Linear Systems and Matrices

Algebra 2 Chapter 3

Algebra II 3

- This Slideshow was developed to accompany the textbook
 - Larson Algebra 2
 - By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.
 - 2011 Holt McDougal
- Some examples and diagrams are taken from the textbook.

Slides created by Richard Wright, Andrews Academy rwright@andrews.edu

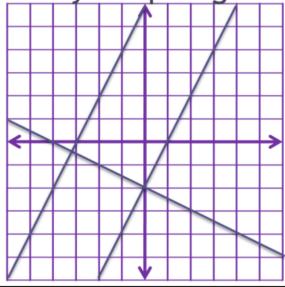
- System of equations
 - More than one equation that share the same solution.
 - Often they involve more than one variable.
 - In order to solve them, you need as many equations as there are variables.

$$3x + 3y = 6$$
$$3x - 4y = 5$$

- Solutions to systems
 - An ordered pair that works in both equations.
 - If the ordered pair works in both equations, then both graphs have to go through that point.
 - Solutions are where the graphs cross.

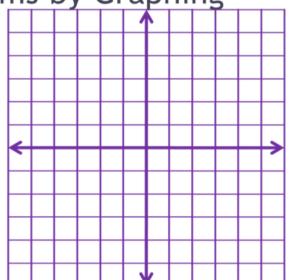
- Solve by graphing
 - Graph both equations on the same graph.
 - Where they cross is the solution.

- Classifying Solutions
 - Many Solutions →
 consistent (has a
 solution), dependant
 - One solution → consistent, independent
 - No solution → inconsistent



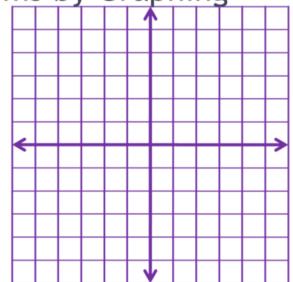
Solve by graphing.
 Classify as consistent and independent, consistent and dependent, or inconsistent.

$$3x + 2y = -4$$
$$x + 3y = 1$$



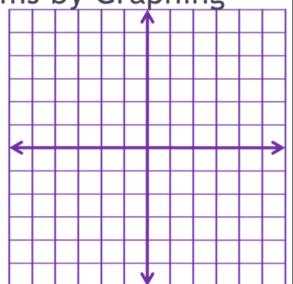
(-2, 1)
Consistent and independent

- Solve by graphing.
 Classify as consistent and independent, consistent and dependent, or inconsistent.
- 3x 2y = 10
- 3x 2y = 2



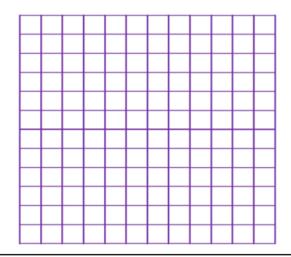
No solution inconsistent

- Solve by graphing.
 Classify as consistent and independent, consistent and dependent, or inconsistent.
- 2x + y = 1
- -4x 2y = -2



Infinitely many solutions Consistent and dependent

 A soccer league offers two options for membership plans. A) \$40 initial fee and \$5 for each game played. B) \$10 for each game played. How many games must you play for both plans to be the same?



Plan A: y = 5x + 40Plan B: y = 10x(8, 80)

(8, 80) 8 games

• 156 #3-31 odd, 35, 37 + 3 choice = 20

Homework Quiz

• 3.1 Homework Quiz

- Graphing to solve systems of equations has some problems.
- Can you guess some?
 - Inaccurate
 - Sometimes hard to graph

- Substitution
 - 1. Solve one equation for one variable
 - 2. Use that expression to replace that variable in the other equation
 - 3. Solve the new equation
 - 4. Substitute back into the first equation
 - 5. Solve for the second variable

