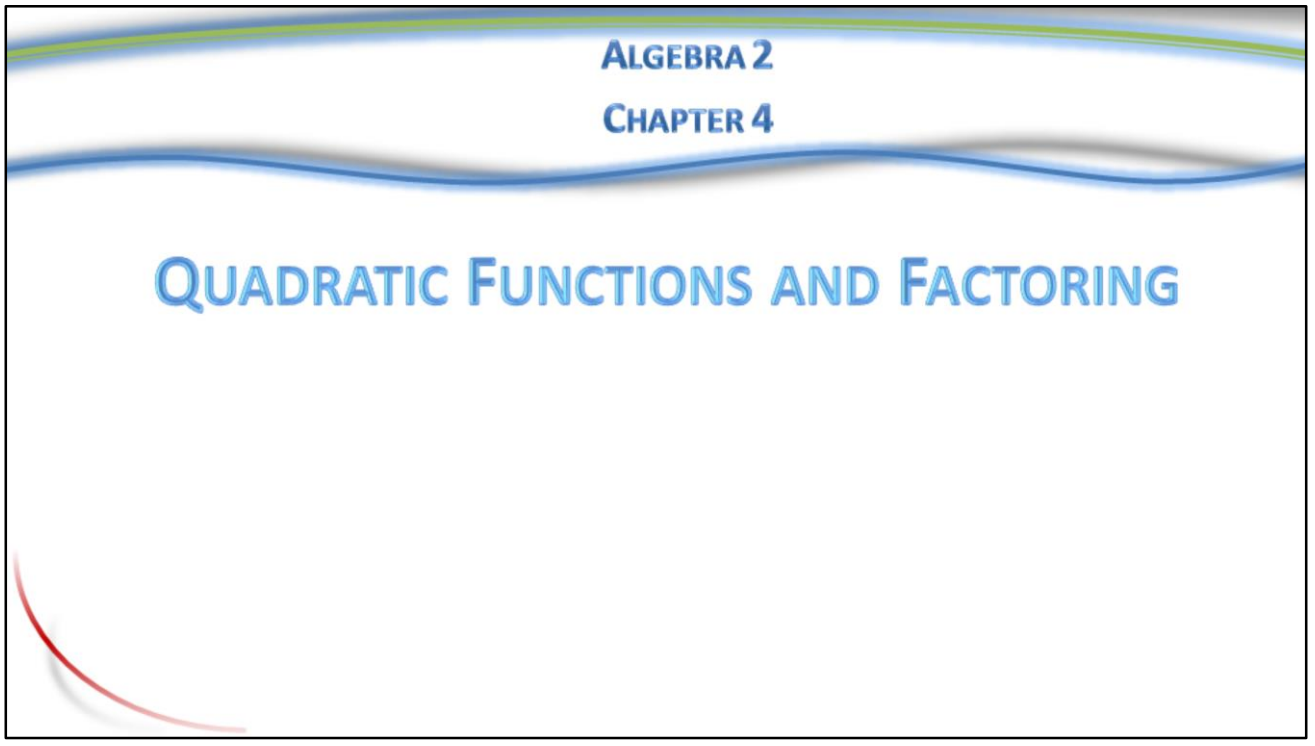

Algebra II 4

- This Slideshow was developed to accompany the textbook
  - *Larson Algebra 2*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

Slides created by
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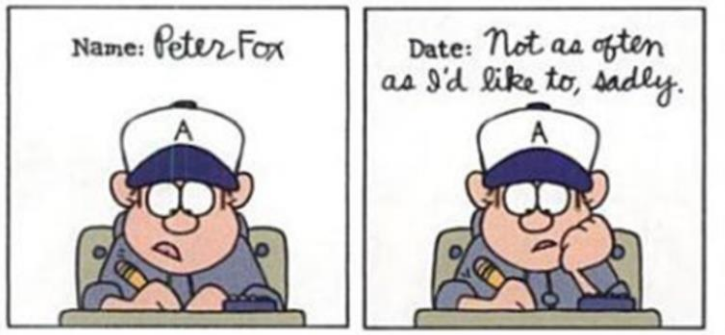
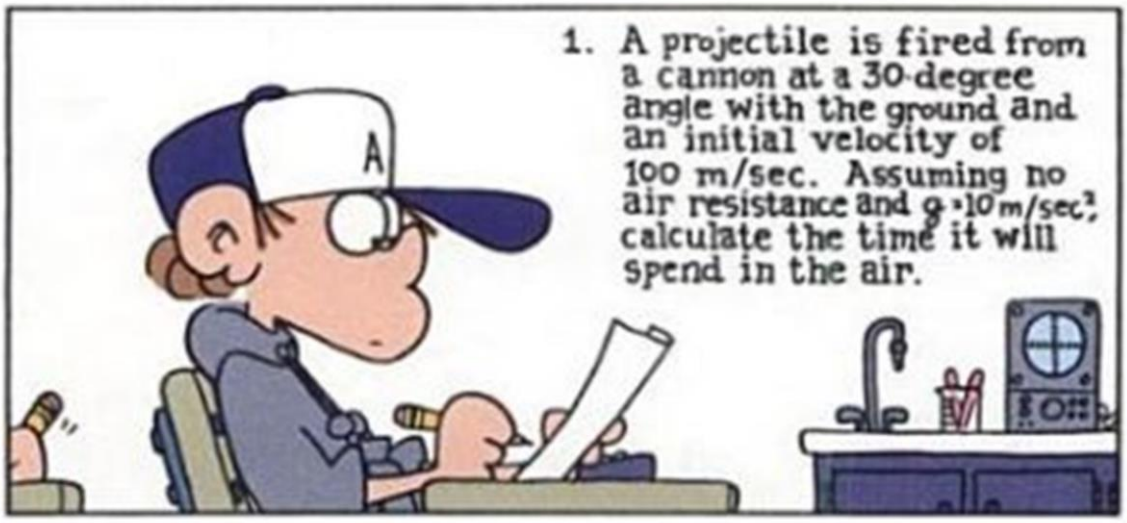
4.1 Graph Quadratic Functions in Standard Form

- Many physical events can be modeled with quadratic equations such as projectile motion.
- The graph of a projectile versus time looks exactly like the path the projectile takes.



4.1 Graph Quadratic Functions in Standard Form



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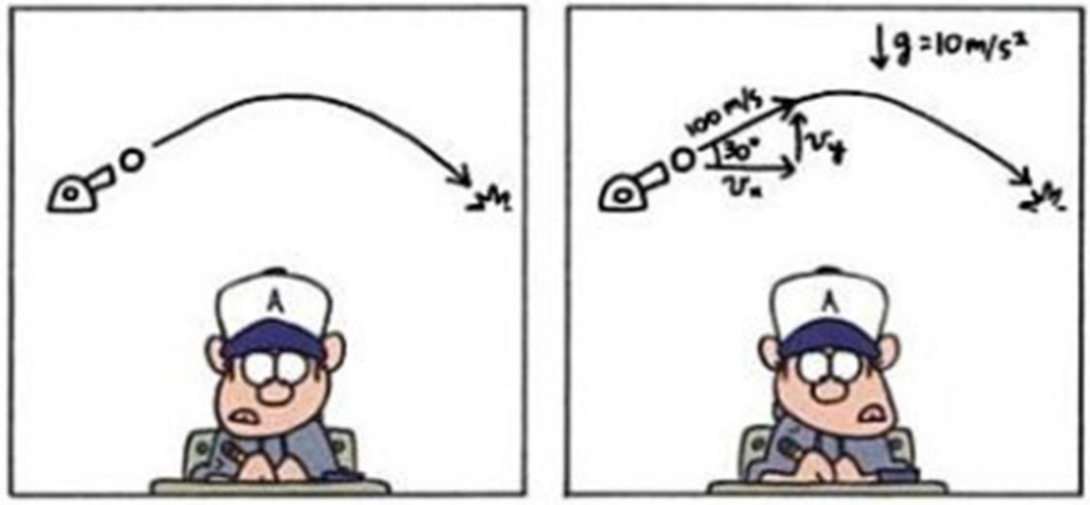

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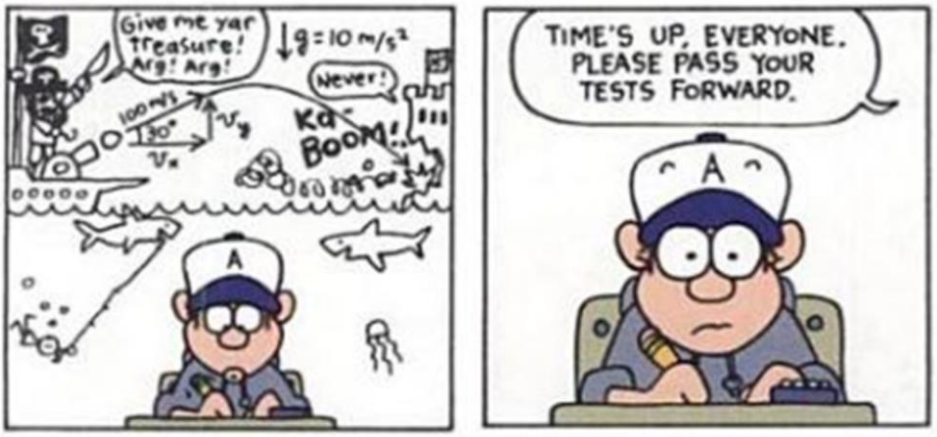
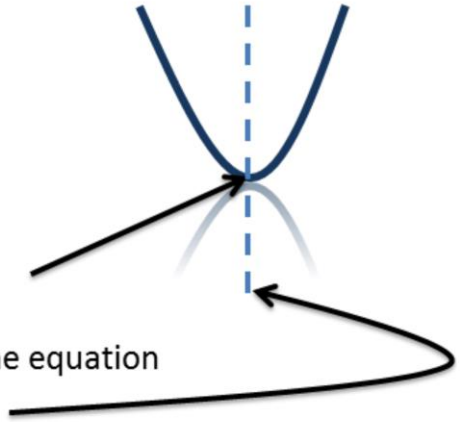


4.1 Graph Quadratic Functions in Standard Form

4.1 Graph Quadratic Functions in Standard Form

- Quadratic Function
  - $y = ax^2 + bx + c$
- Shape is a "u" or **parabola**
  - Opens up if $a > 0$; down if $a < 0$
  - If $|a| > 1$, then narrower than $y = x^2$
  - If $|a| < 1$, then wider than $y = x^2$
- **Vertex** → highest or lowest point
  - X coordinate is found by $-\frac{b}{2a}$
  - Y coordinate found by plugging $-\frac{b}{2a}$ into the equation
- Axis of symmetry → $x = -\frac{b}{2a}$



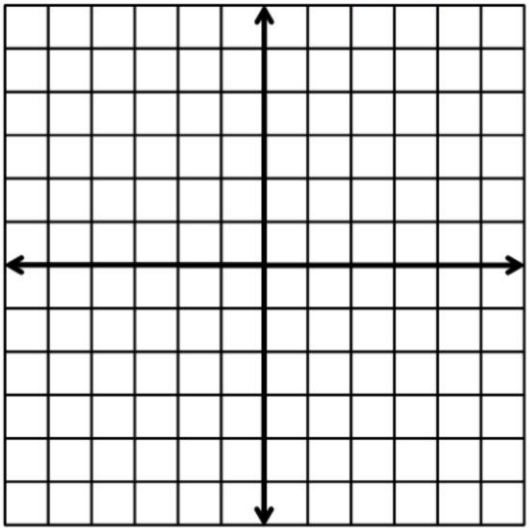
4.1 Graph Quadratic Functions in Standard Form

- How to Graph (Standard Form)
  - Find and plot the vertex
  - Make a table around the vertex
  - Draw the curve through all ***5*** points

4.1 Graph Quadratic Functions in Standard Form

- Graph $y = -x^2 + 2x$

- Graph $y = 2x^2 + 6x + 3$



Vertex at (1, 1) → other points (-2, -8), (-1, -3), (0, 0), (2, 0), (3, -3)

Vertex at (-1.5, -1.5) → other points (-3, 3), (-2, -1), (-1, -1), (0, 3), (1, 11)

4.1 Graph Quadratic Functions in Standard Form

- Find the minimum value of $y = 4x^2 + 16x - 3$

Minimum occurs at vertex: $x = -b/2a \rightarrow x = -16/(2 \cdot 4) = -2$
$y = 4(-2)^2 + 16(-2) - 3 = -19$
The minimum value is -19

4.1 Graph Quadratic Functions in Standard Form

- A video store sells about 150 DVDs a week for $20 each. The owner estimates that for each $1 decrease in price, about 25 more DVDs will be sold each week. How can the owner maximize weekly revenue?

