

$$2x - 1y + 6z = -4$$

$$\bullet \quad 6x + 4y - 5z = -7$$

$$-4x - 2y + 5z = 9$$

$$\bullet \quad y = \frac{\begin{vmatrix} 2 & -4 & 6 & 2 & -4 \\ 6 & -7 & -5 & 6 & -7 \\ -4 & 9 & 5 & -4 & 9 \\ 2 & -1 & 6 & 2 & -1 \\ 6 & 4 & -5 & 6 & 4 \\ -4 & -2 & 5 & -4 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 6 & 2 & -1 \\ 6 & 4 & -5 & 6 & 4 \\ -4 & -2 & 5 & -4 & -2 \end{vmatrix}}$$

$$\bullet \quad y = \frac{2(-7)(5) + (-4)(-5)(-4) + 6(6)(9) - (-4)(-7)(6) - (9)(-5)(2) - 5(6)(-4)}{2(4)(5) + (-1)(-5)(-4) + 6(6)(-2) - (-4)(4)(6) - (-2)(-5)(2) - 5(6)(-1)}$$

$$\bullet \quad y = \frac{-70 + (-80) + 324 - 168 - (-90) - (-120)}{40 + (-20) + (-72) - (-96) - 20 - (-30)}$$

$$\bullet \quad y = \frac{216}{54} = 4$$

3.7 DETERMINANTS AND CRAMER'S RULE

$$2x - 1y + 6z = -4$$

$$\bullet \quad 6x + 4y - 5z = -7$$

$$-4x - 2y + 5z = 9$$

$$\bullet \quad z = \frac{\begin{vmatrix} 2 & -1 & -4 & 2 & -1 \\ 6 & 4 & -7 & 6 & 4 \\ -4 & -2 & 9 & -4 & -2 \\ 2 & -1 & 6 & 2 & -1 \\ 6 & 4 & -5 & 6 & 4 \\ -4 & -2 & 5 & -4 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 6 & 2 & -1 \\ 6 & 4 & -5 & 6 & 4 \\ -4 & -2 & 5 & -4 & -2 \end{vmatrix}}$$

$$\bullet \quad z = \frac{2(4)(9) + (-1)(-7)(-4) + (-4)(6)(-2) - (-4)(4)(-4) - (-2)(-7)(2) - 9(6)(-1)}{2(4)(5) + (-1)(-5)(-4) + 6(6)(-2) - (-4)(4)(6) - (-2)(-5)(2) - 5(6)(-1)}$$

$$\bullet \quad z = \frac{72 + (-28) + 48 - 64 - 28 - (-54)}{40 + (-20) + (-72) - (-96) - 20 - (-30)}$$

$$\bullet \quad z = \frac{54}{54} = 1$$

The solution is $(-3, 4, 1)$

HOMEWORK QUIZ

- [3.7 Homework Quiz](#)

3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

Larson Algebra 2
(2011)

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3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

- You can use matrices to solve linear systems in ways different from Cramer's Rule.
- We will learn how, but it requires that we know how to find inverse matrices.

3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

- The identity Matrix multiplied with any matrix of the same dimension equals the original matrix.
- $A \cdot I = I \cdot A = A$
- This is the matrix equivalent of 1

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

1's on diagonal, everything else is zero

3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

- You cannot divide by a matrix!
- So we multiply by the inverse of a matrix.
- $A \cdot A^{-1} = [1] = I$
- Just like $x (x^{-1}) = x \left(\frac{1}{x}\right) = 1$

3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

- The Rule for 2x2 (Memorize)

- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

- $A^{-1} = \frac{1}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{1(4) - 3(2)} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 \\ 3 & -\frac{1}{2} \end{bmatrix}$$

3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

$$\begin{bmatrix} -2 & -1 \\ 4 & 0 \end{bmatrix}^{-1} = \frac{1}{\begin{vmatrix} -2 & -1 \\ 4 & 0 \end{vmatrix}} \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}$$

$$= \frac{1}{-2(0) - 4(-1)} \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \frac{1}{4} \\ -1 & -\frac{1}{2} \end{bmatrix}$$

3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

- Check by multiplying the two matrices

$$\begin{aligned} & \begin{bmatrix} -2 & -1 \\ 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{1}{4} \\ -1 & -\frac{1}{2} \end{bmatrix} \\ &= \begin{bmatrix} -2(0) + (-1)(-1) & -2\left(\frac{1}{4}\right) + (-1)\left(-\frac{1}{2}\right) \\ 4(0) + 0(-1) & 4\left(\frac{1}{4}\right) + 0\left(-\frac{1}{2}\right) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

- If A, B, and X are matrices, and
 - $A \cdot X = B$
 - $A^{-1} \cdot A \cdot X = A^{-1} \cdot B$
 - $I \cdot X = A^{-1} \cdot B$
 - $X = A^{-1} \cdot B$

3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

- Solve a matrix equation

- $AX = B$

- $\begin{bmatrix} -3 & 4 \\ 5 & -7 \end{bmatrix} X = \begin{bmatrix} 3 & 8 \\ 2 & -2 \end{bmatrix}$

- Continued on next slide

- Find A^{-1}

- $\begin{bmatrix} -3 & 4 \\ 5 & -7 \end{bmatrix}^{-1}$

- $= \frac{1}{\begin{vmatrix} -3 & 4 \\ 5 & -7 \end{vmatrix}} \begin{bmatrix} -7 & -4 \\ -5 & -3 \end{bmatrix}$

- $= \frac{1}{-3(-7)-5(4)} \begin{bmatrix} -7 & -4 \\ -5 & -3 \end{bmatrix}$

- $= \frac{1}{1} \begin{bmatrix} -7 & -4 \\ -5 & -3 \end{bmatrix}$

- $A^{-1} = \begin{bmatrix} -7 & -4 \\ -5 & -3 \end{bmatrix}$

3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

- $A^{-1}AX = A^{-1}B$ $A^{-1}A$ cancel

- $X = A^{-1}B$

$$X = \begin{bmatrix} -7 & -4 \\ -5 & -3 \end{bmatrix} \begin{bmatrix} 3 & 8 \\ 2 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} -7(3) + -4(2) & -7(8) + -4(-2) \\ -5(3) + -3(2) & -5(8) + -3(-2) \end{bmatrix}$$

$$X = \begin{bmatrix} -29 & -48 \\ -21 & -34 \end{bmatrix}$$

3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

- $2x + y = -13$
 $x - 3y = 11$
- Take your equation and write it as matrices ($A \cdot X = B$)
 - $\begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -13 \\ 11 \end{bmatrix}$
- Find the coefficient matrix inverse
 - $\begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}^{-1} = \frac{1}{-7} \begin{bmatrix} -3 & -1 \\ -1 & 2 \end{bmatrix}$

3.8 USE INVERSE MATRICES TO SOLVE LINEAR SYSTEMS

- Multiply the front of both sides by the inverse ($A^{-1}A \cdot X = A^{-1}B$)

- Left side cancels ($X = A^{-1}B$)

- $$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-7} \begin{bmatrix} -3 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -13 \\ 11 \end{bmatrix}$$

- $$= \frac{1}{-7} \begin{bmatrix} 39 + -11 \\ 13 + 22 \end{bmatrix}$$

- $$= \frac{1}{-7} \begin{bmatrix} 28 \\ 35 \end{bmatrix}$$

- $$= \begin{bmatrix} -4 \\ -5 \end{bmatrix}$$

(-4, -5)

HOMEWORK QUIZ

- [3.8 Homework Quiz](#)