3.2 Solve Lip ar Systems Algebraically y is the same thing

$$y = x + 2$$

$$2x + y = 8$$

$$y \text{ is the same thing as } x + 2$$

$$y = 2 + 2$$

$$y = 4$$

$$2x + x + 2 = 8$$
$$3x + 2 = 8$$
$$3x = 6$$
$$x = 2$$

(2,4)

$$3x + 2y = 8$$
$$x + 4y = -4$$

$$x = -3 \frac{1}{2} \frac{1}{$$

$$3 (-4y-4) + 2y = 8$$

$$-12y - 12 + 2y = 8$$

$$-10y - 12 = 8$$

$$-10y = 20$$

$$y = -2$$

$$x = -4 (-2) - 4$$
$$x = 4$$

$$(4, -2)$$

- Elimination
 - 1. Line up the equations into columns
 - Multiply one or both equations by numbers so that one variable has the same coefficient, but opposite sign
 - 3. Add the equations
 - 4. Solve the resulting equation
 - 5. Substitute the value into one original equation and solve

$$2x - 3v = -14$$

$$(-3)(3x - y) = -7(-3)$$

$$2x - 3y = -14$$

$$-9x + 3y = 21$$

$$-7x = 7$$

$$x = -1$$

$$2 (-1) - 3y = -14$$

$$-2 - 3y = -14$$

$$-3y = -12$$

$$y = 4$$

$$(-1,4)$$

$$(2)(3x + 11y) = 4(2) -2x - 6(2) = 0$$

$$(3)(-2x - 6y) = 0(3) -2x - 12 = 0$$

$$-2x = 12$$

$$6x + 22y = 8$$

$$-6x - 18y = 0$$

$$4y = 8$$

y = 2

(-6, 2)

- Number of Solutions
 - If both variables disappear after you substitute or combine and
 - You get a true statement like 2 = 2 → infinite solutions
 - You get a false statement like $2 = 5 \rightarrow$ no solution

- · Summary of Solving Techniques
 - When to graph?
 - · To get general picture and estimate
 - When to use substitution?
 - · When one of the coefficients is 1
 - When to use elimination?
 - · When none of the coefficients is 1
- 164 #3-55 every other odd, 61 + 5 choice = 20

Homework Quiz

• 3.2 Homework Quiz

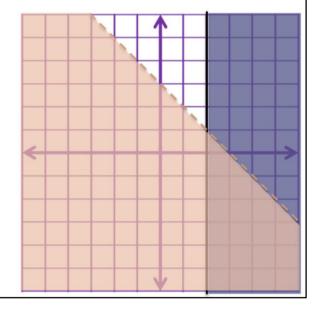
3.3 Graph Systems of Linear Inequalities

- To solve systems of inequalities, graph them all on one graph.
- · Solution is where all graphs overlap

3.3 Graph Systems of Linear

Inequalities

- Solve the system of inequalities
- x ≥ 2
- \odot x + y < 3



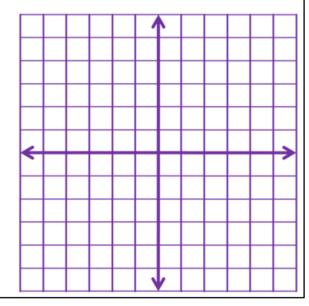
3.3 Graph Systems of Linear

Inequalities

• Solve the system of inequalities

$$y < -\frac{4x}{5} - 4$$

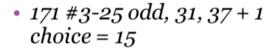
$$y > -\frac{4x}{5} + 2$$

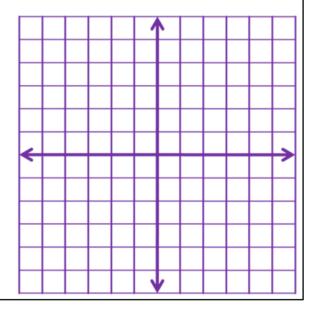


3.3 Graph Systems of Linear

Inequalities
• Solve the system of

- y ≤ 3
- \circ o \leq x \leq 5
- \bullet x > -y





Homework Quiz

• 3.3 Homework Quiz

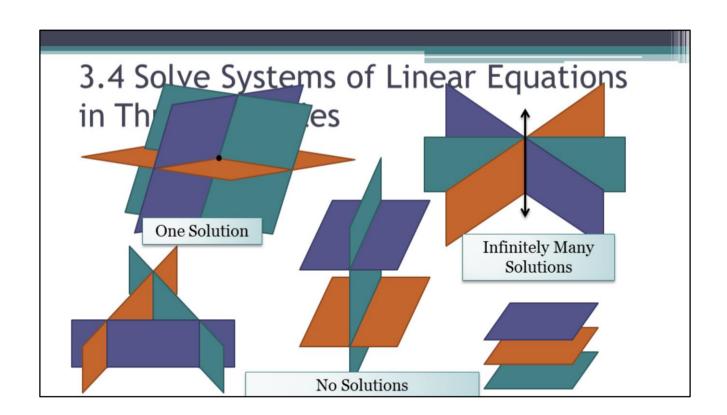
 We have now worked with 2 variables and 2 dimensions, but sometimes there are more