Revenue is how much money comes in.
Revenue = Price · Number sold
$R = (20 - x)(150 + 25x) = 3000 + 500x - 150x - 25x^2$
$R = -25x^2 + 350x + 3000$
Maximum occurs at vertex
$x = -350/(2(-25)) = 7$
$y = -25(7)^2 + 350(7) + 3000 = 4225$
The owner should drop the price $7 making the revenue $4225

4.1 Graph Quadratic Functions in Standard Form

- *240 #1-57 every other odd + 5 choice = 20*

Homework Quiz

- 4.1 Homework Quiz

4.2 Graph Quadratic Functions in Vertex or Intercept Form

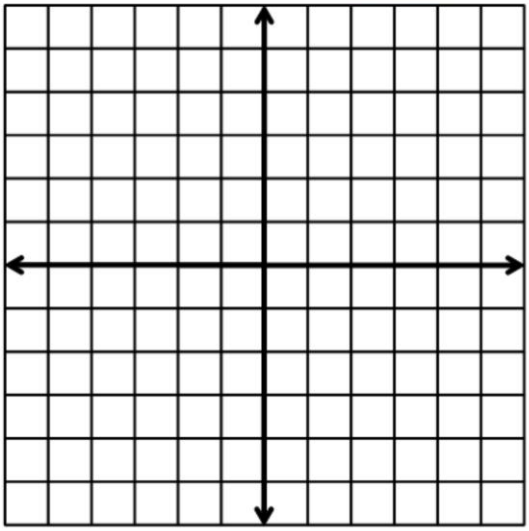
- Vertex Form
  - $y = a(x - h)^2 + k$
  - (h, k) is vertex
  - x = h is axis of symmetry

- (h, k) is the vertex because, if h and k = 0, then $y = ax^2$
  - The vertex of this is zero (from yesterday)
  - We learned before that
    - h is how far the graph moves right
    - k is how far the graph moves up

- Graph same way as before

4.2 Graph Quadratic Functions in Vertex or Intercept Form

- Graph $y = 2(x - 1)^2 + 3$



Vertex at (1, 3)→ other points (2, 5), (0, 5), (3, 11), (-1, 11)

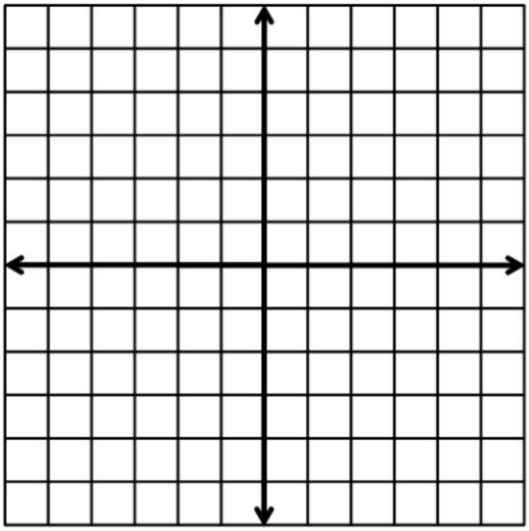
4.2 Graph Quadratic Functions in Vertex or Intercept Form

- Intercept Form
  - $y = a(x - p)(x - q)$
  - x-intercepts are p and q
  - axis of symmetry is halfway between p and q

- Graph the intercepts and find the axis of symmetry.
  - Vertex → x coordinate is axis of symmetry, plug in to equation to find y-value

4.2 Graph Quadratic Functions in Vertex or Intercept Form

- Graph $y = (x - 3)(x + 1)$



Vertex at (1, -4) → x-int at (3, 0) and (-1, 0)

4.2 Graph Quadratic Functions in Vertex or Intercept Form

- If an object is propelled straight upward from Earth at an initial velocity of 80 feet per second, its height after *t* seconds is given by h(t) = -16t(t – 5).
  - How many seconds after it is propelled will the object hit the ground?
  - What is the object's maximum height?

This is in intercept form y = -16(t – 0)(t – 5)
Thus it will be at zero height when either (t – 0) = 0 or (t – 5) = 0. t – 5 = 0 → t = 5 seconds

Maximum height occurs at the vertex: intercepts are 0 and 5, so vertex is at t = 2.5
h(2.5) = -16(2.5)(2.5 – 5) = 100 feet

4.2 Graph Quadratic Functions in Vertex or Intercept Form

FOIL

- To multiply $(x + 2)(x – 3)$
  - First → $x \cdot x$
  - Outer → $-3x$
  - Inner → $2x$
  - Last → $2 \cdot -3 = -6$
  - Add together → $x^2 – x – 6$

- To multiply $(x – 2)^2$
  - $(x – 2)(x – 2)$
  - FOIL
  - $x^2 – 4x + 4$

4.2 Graph Quadratic Functions in Vertex or Intercept Form

- Write the quadratic function in standard form
  - $y = -(x - 2)(x - 7)$
  - $f(x) = -(x + 2)^2 + 4$

- 249 #1-53 every other odd + 1 choice = 15

$y = -(x - 2)(x - 7) \rightarrow y = -(x^2 - 7x - 2x + 14) \rightarrow y = -(x^2 - 9x + 14) \rightarrow y = -x^2 + 9x - 14$

$f(x) = -(x + 2)^2 + 4 \rightarrow f(x) = -(x + 2)(x + 2) + 4 \rightarrow f(x) = -(x^2 + 2x + 2x + 4) + 4 \rightarrow f(x) = -(x^2 + 4x + 4) + 4 \rightarrow f(x) = -x^2 - 4x - 4 + 4 \rightarrow f(x) = -x^2 - 4x$

Homework Quiz

- 4.2 Homework Quiz

4.3 Solve $x^2 + bx + c = 0$ by Factoring

- Factoring is the opposite of FOILing
- Factoring undoes multiplication
- $(x + 2)(x + 5) = x^2 + 7x + 10$
  - $x + 2$ called **binomial**
  - $x^2 + 7x + 10$ called **trinomial**

4.3 Solve $x^2 + bx + c = 0$ by Factoring

- Factoring trinomial
  - $ax^2 + bx + c$

1. Write two sets of parentheses ( )( )
2. Guess and Check
3. The Firsts multiply to make ax^2
4. The Lasts multiply to make c
5. Check to make sure the Outers + Inners make bx

4.3 Solve $x^2 + bx + c = 0$ by Factoring

- Factor the expression
  - $x^2 - 3x - 18$
  - $n^2 - 3n + 9$
  - $r^2 + 2r - 63$

$(x - 6)(x + 3)$

Cannot be factored

$(r + 9)(r - 7)$

4.3 Solve $x^2 + bx + c = 0$ by Factoring

- Special patterns
  - Difference of Squares
    - $a^2 - b^2 = (a - b)(a + b)$
  - Perfect Squares
    - $a^2 \pm 2ab + b^2 = (a \pm b)^2$
- Factor
  - $x^2 - 9$
  - $w^2 - 18w + 81$

Difference of Squares: $x^2 - 9 = (x - 3)(x + 3)$

Perfect Squares: $w^2 - 18w + 81 = (w - 9)^2$

4.3 Solve $x^2 + bx + c = 0$ by Factoring

- Solving quadratic equations by factoring
  - The solutions to quadratic equation are called zeros
- Zero Product Property
  - Zero times anything = 0
  - If ab = 0, then a = 0, b = 0, or both.
  - Thus we can factor a quadratic equation (remember factoring gives you at least two pieces multiplied together) and set each factor equal to zero to solve.
- Example:
  - Solve $(3y - 5)(2y + 7) = 0$

ANS: take each factor = 0

$3y - 5 = 0 \rightarrow 3y = 5 \rightarrow y = 5/3$

$2y + 7 = 0 \rightarrow 2y = -7 \rightarrow y = -7/2$

4.3 Solve $x^2 + bx + c = 0$ by Factoring

- Solve
  - $x^2 - x - 42 = 0$
  - $x^2 - 8x + 16 = 0$

$x^2 - x - 42 = 0 \rightarrow (x - 7)(x + 6) = 0 \rightarrow$

$x - 7 = 0 \rightarrow x = 7$

$x + 6 = 0 \rightarrow x = -6$

Solutions are $x = -6, 7$

$x^2 - 8x + 16 = 0 \rightarrow (x - 4)(x - 4) = 0 \rightarrow$

$x - 4 = 0 \rightarrow x = 4$

Solutions are $x = 4$

4.3 Solve $x^2 + bx + c = 0$ by Factoring

- Finding Zeros
  - Zeros are the values of x when y = 0
    - ➢ Also called x-intercepts or roots
  - When you find zeros make y = 0 and solve
- Find the zeros of $y = x^2 - 7x - 30$ by rewriting the function in intercept form.

$y = x^2 - 7x - 30 \rightarrow y = (x - 10)(x + 3)$
Zeros are -3, 10

4.3 Solve $x^2 + bx + c = 0$ by Factoring

- *255 #1-21 every other odd, 27-59 every other odd, 65 + 4 choice = 20*

Homework Quiz

- 4.3 Homework Quiz

4.4 Solve $ax^2 + bx + c = 0$ by Factoring

- Very similar to yesterday's lesson
- Two differences
  - Factor monomial first
  - Make *a* work just like *c*

4.4 Solve $ax^2 + bx + c = 0$ by Factoring

- Monomial First
  - Factor out any common terms first, then factor what's left
- $14x^2 + 2x - 12$

- $3x^2 - 18x$

→ $2(7x^2 + x - 6)$ → $2(7x - 6)(x + 1)$

→ $3x(x - 6)$

4.4 Solve $ax^2 + bx + c = 0$ by Factoring

- Factor
  - $12x^2 + 3x + 3$
  - $2x^2 - 32$

$3(4x^2 + x + 1)$

$2(x^2 - 16) \rightarrow 2(x - 4)(x + 4)$

4.4 Solve $ax^2 + bx + c = 0$ by Factoring

- Solve
  - $9t^2 - 12t + 4 = 0$
  - $3x - 6 = x^2 - 10$

ANS: $(3t - 2)^2 = 0 \rightarrow 3t - 2 = 0 \rightarrow 3t = 2 \rightarrow t = 2/3$

ANS: Put in standard form

$x^2 - 3x - 4 = 0$

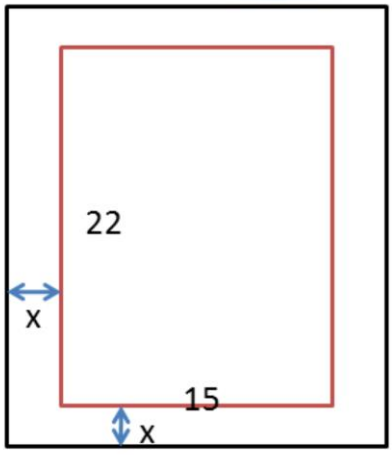
$(x - 4)(x + 1) = 0$

$x - 4 = 0 \rightarrow x = 4$

$x + 1 = 0 \rightarrow x = -1$

4.4 Solve $ax^2 + bx + c = 0$ by Factoring

- You are designing a garden. You want the garden to be made up of a rectangular flower bed surrounded by a border of uniform width to be covered with decorative stones. You have decided that the flower bed will be 22 feet by 15 feet, and your budget will allow for enough stone to cover 120 square feet. What should be the width of the border?



Outer rectangle area – inner rectangle area = 120

$(2x + 22)(2x + 15) – (22)(15) = 120$

$4x^2 + 30x + 44x + 330 – 330 = 120$

$4x^2 + 74x = 120$

$4x^2 + 74x – 120 = 0$

Divide by 2: $2x^2 + 37x – 60 = 0$

Factor: $(2x – 3)(x + 20) = 0$

$2x – 3 = 0 \rightarrow 2x = 3 \rightarrow x = 3/2$

$x + 20 = 0 \rightarrow x = -20$ can't use negative numbers

Border is 3/2 feet or 1.5 feet or 18 inches

4.4 Solve $ax^2 + bx + c = 0$ by Factoring

- *263 #3-27 every other odd, 31-47 odd, 51, 55, 59, 65 + 5 choice = 25*

Homework Quiz

- 4.4 Homework Quiz

4.5 Solve Quadratic Equations by Finding Square Roots

- Find
  - $\sqrt{36}$
  - $\sqrt{9}$
  - $\sqrt{4}$
- What do you notice?

√36 = √9√4

4.5 Solve Quadratic Equations by Finding Square Roots

- Square Root Definition
  - If $a^2 = b$, then a is the square root of b
  - A positive number has 2 square roots shown by $\sqrt{b}$ and $-\sqrt{b}$

radical sign → $\sqrt{36}$ ← radicand

  - Expression with radical sign is called a radical expression
- Simplifying
  - All perfect squares taken out
  - No radicals in denominator

The term radical comes from Latin “radix” which means “root”. Other terms with same root: “Radish”, “eradicate” (pull out be roots)

4.5 Solve Quadratic Equations by Finding Square Roots

- Properties of square roots
- Product Property
  - $\sqrt{ab} = \sqrt{a}\sqrt{b}$
- Quotient Property
  - $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

- Examples: Simplify
  - $\sqrt{500}$
  - $3\sqrt{12}\sqrt{6}$

→√5 √100 → 10√5
→ 3√72 → 3√36 √2 → (3)(6)√2 → 18√2

4.5 Solve Quadratic Equations by Finding Square Roots

- Simplify
  - $\sqrt{\frac{25}{3}}$
- $\frac{5}{2+\sqrt{3}}$

→√25 / √3 → 5 / √3 → 5√3 / (√3 √3) → 5√3 / 3

→$\frac{5}{2+\sqrt{3}} = \frac{5(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{10-5\sqrt{3}}{4-3} = 10-5\sqrt{3}$

4.5 Solve Quadratic Equations by Finding Square Roots

- Solving Quadratic Equations by finding square roots
  - When? Only 1 term with x and it is squared
  - Isolate the square and then take the $\pm\sqrt{}$
- Solve
  - $3 - 5x^2 = -9$
  - $3(x-2)^2 = 21$

- *269 #1-39 odd + 5 choice = 25*

ANS: $-5x^2 = -12 \rightarrow x^2 = 12/5 \rightarrow x = \pm\sqrt{(12/5)} \rightarrow x = \pm\sqrt{12}/\sqrt{5} \rightarrow x = \pm 2\sqrt{3}\ \sqrt{5} / \sqrt{5}\ \sqrt{5} \rightarrow x = \pm 2\sqrt{15} / 5$

ANS: $(x-2)^2 = 7 \rightarrow x - 2 = \pm\sqrt{7} \rightarrow x = 2 \pm \sqrt{7}$

Homework Quiz

- 4.5 Homework Quiz

4.6 Perform Operations with Complex Numbers

- When we were young we learned to count.
- Then as we got older we learned to operate with combining those counting numbers.
- Next we learned about negative numbers and fractions. With this came more rules for the operations.
- Finally we are going to learn about complex numbers and the rules for dealing with them.