· Linear equation in 3 variables graphs a plane

- · Solution to system in 3 variables
 - Ordered triple (x, y, z)
- Example: Is (2, -4, 1) a solution of x + 3y z = -11• 2x + y + z = 1• 5x - 2y + 3z = 21

Plug it in (yes)



Elimination Method

- · Like two variables, you just do it more than once.
 - 1. Combine first and second to eliminate a variable
 - 2. Combine second and third to eliminate the same variable as before
 - 3. Combine these new equations to find the two variables
 - 4. Substitute those two variables into one of the original equations to get the third variable
- If you get a false statement like 8=0 → no solution
- If you get an identity like o=o → infinitely many solutions

$$2x + 3y + 7z = -3$$
$$x - 6y + z = -4$$
$$-x - 3y + 8z = 1$$

Combine first two equations (multiply second by -2): 2x+3y+7z=-3 -2x+12y-2z=8 $15y + 5z = 5 \rightarrow 3y + z = 1$

Combine last two equation (just add):

$$x-6y+z=-4$$

 $-x-3y+8z=1$
 $-9y + 9z = -3 \rightarrow -3y + 3z = -1$

Combine the combinations (just add):

$$3y + z = 1$$

 $-3y + 3z = -1$
 $4z = 0 \rightarrow z = 0$

Substitute into 3y + z = 1: $3y + 0 = 1 \rightarrow y = 1/3$

Substitute into
$$x - 6y + z = -4$$
:
 $x - 6(1/3) + 0 = -4 \rightarrow x - 2 = -4 \rightarrow x = -2$

(-2, 1/3, 0)

$$-x + 2y + z = 3$$

 $2x + 2y + z = 5$
 $4x + 4y + 2z = 6$

No Solution

- Substitution
 - 1. Solve one of the equations for one variable
 - 2. Substitute that into both of the other equations
 - 3. Solve the resulting system of two variables

$$x + y + z = 6$$

$$x - y + z = 6$$

$$4x + y + 4z = 24$$

```
ANS: Solve 1^{st} for y 	oup y = -x - z + 6

Substitute this into 2^{nd} 	oup x - (-x - z + 6) + z = 6 	oup 2x + 2z - 6 = 6 	oup 2x + 2z = 12 	oup x + z = 6

Substitute the solve equation into the 3^{rd} 	oup 4x + (-x - z + 6) + 4z = 24 	oup 3x + 3z + 6 = 24 	oup 3x + 3z = 18 	oup x + z = 6

Write the new system 	oup

Solve the 1^{st} for x 	oup x = 6 - z

Substitute into 2^{nd} and solve 	oup (6 - z) + z = 6 	oup 6 = 6 many solutions

Let z = z

Substitute into x + z = 6 and find x: x = 6 - z

Substitute back into first and find y: y = -x - z + 6 	oup y = -(6 - z) - z + 6 	oup y = 0

Solution: (6-z, 0, z)
```

- · If there are infinitely many solutions
 - Let x = a
 - Solve for y in terms of a
 - Substitute those to find z in terms of a
 - Sample answer (a, a + 4, 2a)
- 182 #1, 5, 9, 13, 15, 19, 21, 25, 29, 33, 43 + 4 choice = 15

Homework Quiz

• 3.4 Homework Quiz

3.5 Perform Basic Matrix Operations

- No matter what you might think, "The Matrix" was not really a matrix as far as math is concerned.
- Matrices are simply a way to organize data.
- For example, a computer desktop wallpaper (bitmap) is a matrix. Each element tells what color pixel goes in that spot.

3.5 Perform Basic Matrix Operations

A matrix is a rectangular arrangement of things (variables or numbers in math)

$$\begin{bmatrix} 2 & -1 & 5 & a \\ 2 & y & 6 & b \\ 3 & 14 & x & c \end{bmatrix}$$

- Dimensions
 - ζ Rows by columns
 - ζ 3 x 4 for the above matrix

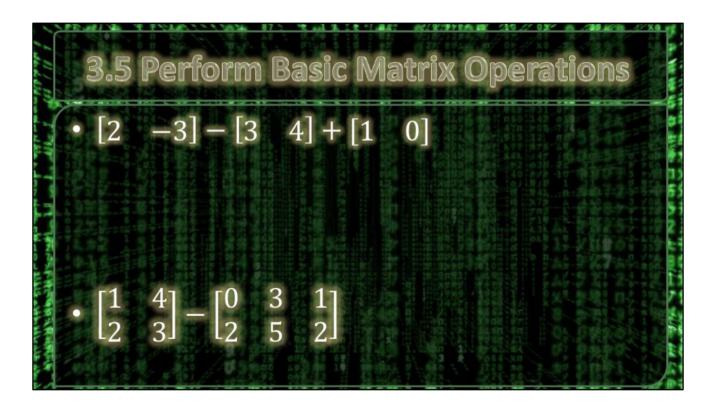
• In order for two matrices to be equal, they must be the same dimensions and corresponding elements must be the same • $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ • Examples ζ Find the variables $\begin{bmatrix} 2 & y+1 \\ x/3 & 4 \end{bmatrix} = \begin{bmatrix} w & -4 \\ 5 & z-4 \end{bmatrix}$

ANS:
$$2 = w$$
,
 $x/3 = 5 \rightarrow x = 15$,
 $y + 1 = -4 \rightarrow y = -5$,
 $4 = z - 4 \rightarrow z = 8$

3.5 Perform Basic Matrix Operations • Adding and Subtracting ζ You can only add and subtract matrices that are the same dimensions ζ When you add or subtract, add the corresponding elements. $\zeta \begin{bmatrix} 1 & 2 \\ -5 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 5 \\ 4 & -3 \end{bmatrix}$

[-1,3]

[-1,1]



[0,-7]
Can't add because different dimensions

```
3.5 Perform Basic Matrix Operations

• Scalar Multiplication

\zeta Multiply each element by the scalar

\zeta Distribute

• 3\begin{bmatrix} 5 & -2 & 7 \\ -3 & 8 & 4 \end{bmatrix}
```

[15,-6,21] [-9,24,12]

3.5 Perform Basic Matrix Operations

The National Weather Service keeps track of weather.

June 2014	Benton Harbor	South Bend	
Precip Days	13	18	
Clear Days	16	13	
Ab Norm T	12	19	

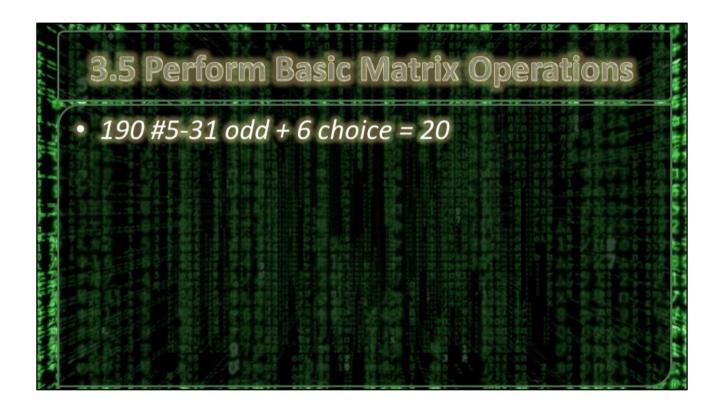
July 2014	Benton Harbor	South Bend	
Precip Days	14	15	
Clear Days	18	18	
Ab Norm T	2	8	