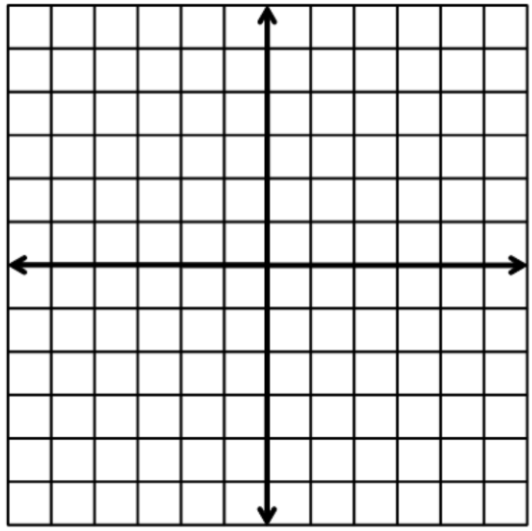
4.6 Perform Operations with Complex Numbers

- Imaginary Number (imaginary unit)
  - i
  - $i = \sqrt{-1}$
  - $i^2 = -1$
- Examples
  - $\sqrt{-9}$
  - $\sqrt{-12}$

→ $3i$
→ $i\sqrt{4}\sqrt{3}$ → $2i\sqrt{3}$

4.6 Perform Operations with Complex Numbers

- Complex Number
  - Includes real numbers and imaginary numbers
- Imaginary numbers are any number with *i*
  - (a + bi) where a and b are real
- Plotting
  - Complex plane → x-axis is the real axis; y-axis is the imaginary axis
- Examples: plot
  - -4-i
  - 5
  - 1 + 3i



4.6 Perform Operations with Complex Numbers

- Adding and Subtracting Complex Numbers
  - Add the same way you add (x + 4) + (2x – 3) = 3x+1
  - Combine like terms
- Examples: simplify
  - (-1 + 2*i*) + (3 + 3*i*)
  - (2 – 3*i*) – (3 – 7*i*)
  - 2*i* – (3 + *i*) + (2 – 3*i*)

2 + 5i
-1 + 4i
-1

4.6 Perform Operations with Complex Numbers

- Multiplying complex numbers
  - FOIL
  - Remember $i^2 = -1$
- Examples: multiply
  - $-i(3 + i)$
  - $(2 + 3i)(-6 - 2i)$
  - $(1 + 2i)(1 - 2i)$

$-3i - i^2 = 1 - 3i$
$-12 - 4i - 18i - 6i^2 = -6 - 22i$
$1 - 2i + 2i - 4i^2 = 5$

4.6 Perform Operations with Complex Numbers

- Notice on the last example that the answer was just real

- Complex conjugate → same numbers just opposite sign on the imaginary part
  - When you multiply complex conjugates, the product is real
- Dividing Complex Numbers
  - To divide, multiply the numerator and denominator by the complex conjugate of the denominator
  - No imaginary numbers are allowed in the denominator when simplified

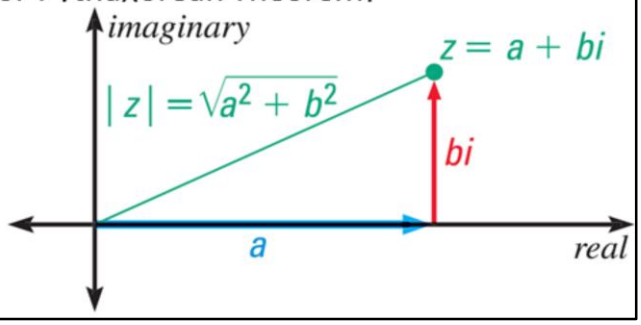
4.6 Perform Operations with Complex Numbers

- Divide
- $\frac{2-7i}{1+i}$

(-5-9i)/2

4.6 Perform Operations with Complex Numbers

- Absolute Value
  - Distance a number is from the origin
  - $| z | = \sqrt{a^2 + b^2}$ where z = a + bi
    - This is the distance formula (or Pythagorean Theorem)
- Example:
  - Find | 2 – 4i |

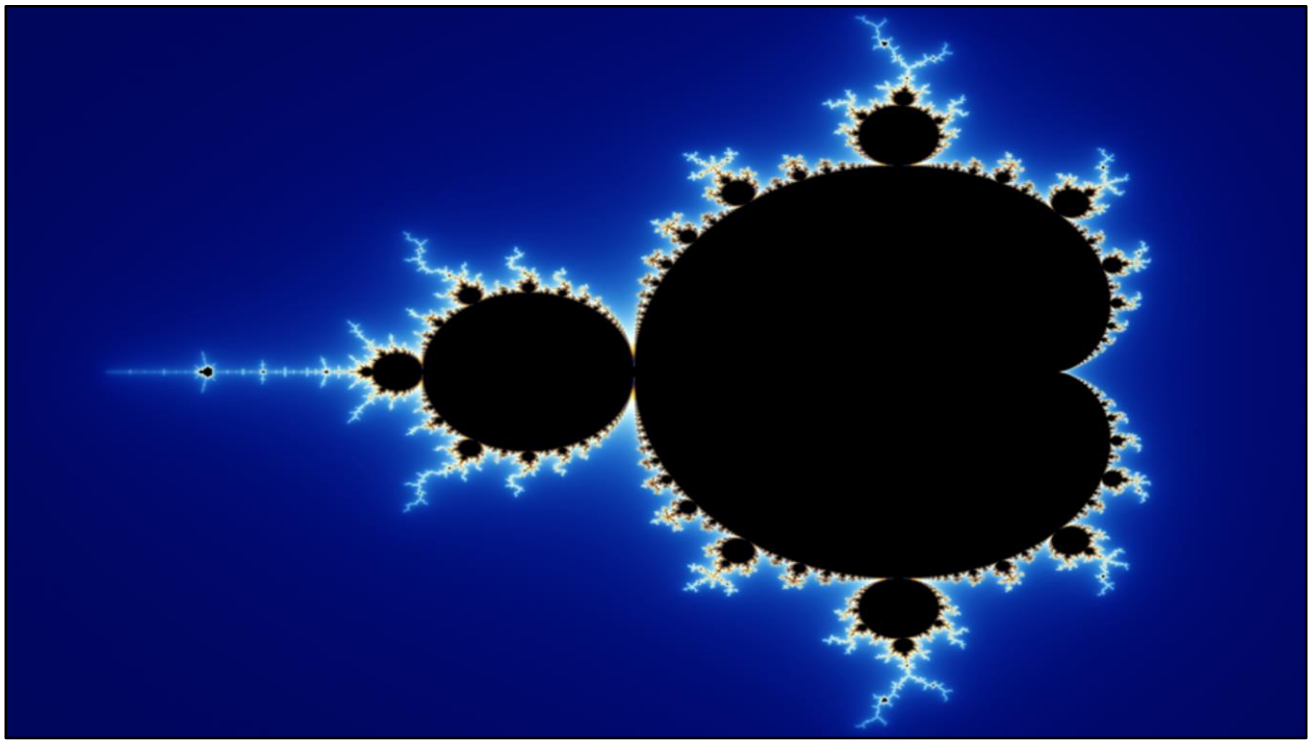


$\sqrt{(2^2 + (-4)^2)} = \sqrt{(4 + 16)} = \sqrt{20} = 2\sqrt{5}$

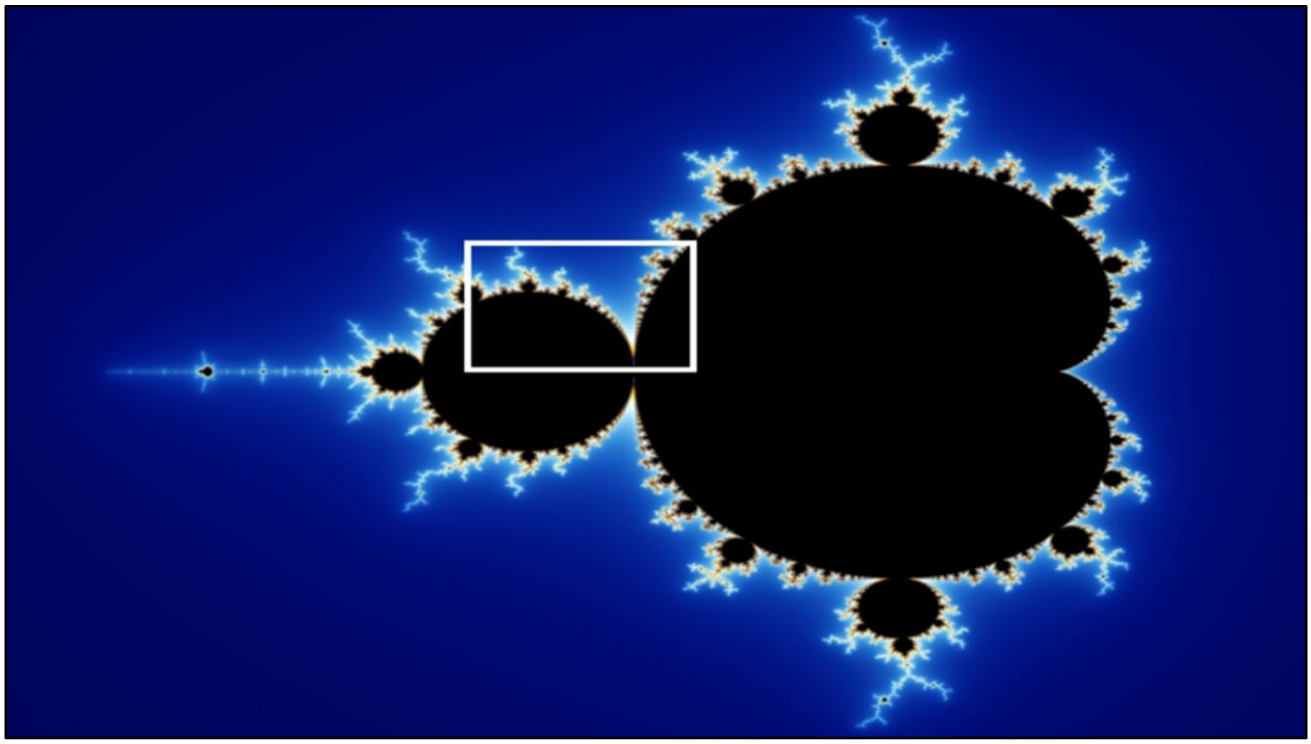
4.6 Perform Operations with Complex Numbers

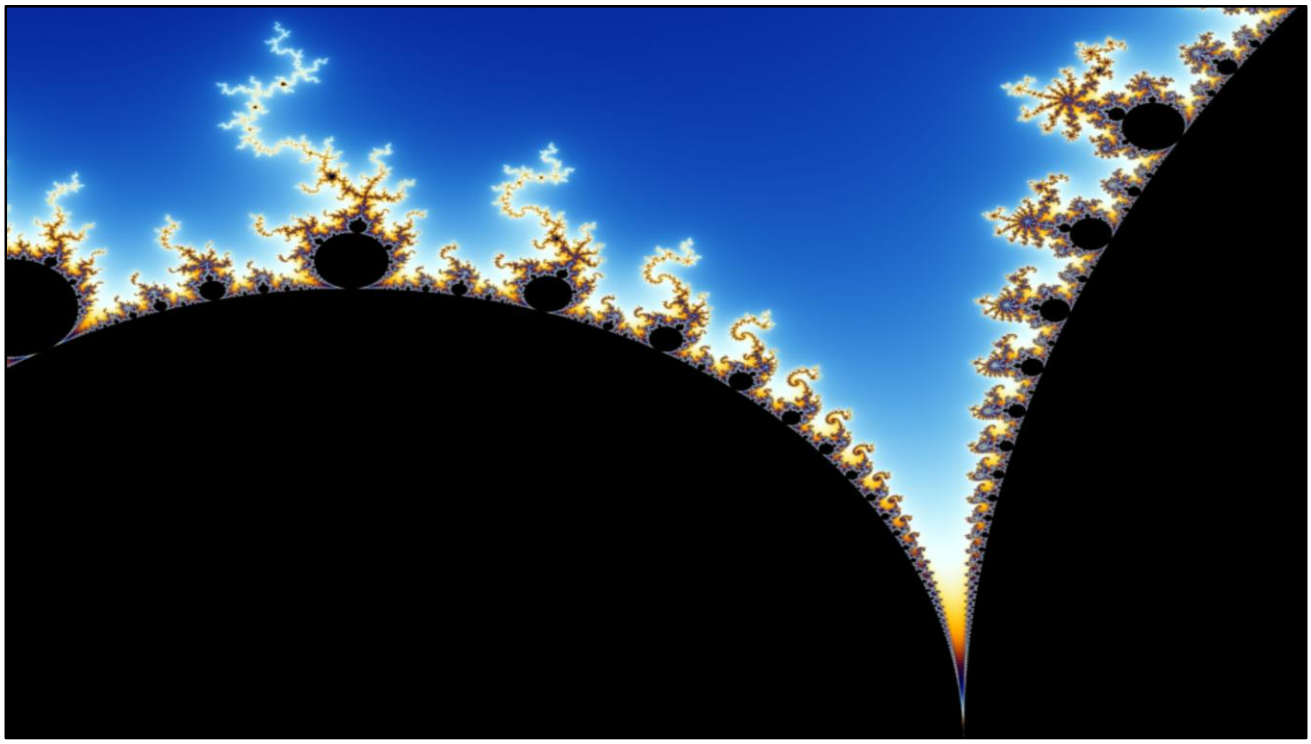
- Mandelbrot Set
  - Fractal picture created using complex numbers
- In the following pictures
  - the black is part of the Mandelbrot Set
  - The color is based on the number of iterations before $z > \sqrt{5}$