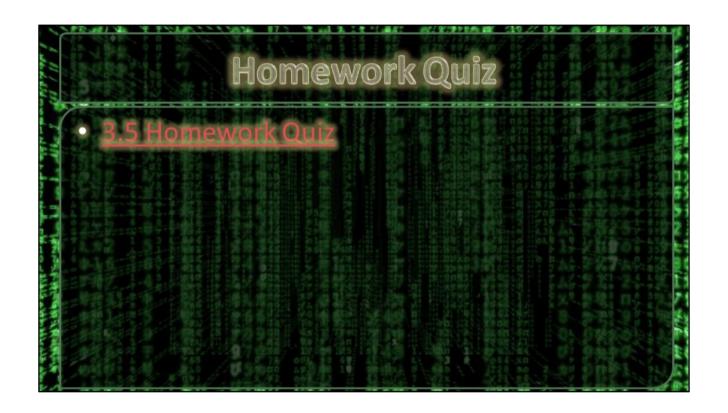
- What is meaning of the first matrix + second matrix?
- Use matrix operations to find the total weather stats of each city

the total number of days of each type for each city

[27 33] [34 31] [14 27]

^{*}Precip Days = Days with precipitation Clear Days = Days with no clouds Ab Norm T = Days with Above Normal Temperature





3.6 Multiply Matrices

- Yesterday we learned all about matrices and how to add and subtract them. But how do you multiply or divide matrices?
- Today we will multiply matrices.
- Later we will find out that you can't divide by a matrix.

3.6 Multiply Matrices

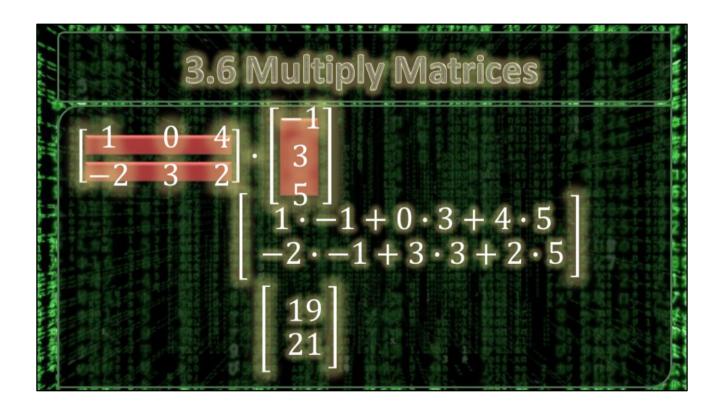
- Matrix multiplication can only happen if the number of columns of the first matrix is the same as the number of rows on the second matrix.
- You can multiply a 3x5 with a 5x2.

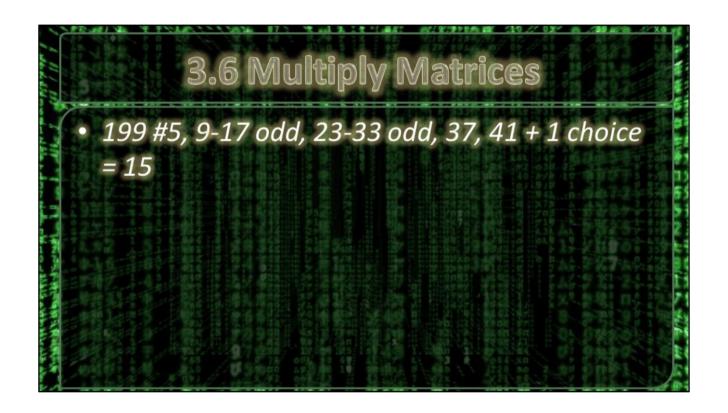
 ζ 3x5 ⋅ 5x2 → 3x2 will be the dimensions of the answer.
- Because of this order does matter!

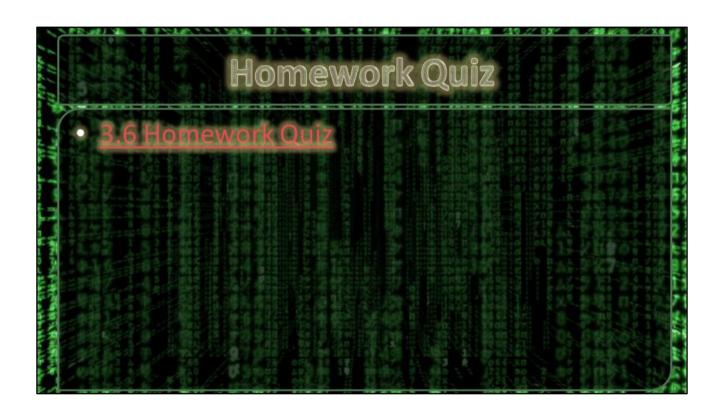
3.6 Multiply Matrices
$$\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} -2 & -1 \\ 4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 \cdot -2 + 2 \cdot 4 & 1 \cdot 1 + 2 \cdot 3 \\ 0 \cdot -2 + -3 \cdot 4 & 0 \cdot 1 + -3 \cdot 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 7 \\ -12 & -9 \end{bmatrix}$$



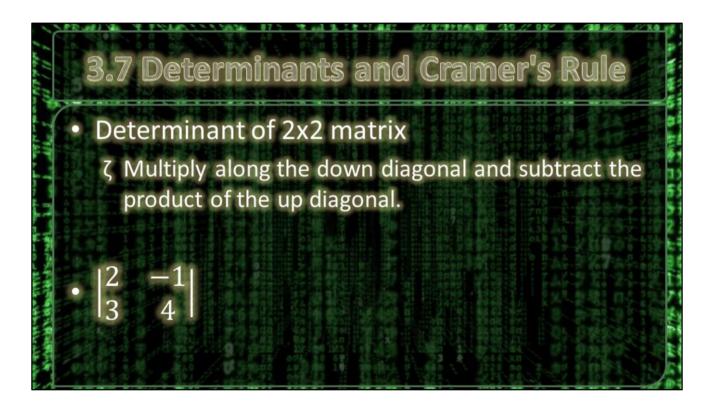




3.7 Determinants and Cramer's Rule

- You had to know that all this matrix stuff must have some purpose.
- Uses of matrices (that we will investigate today)
 - ζ Solve systems of equations
 - ζ Find the area of a triangle when we only know the coordinates of its vertices

3.7 Determinants and Cramer's Rule • Determinant ζ Number associated with square matrices ζ Symbolized by det A or | A | • Vertical lines mean determinant • I won't answer that question on the test for you!



ANS: 2(4) - 3(-1) = 8 + 3 = 11

3.7 Determinants and Cramer's Rule Determinant of 3x3 Matrix ζ Copy the first 2 columns behind the matrix and then add the products of the down diagonals and subtract the product of the up diagonals. |1 2 3 | 1 2 |4 5 6 | 4 5 |7 8 9 | 7 8

ANS: \rightarrow 1·5·9 + 2·6·7 + 3·4·8 - 7·5·3 - 8·6·1 - 9·4·2 = 45 + 84 + 96 - 105 - 48 - 72 = 225 - 225 = 0

3.7 Determinants and Cramer's Rule

• Area of a Triangle
$$Area = \pm \frac{1}{2} \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$
where x's and y's are the coordinates of the vertices

3.7 Determinants and Cramer's Rule • Find the area of a triangle with vertices of (2,4), (5,1), and (2,-2) • $Area = \pm \frac{1}{2} \begin{bmatrix} 2 & 4 & 1 \\ 5 & 1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$

ANS:
$$\rightarrow \frac{1}{2} (2 \cdot 1 \cdot 1 + 4 \cdot 1 \cdot 2 + 1 \cdot 5 \cdot -2 - 2 \cdot 1 \cdot 1 - -2 \cdot 1 \cdot 2 - 1 \cdot 5 \cdot 4)$$