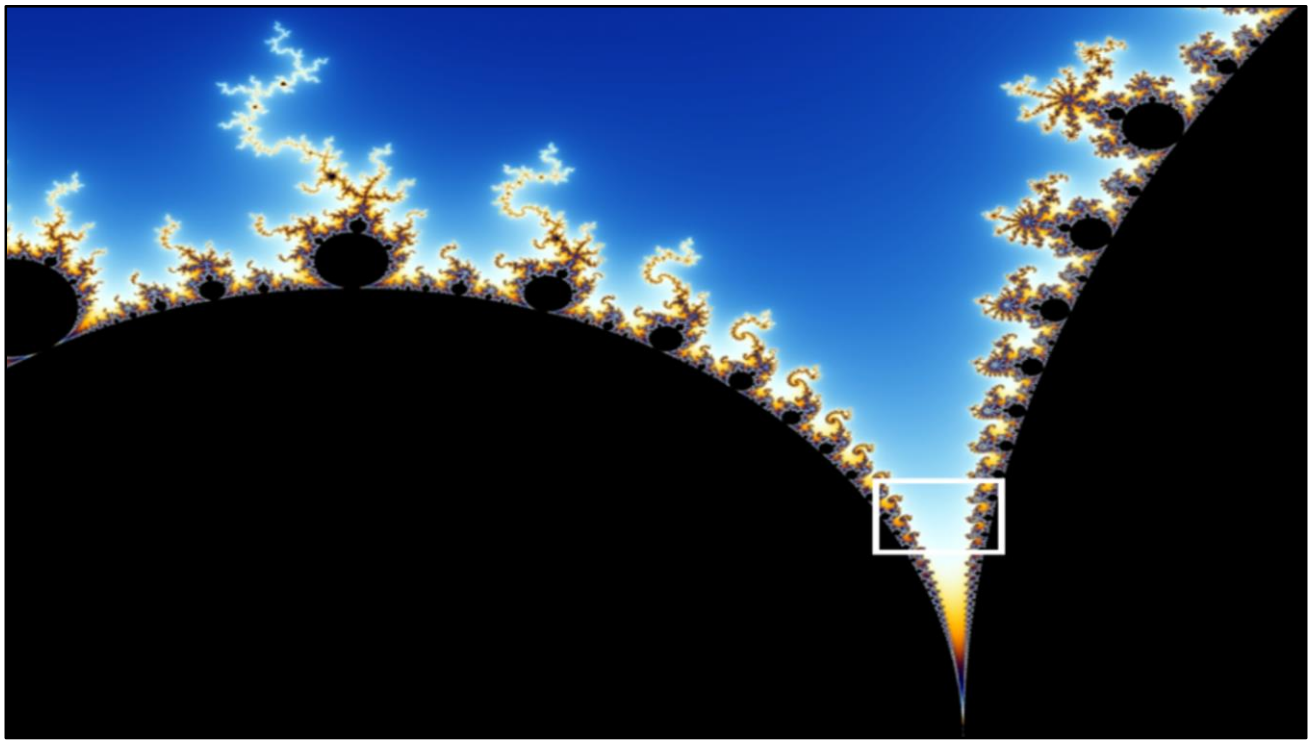
We'll find out about z later

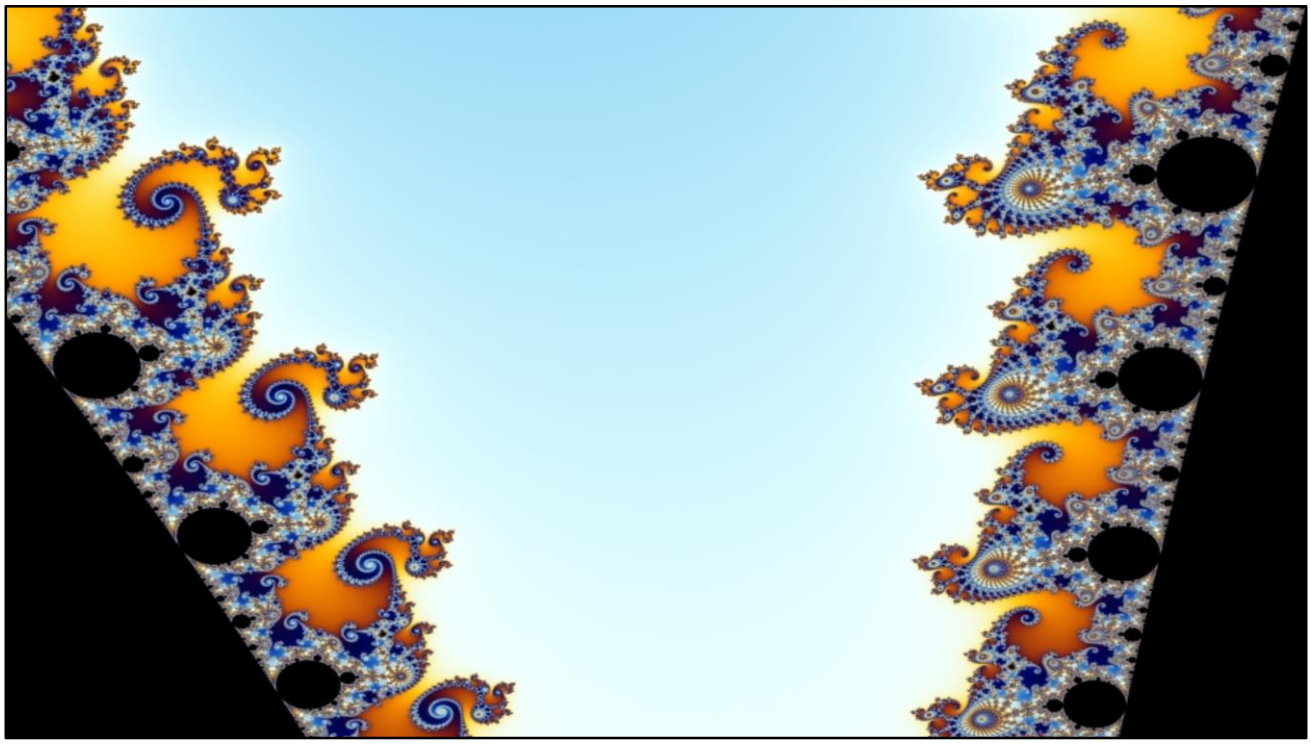


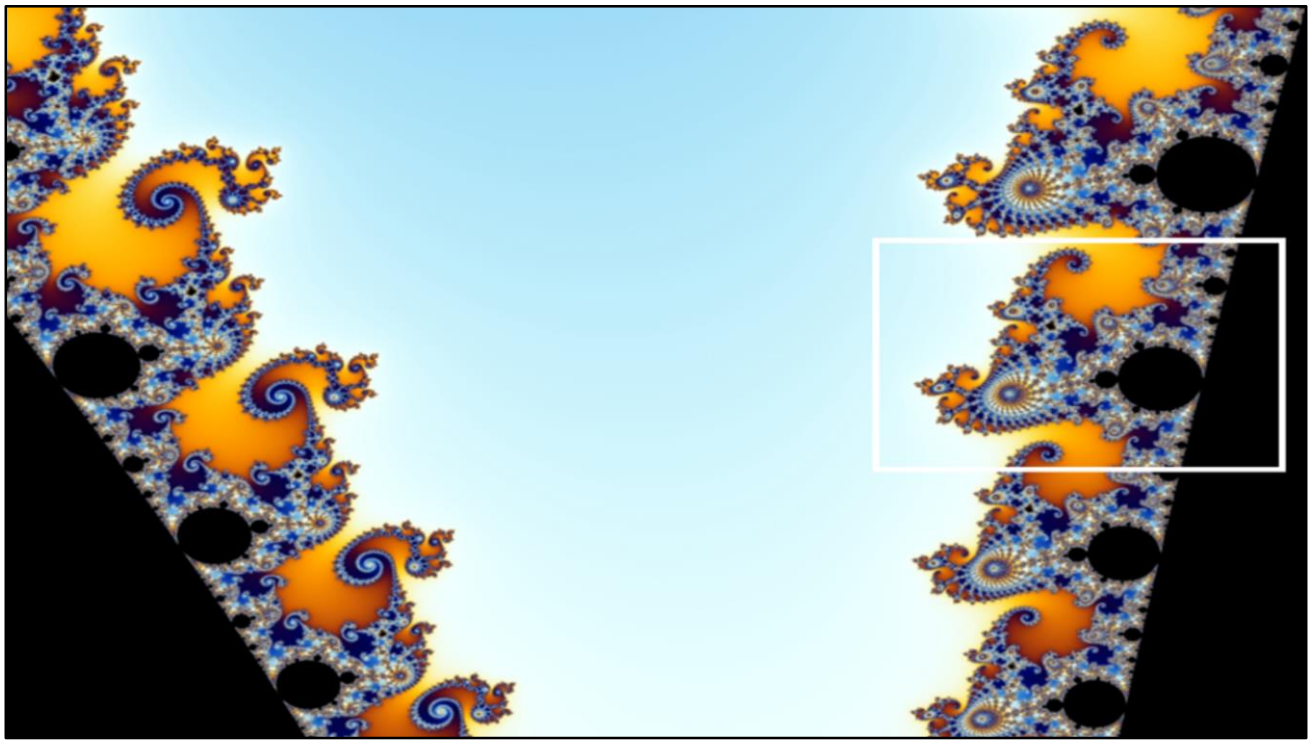
Black is part of the set
Color indicates how many iterations were required before z became larger than √5