 $\frac{1}{2} (2 + 8 + -10 - 2 - -4 - 20) = \frac{1}{2} (-18) = -9$
Area = 9

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• Cramer's Rule

• Cramer's Rule

\zeta \text{ Write the equations in standard form}

\zeta \text{ Make a matrix out of the coefficients}

• 2x2

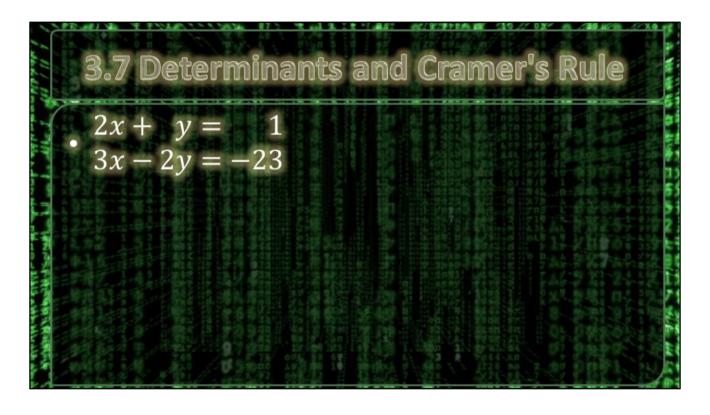
ax + by = e

• cx + dy = f

gives

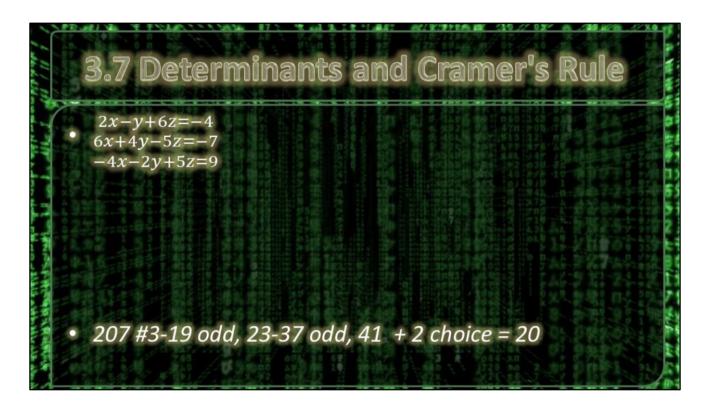
x = \frac{\begin{vmatrix} e & b \\ f & d \\ a & b \end{vmatrix}}{\begin{vmatrix} c & d \\ c & d \end{vmatrix}}, y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}
```

Notice that the numerator and denominator are the same except for the columns containing the coefficients of the variable you are solving for is replaced with the numbers from the constants column.

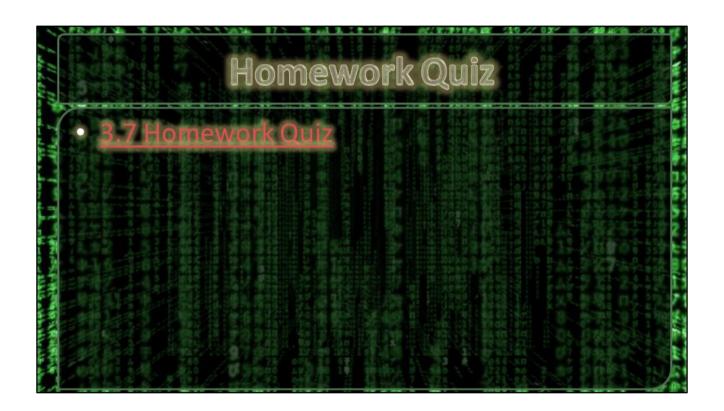


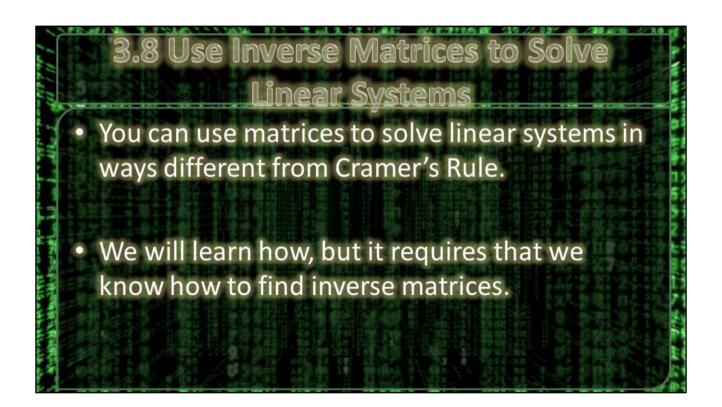
(-3, 7)

3.7 Determinants and Cramer's Rule • Cramer's Rule on a 3x3 System ζ Same as 2x2 system ζ The denominator is the determinant of the coefficient matrix and the numerator is the same only with the column of the variable you are solving for replaced with the = column.



(-3, 4, -1)





	THE RESERVE	100	latrices to Solve stems		
 The identity M 	atrix mu	ıltipli	ed with any matrix of the original matrix.		
• This is the matrix equivalent of 1					
$\begin{bmatrix} 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$	0	0]		
	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	$\begin{array}{c} 1 \\ 0 \end{array}$			

1's on diagonal, everything else is zero

3.8 Use Inverse Matrices to Solve Linear Systems You cannot divide by a matrix! So we multiply by the inverse of a matrix. A·A⁻¹ = [1] = I ζ Just like x (x⁻¹) = x (¹/_x) = 1 If A, B, and X are matrices, and ζ A·X = B ζ A·X = B ζ I·X = A⁻¹·B ζ I·X = A⁻¹·B ζ X = A⁻¹·B

3.8 Use Inverse Matrices to Solve Linear Systems • The Rule for 2x2 (Memorize) • If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ • $A^{-1} = \frac{1}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

3.8 Use Inverse Matrices to Solve

Linear Systems
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{1(4) - 3(2)} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

3.8 Use Inverse Matrices to Solve

Linear Systems
$$\begin{bmatrix}
-2 & -1 \\ 4 & 0
\end{bmatrix}^{-1} = \frac{1}{\begin{vmatrix} -2 & -1 \\ 4 & 0 \end{vmatrix}} \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}$$

$$= \frac{1}{-2(0) - 4(-1)} \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \frac{1}{4} \\ -1 & -\frac{1}{2} \end{bmatrix}$$

3.8 Use Inverse Matrices to Solve
Linear Systems

• Check by multiplying the two matrices
$$\begin{bmatrix} -2 & -1 \\ 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{1}{4} \\ -1 & -\frac{1}{2} \end{bmatrix}$$

3.8 Use Inverse Matrices to Solve Linear Systems • Solve a matrix equation $\zeta \text{ AX = B}$ $\zeta \begin{bmatrix} -3 & 4 \\ 5 & -7 \end{bmatrix} X = \begin{bmatrix} 3 & 8 \\ 2 & -2 \end{bmatrix}$ • Find A-1 • $\begin{bmatrix} -3 & 4 \\ 5 & -7 \end{bmatrix}^{-1} = \frac{1}{-3(-7)-5(4)} \begin{bmatrix} -7 & -4 \\ -5 & -3 \end{bmatrix}$ • $A^{-1} = \begin{bmatrix} -7 & -4 \\ -5 & -3 \end{bmatrix}$

3.8 Use Inverse Matrices to Solve

Linear Systems

• A-1AX = A-1B
$$\begin{bmatrix}
-7 & -4 \\
-5 & -3
\end{bmatrix}
\begin{bmatrix}
-3 & 4 \\
5 & -7
\end{bmatrix}
X = \begin{bmatrix}
-7 & -4 \\
-5 & -3
\end{bmatrix}
\begin{bmatrix}
3 & 8 \\
2 & -2
\end{bmatrix}$$

$$I \cdot X = \begin{bmatrix}
-7(3) + -4(2) & -7(8) + -4(-2) \\
-5(3) + -3(2) & -5(8) + -3(-2)
\end{bmatrix}$$

$$X = \begin{bmatrix}
-29 & -48 \\
-21 & -34
\end{bmatrix}$$

3.8 Use Inverse Matrices to Solve Linear Systems 2x + y = -13 x - 3y = 11• Take your equation and write it as matrices $\zeta \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -13 \\ 11 \end{bmatrix}$ • Find the coefficient matrix inverse $\zeta \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}^{-1} = \frac{1}{-7} \begin{bmatrix} -3 & -1 \\ -1 & 2 \end{bmatrix}$

3.8 Use Inverse Matrices to Solve Linear Systems • Multiply the front of both sides by the inverse $\begin{bmatrix} x \\ y \end{bmatrix}$ $\zeta \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-7} \begin{bmatrix} -3 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -13 \\ 11 \end{bmatrix}$ $\zeta = \frac{1}{-7} \begin{bmatrix} 39 + -11 \\ 13 + 22 \end{bmatrix}$ $\zeta = \begin{bmatrix} -4 \\ -5 \end{bmatrix}$ $\zeta = \begin{bmatrix} -4 \\ -5 \end{bmatrix}$ (-4, -5)