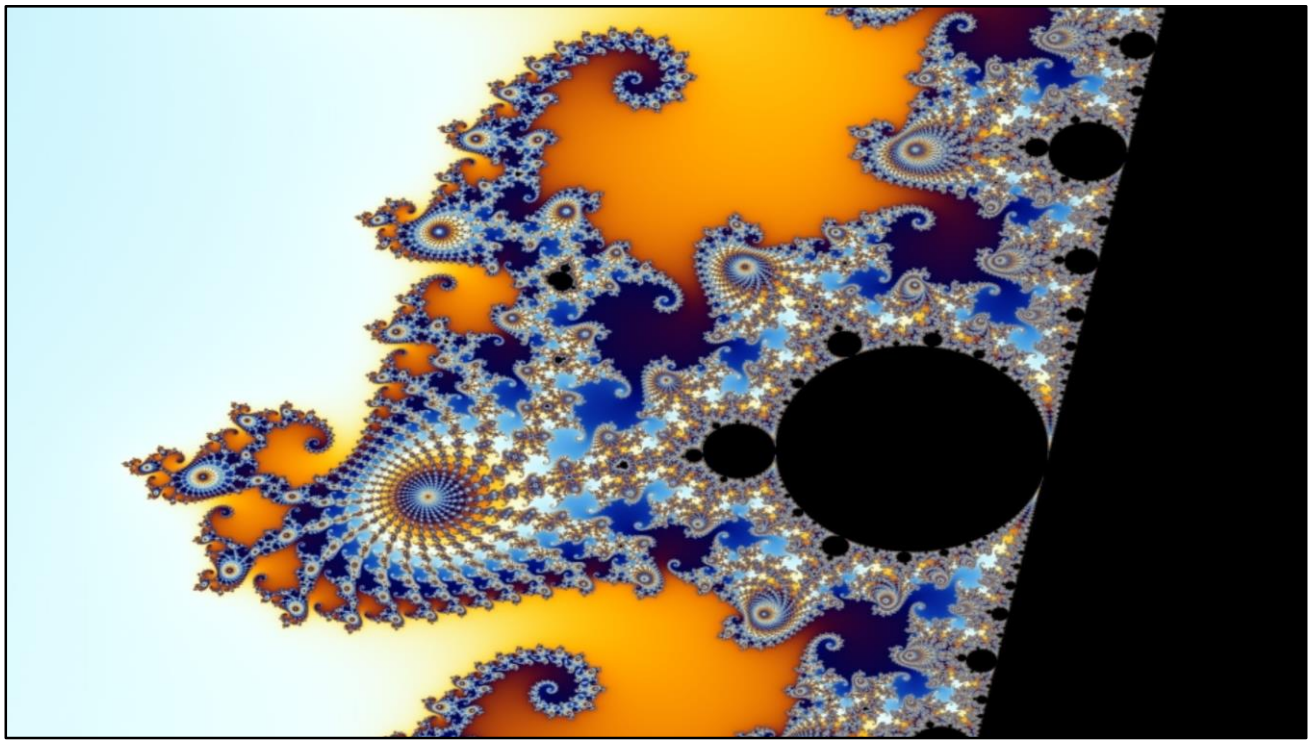


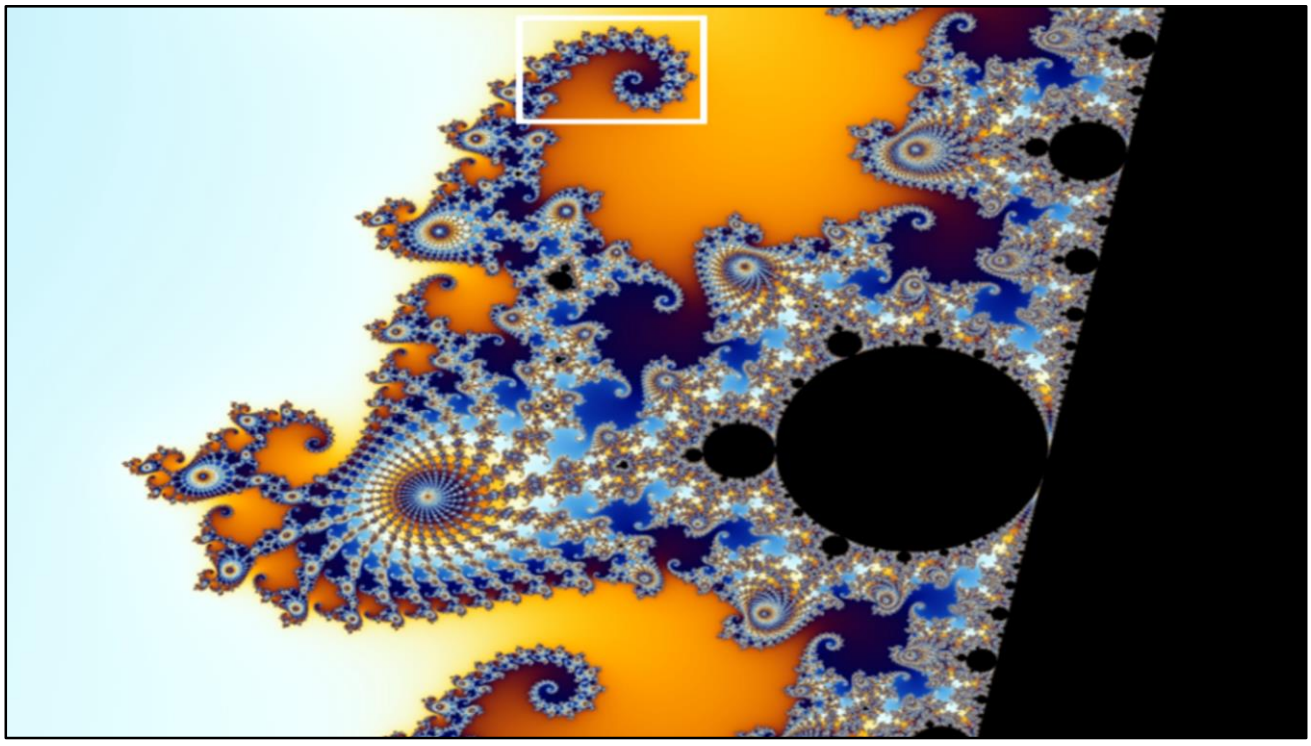


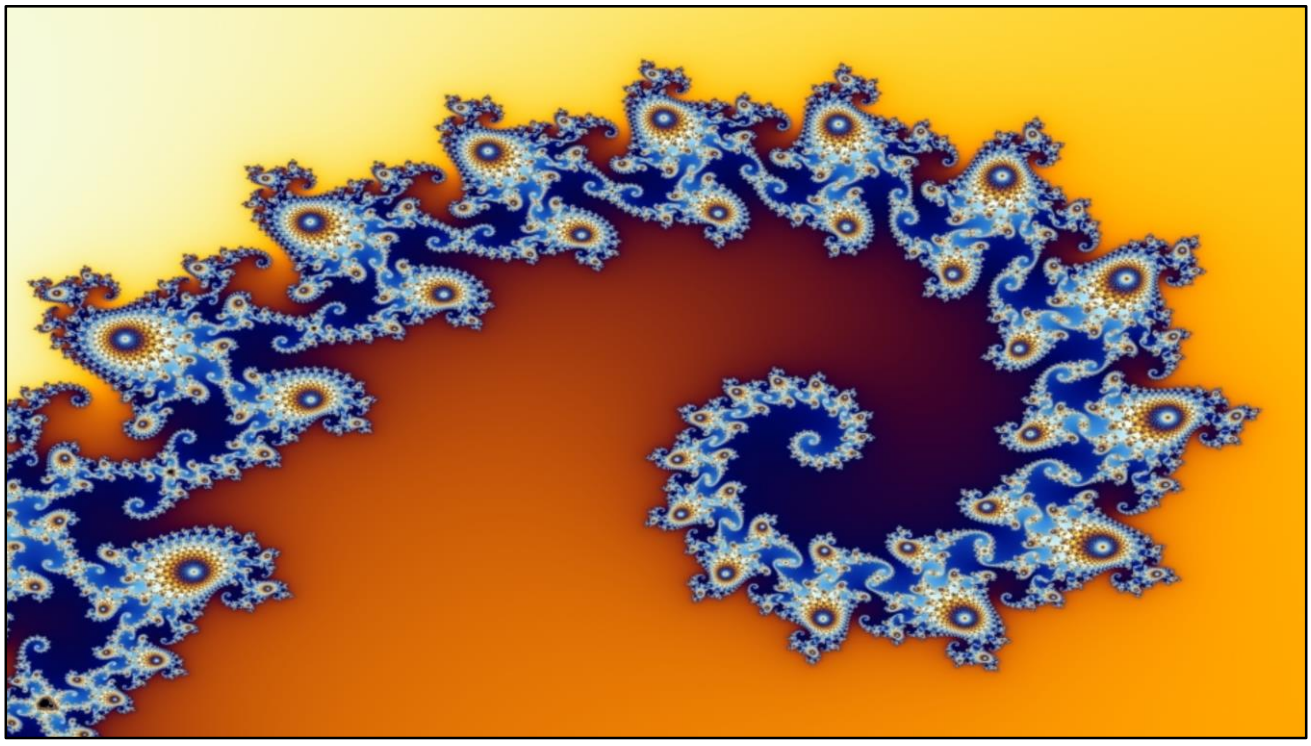


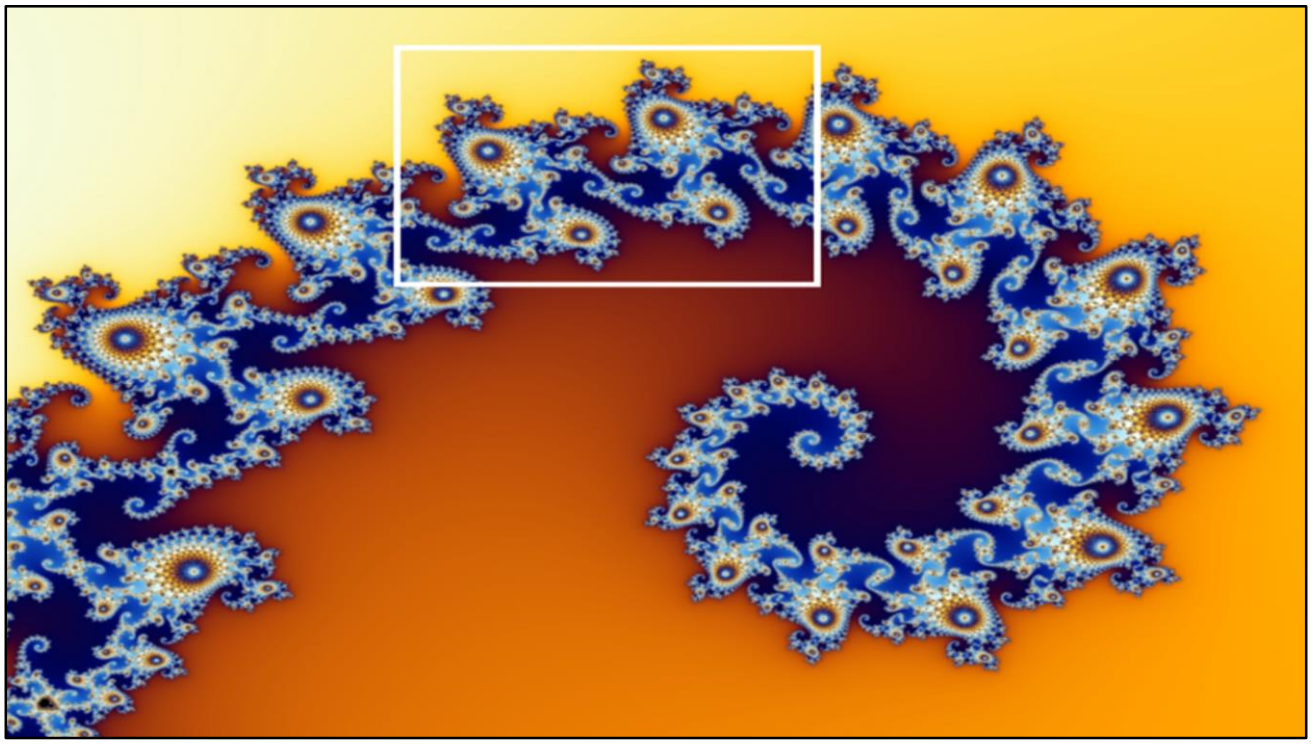


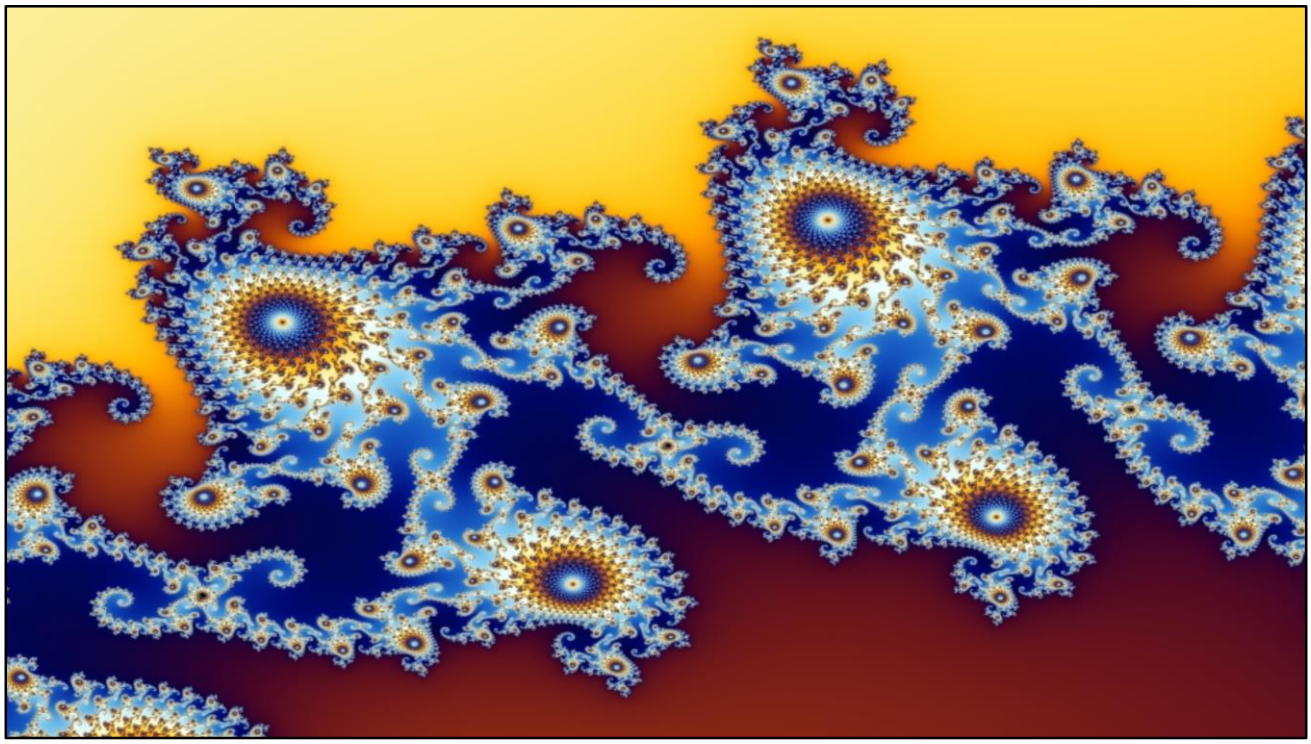


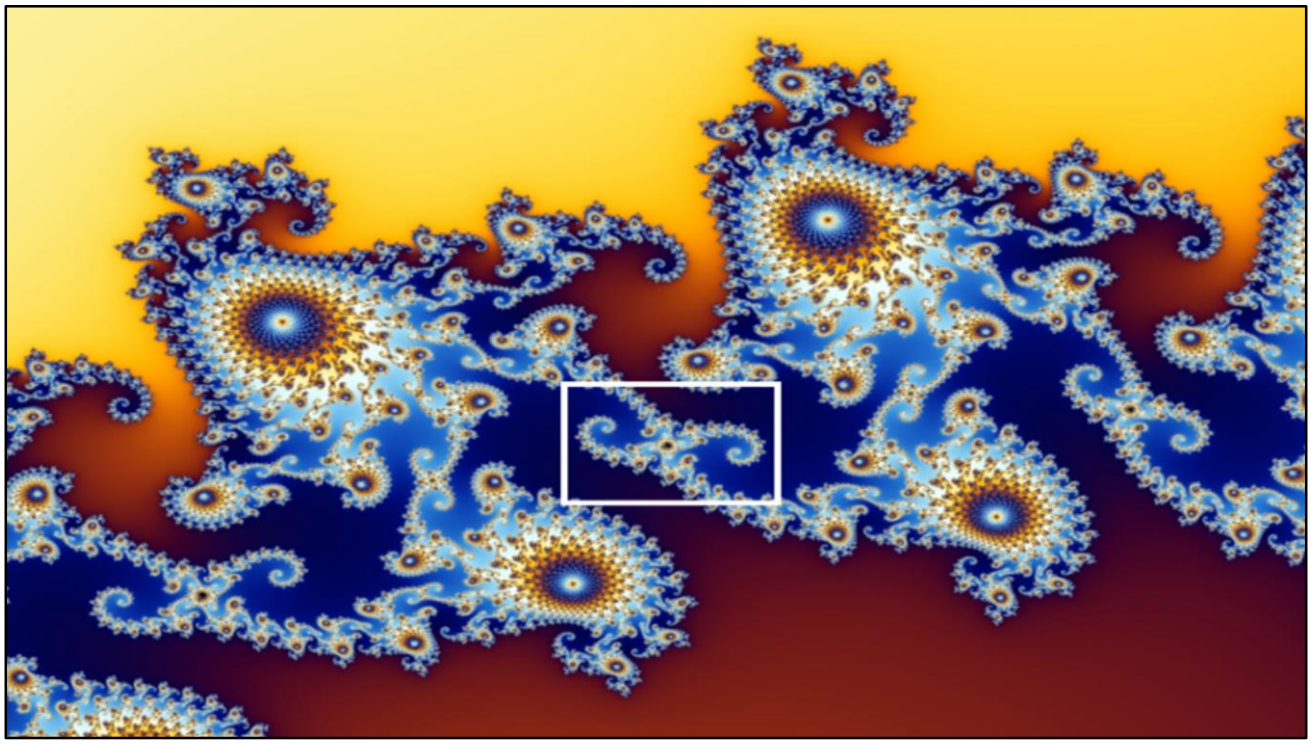


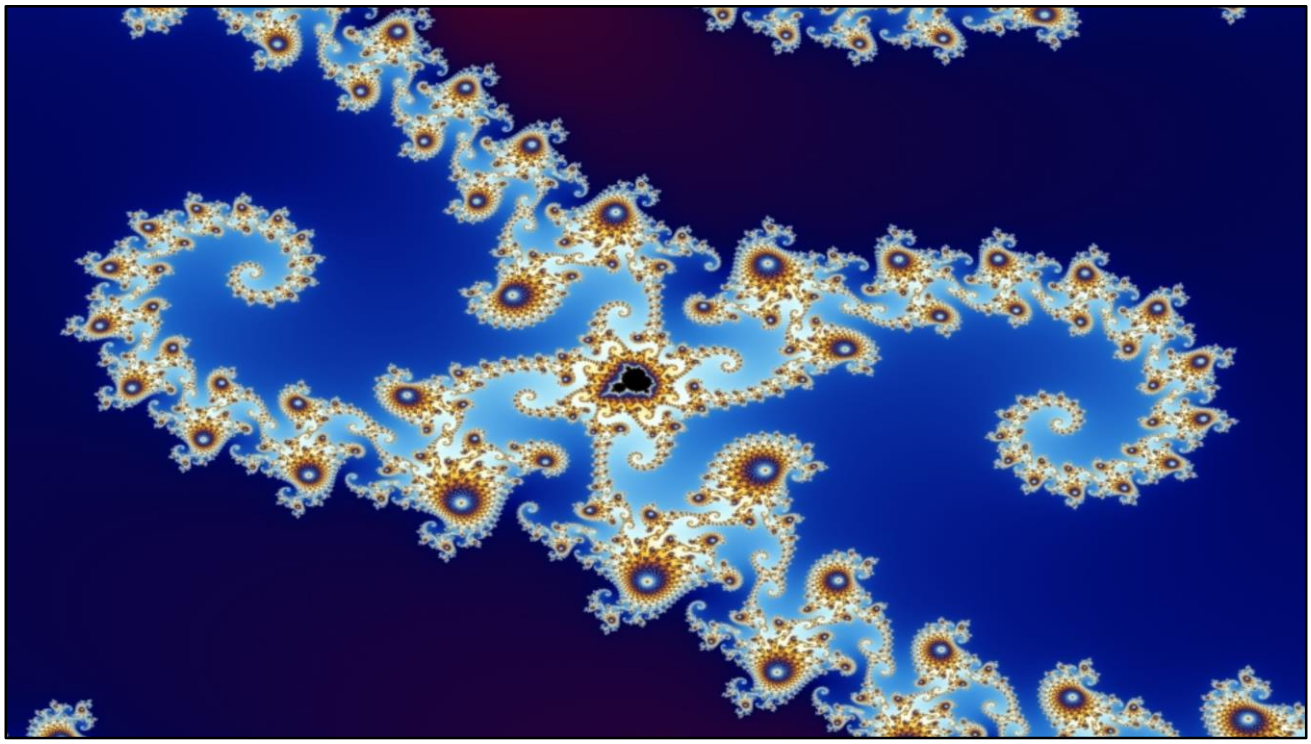


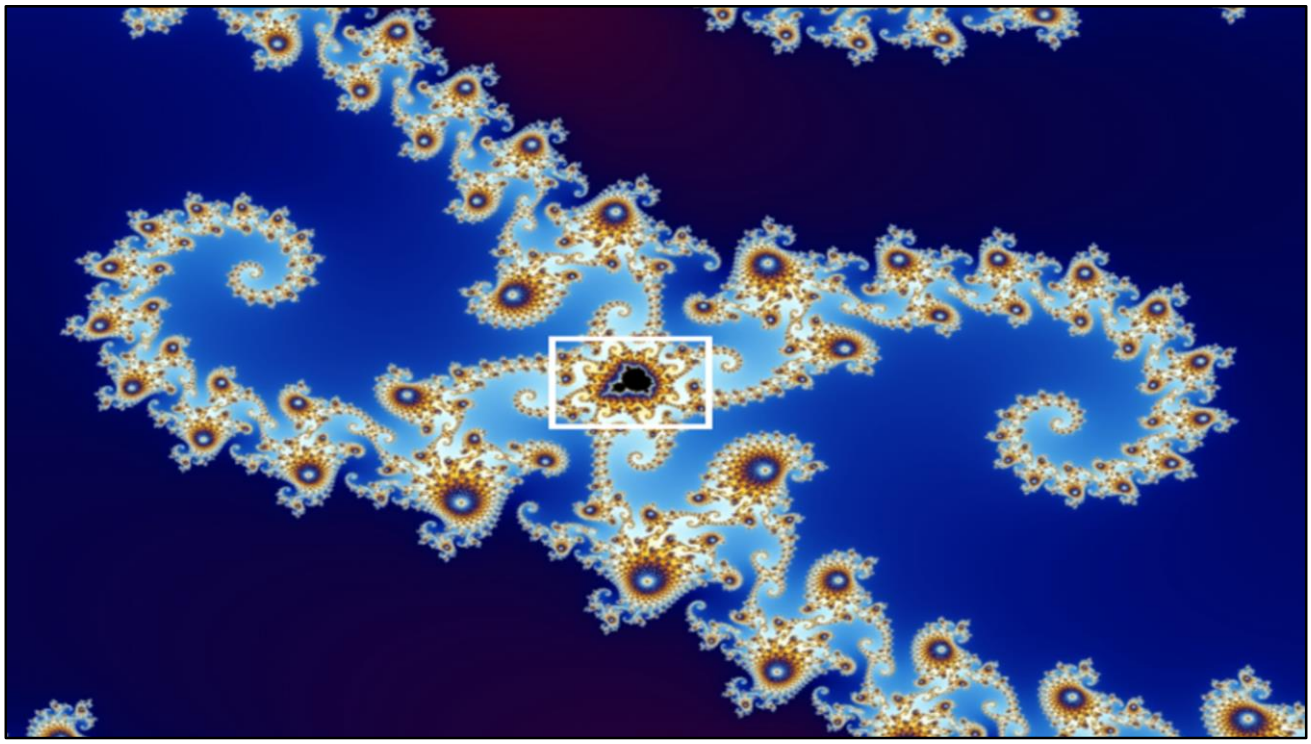


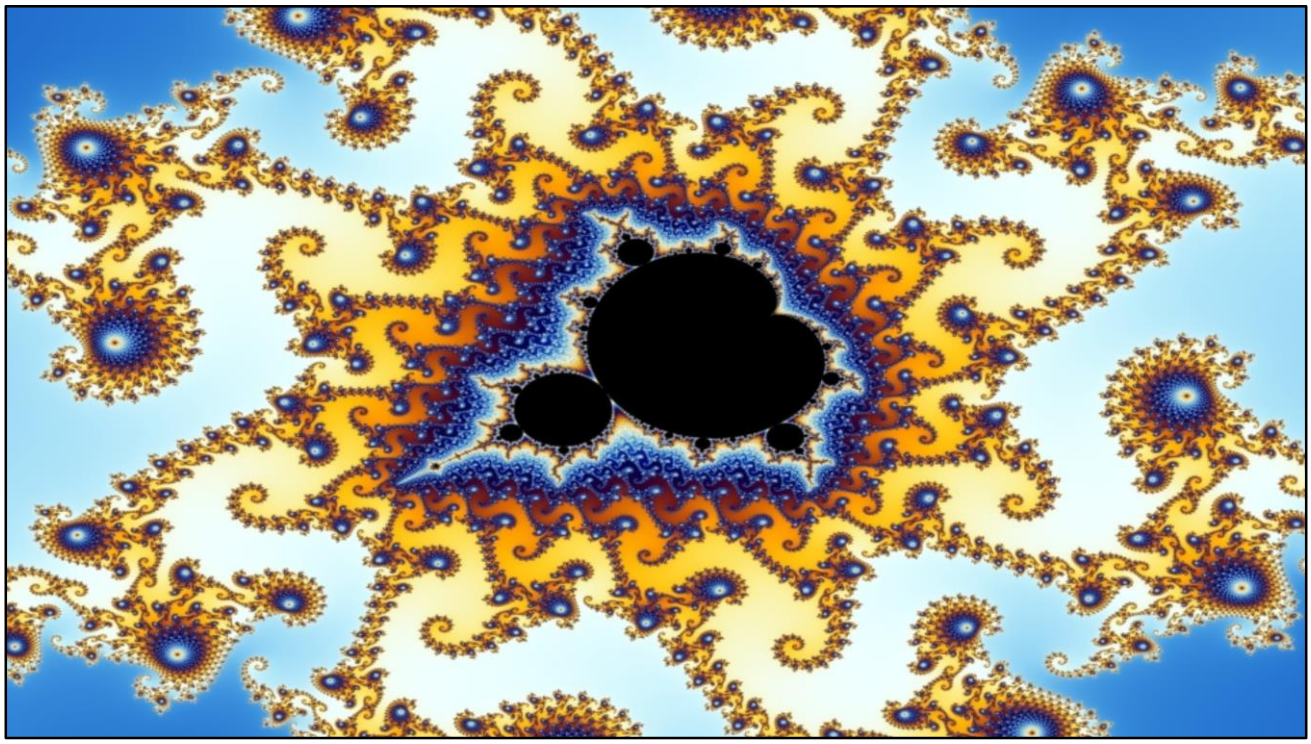


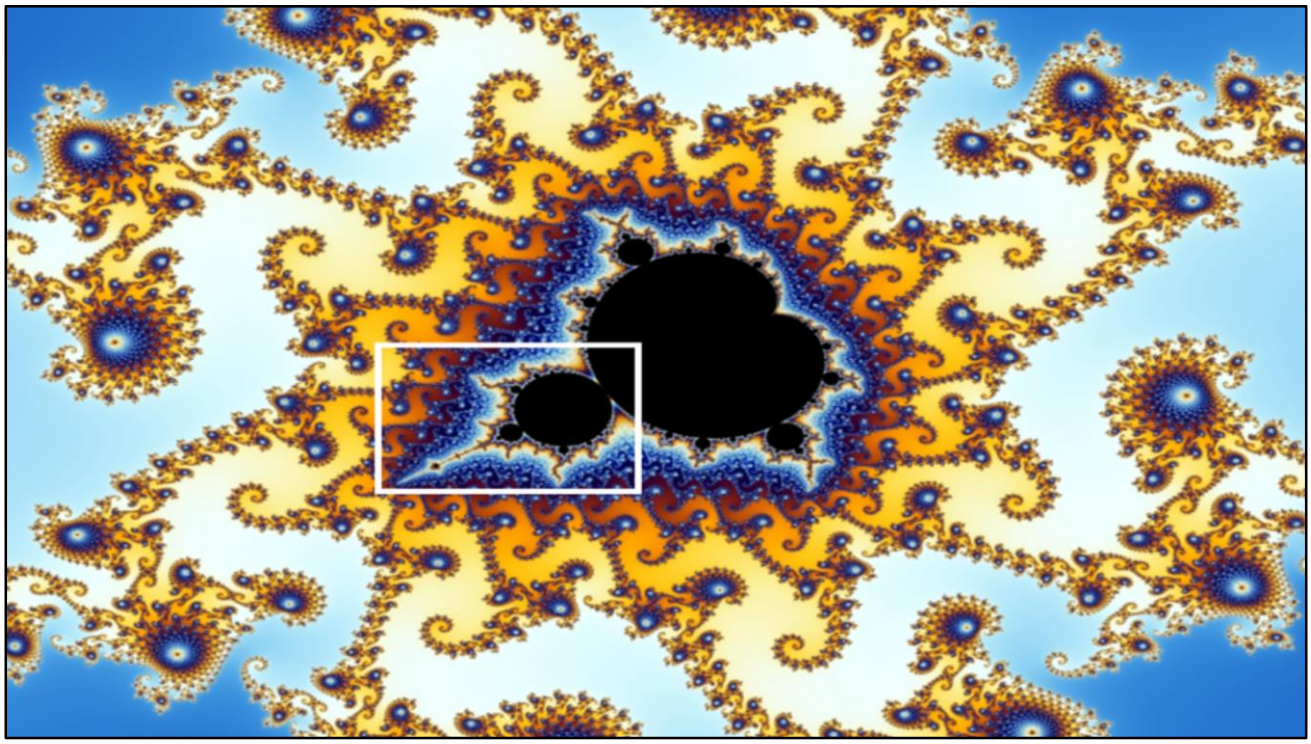


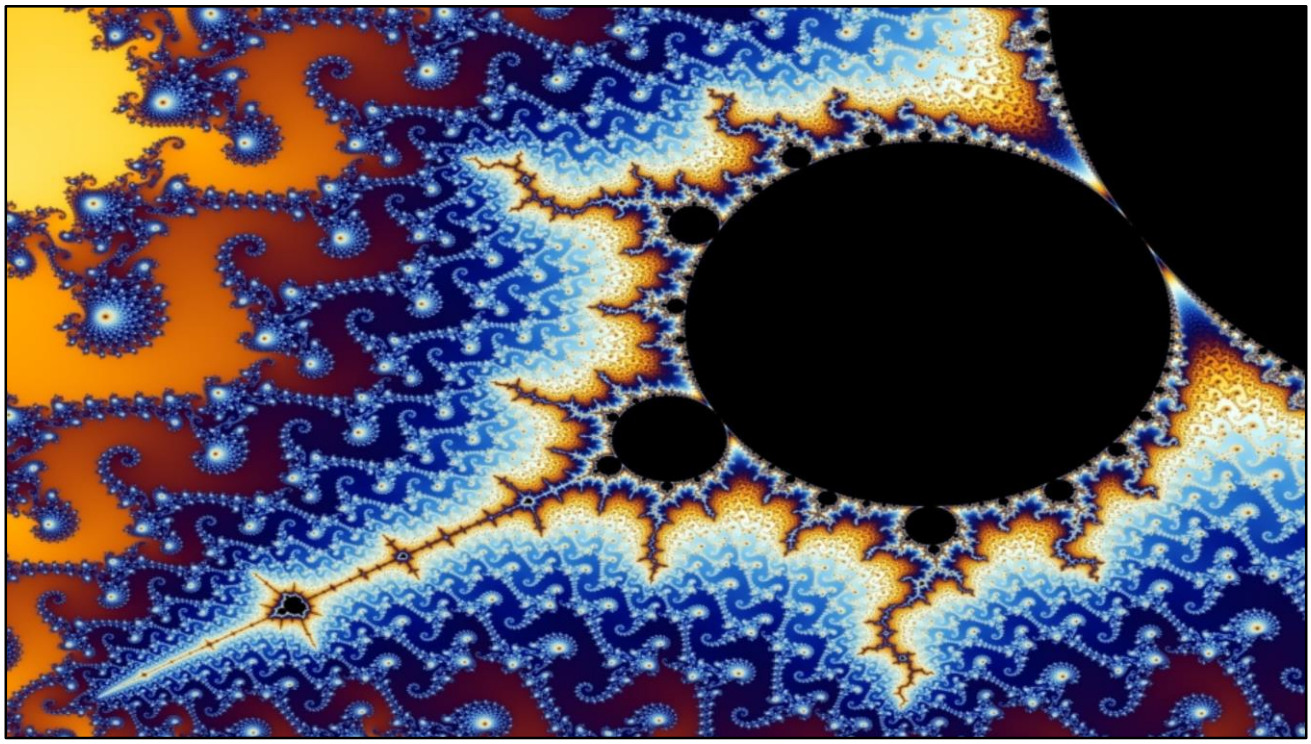


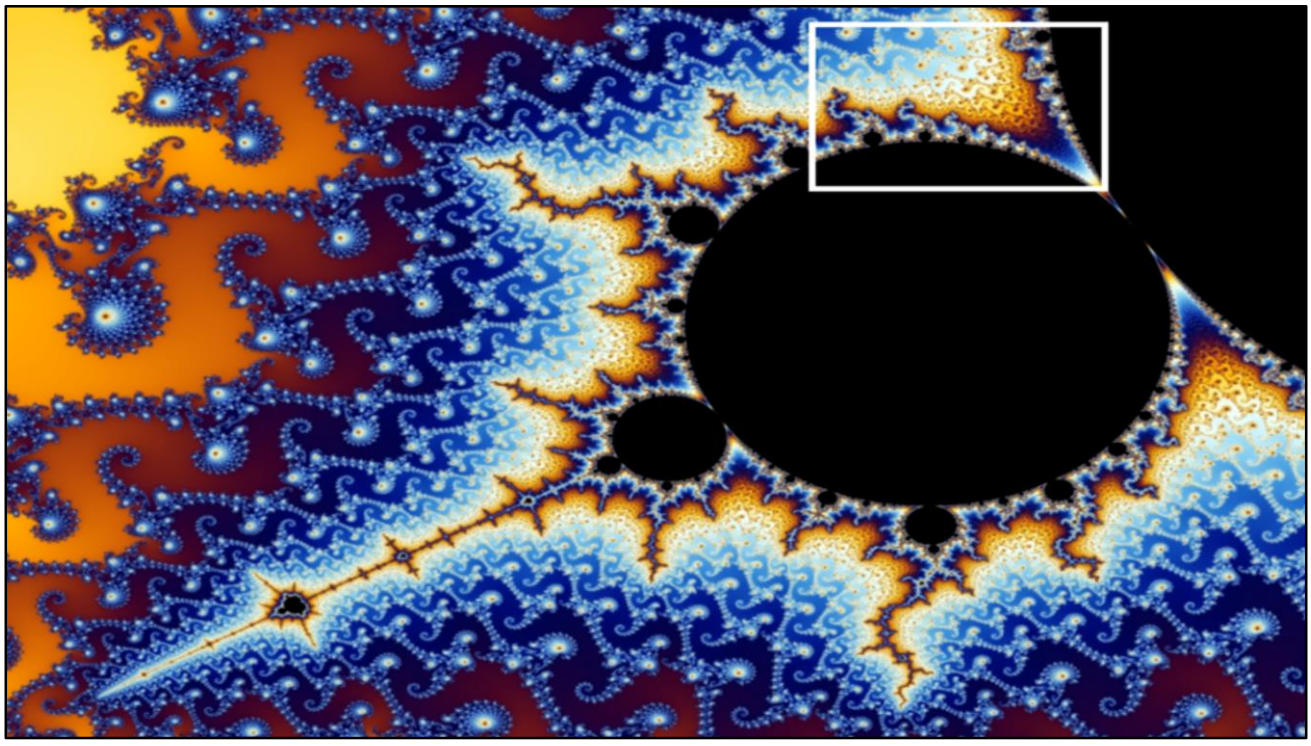


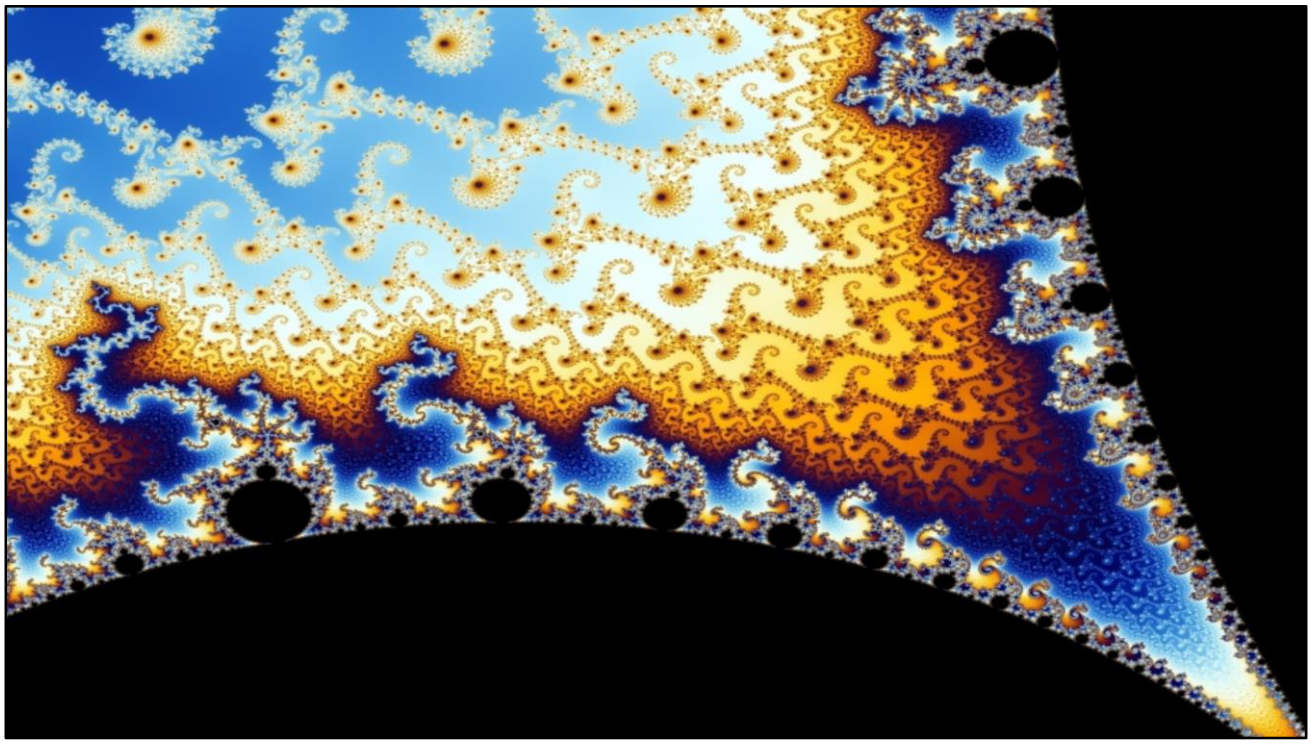


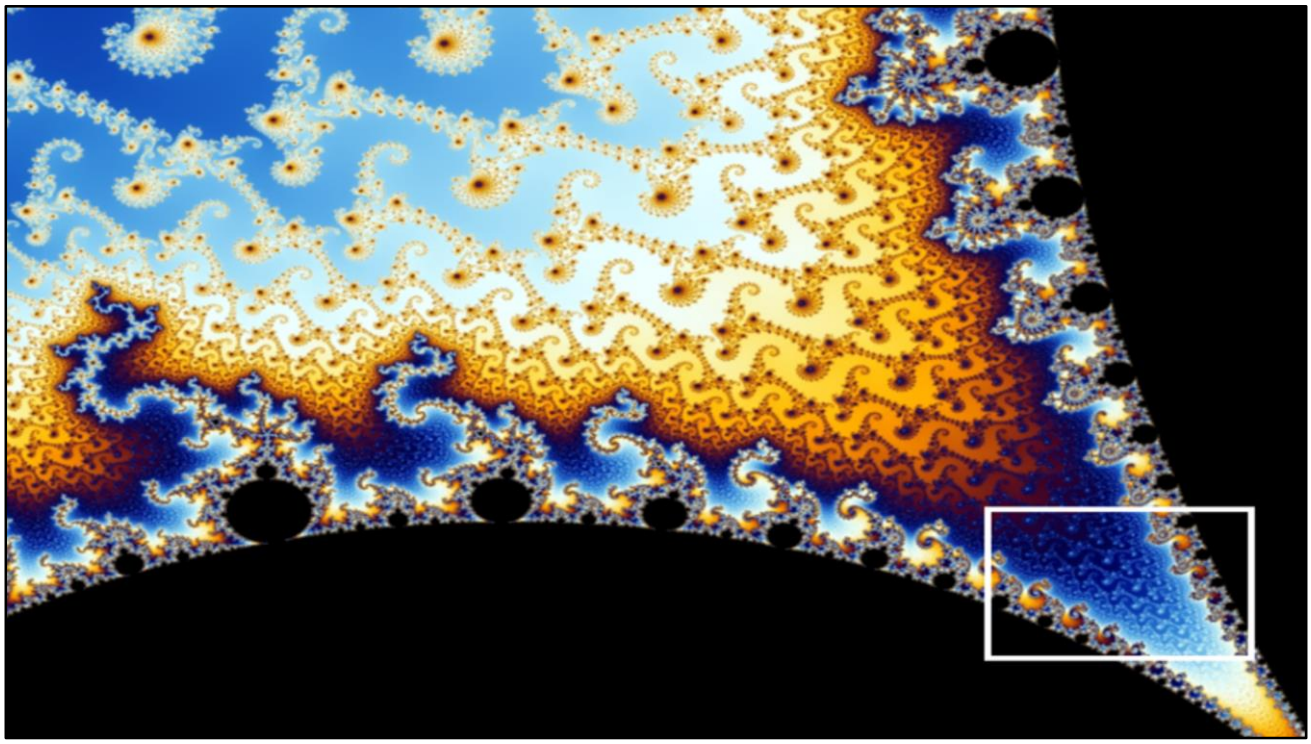


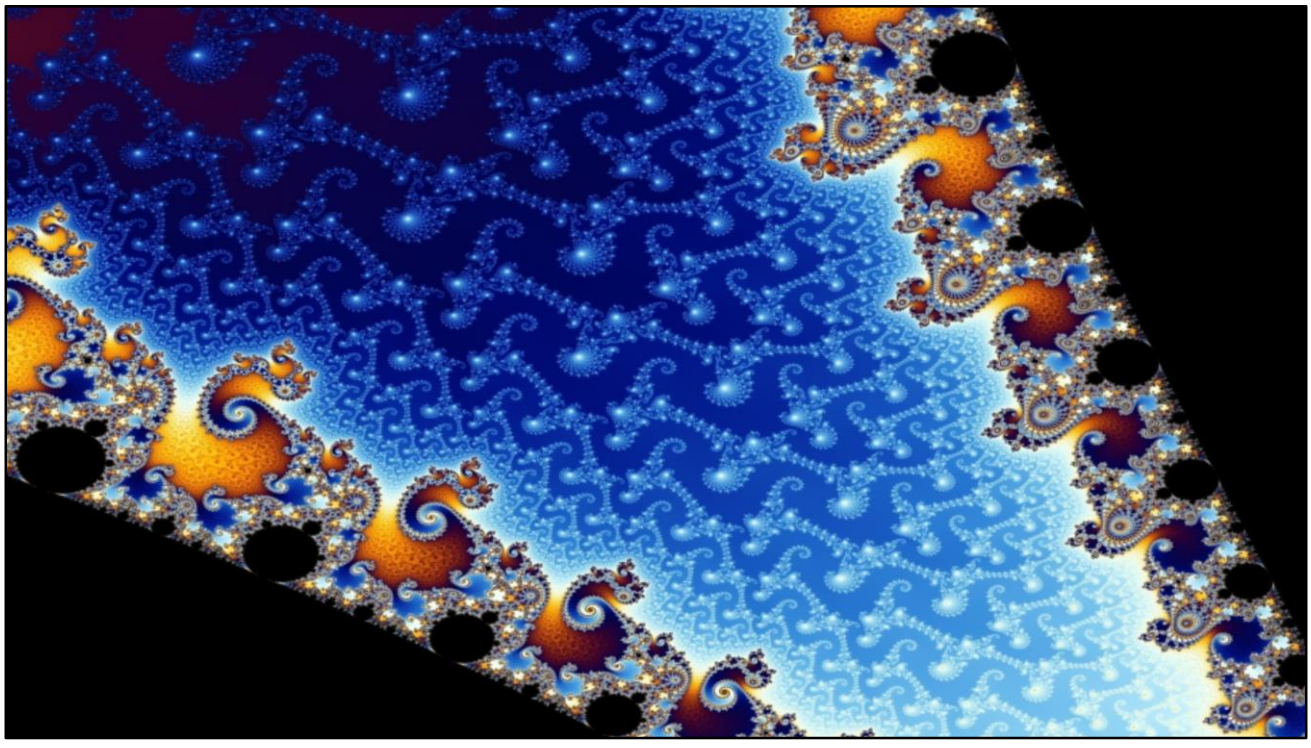


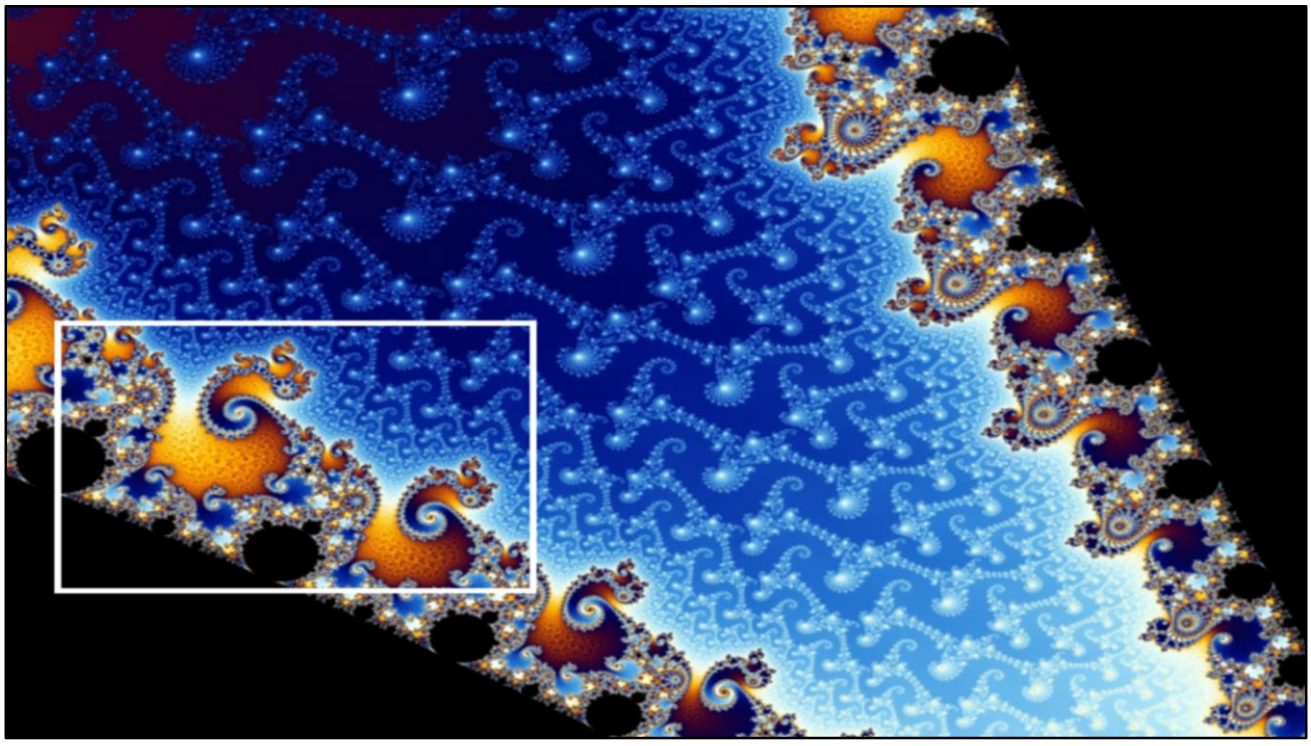


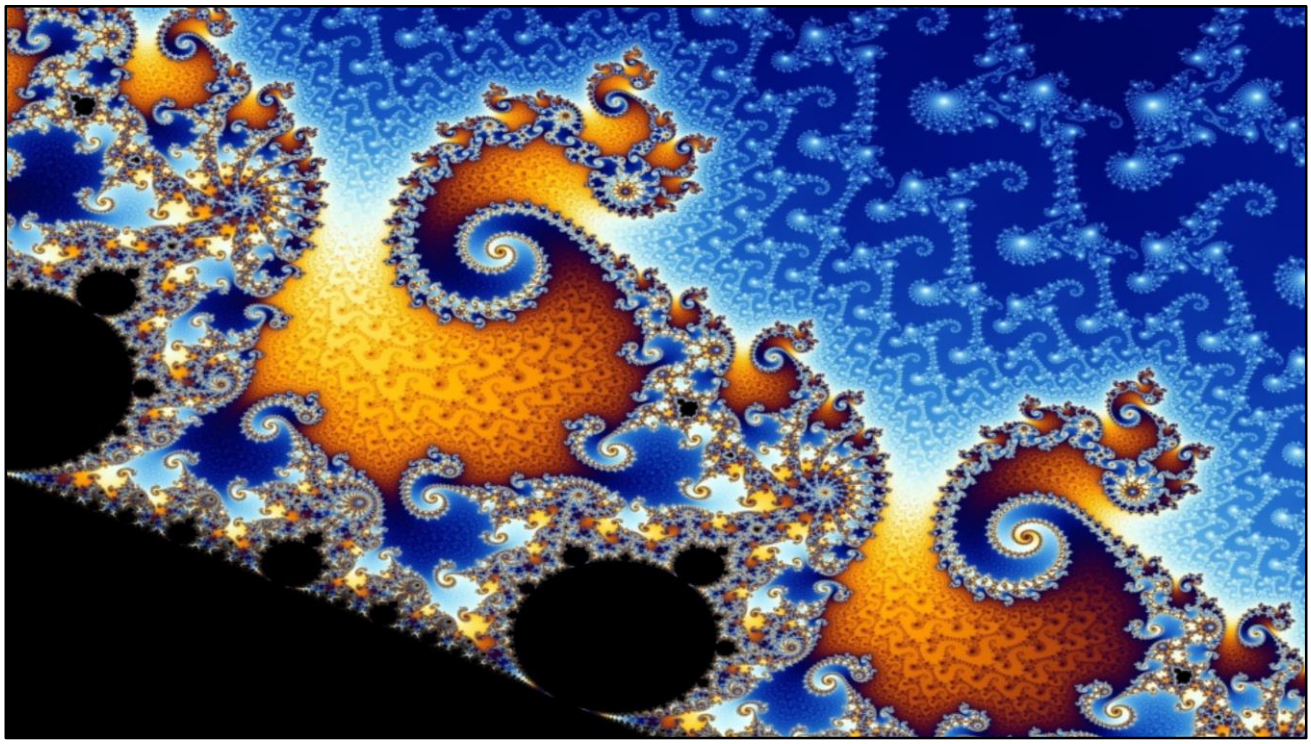


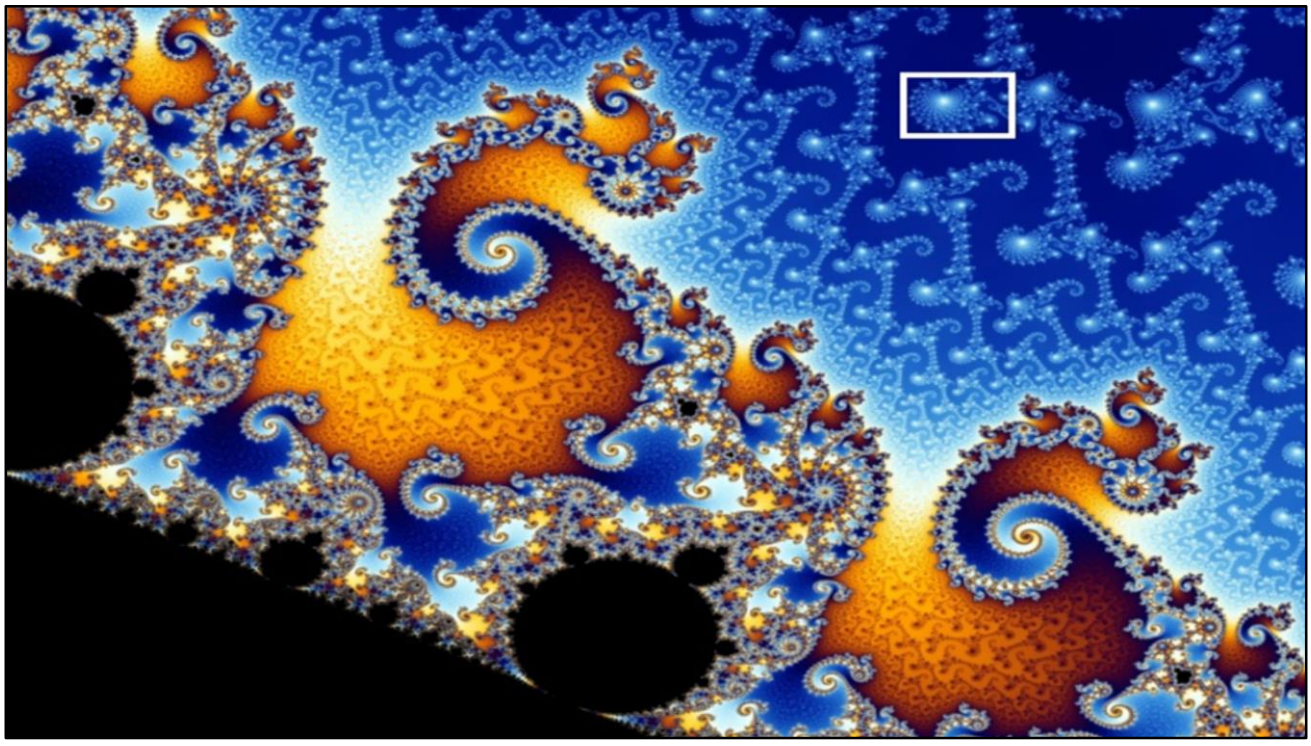


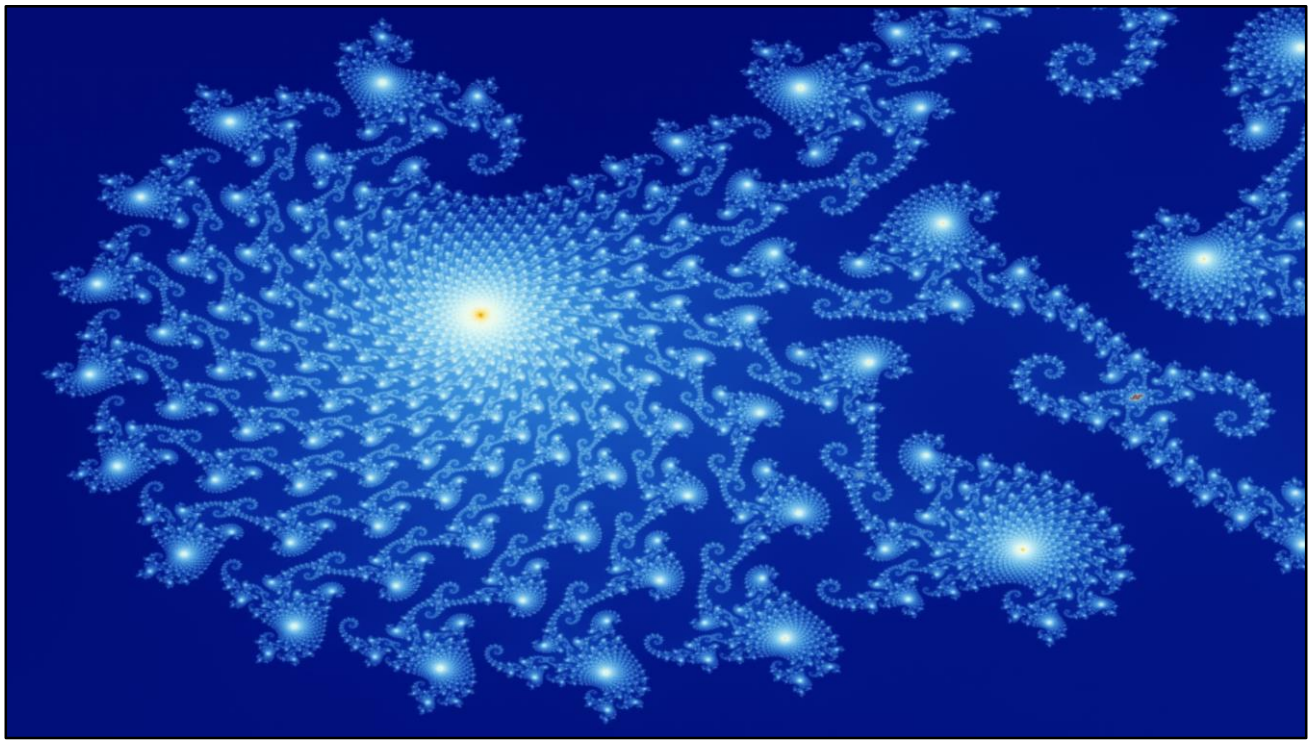


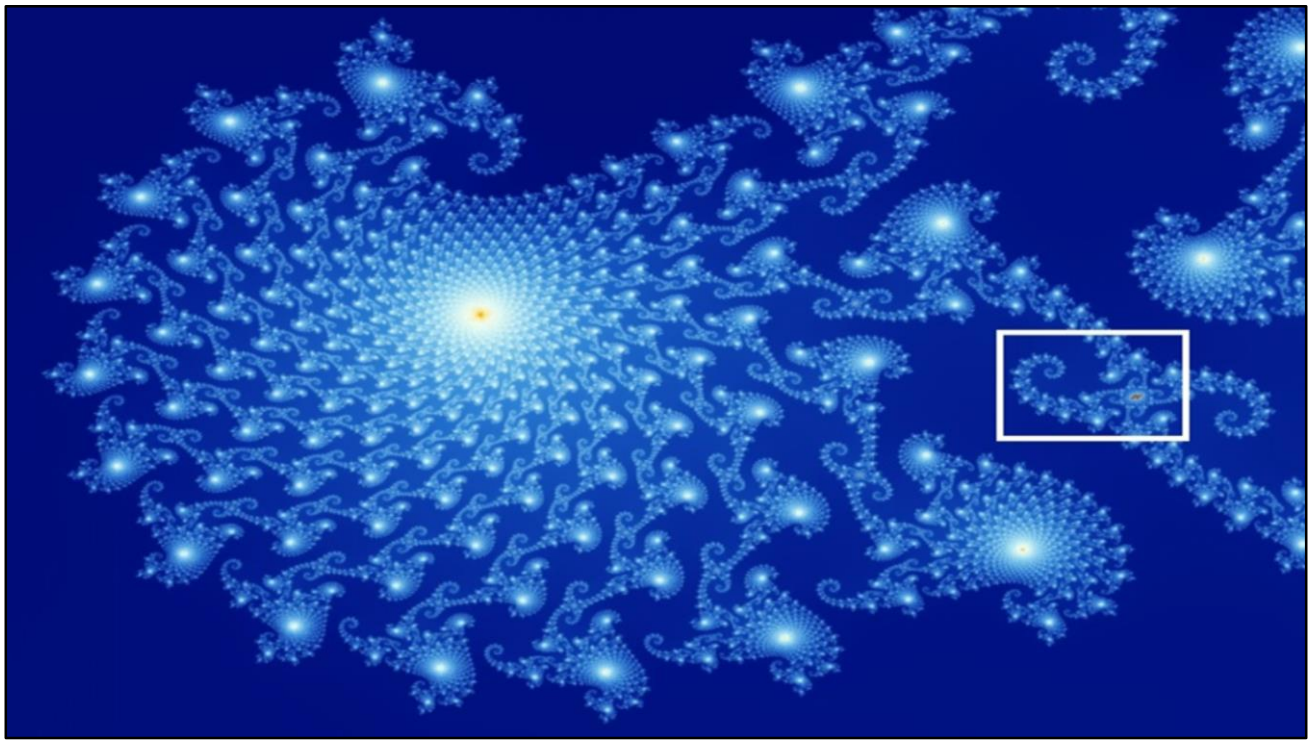


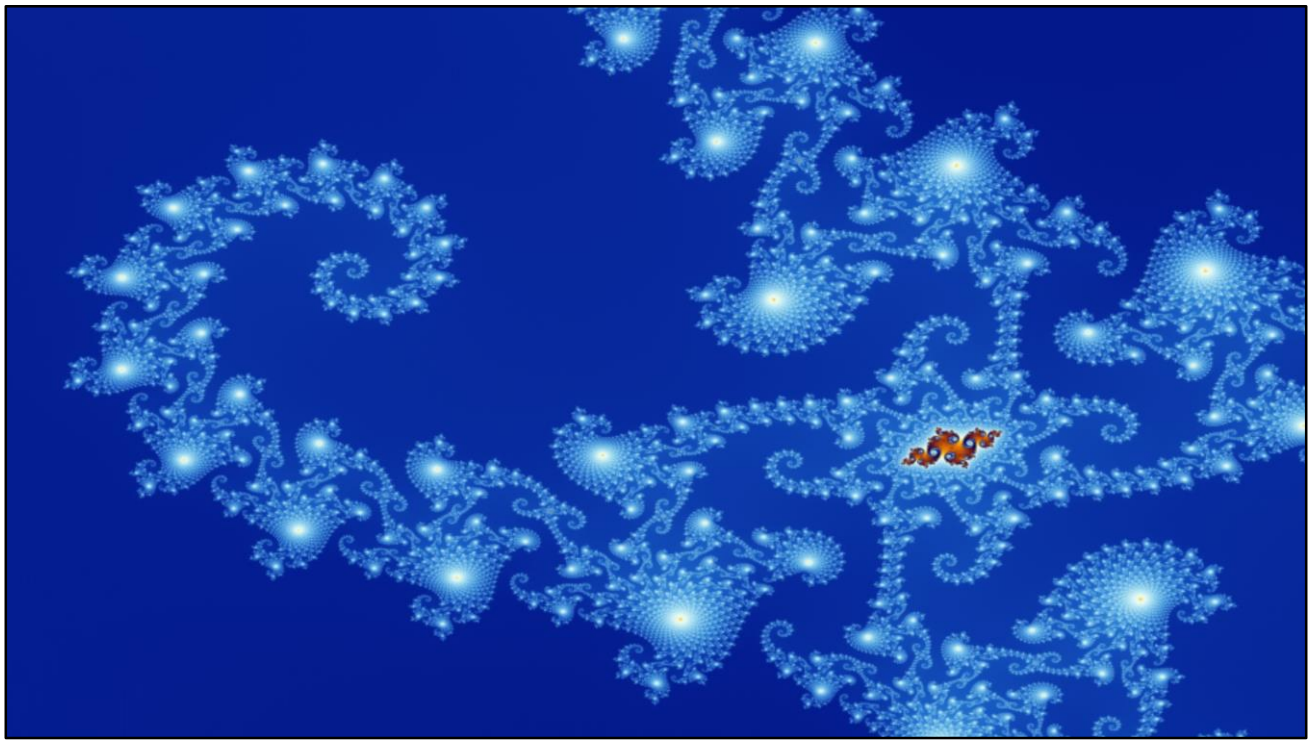


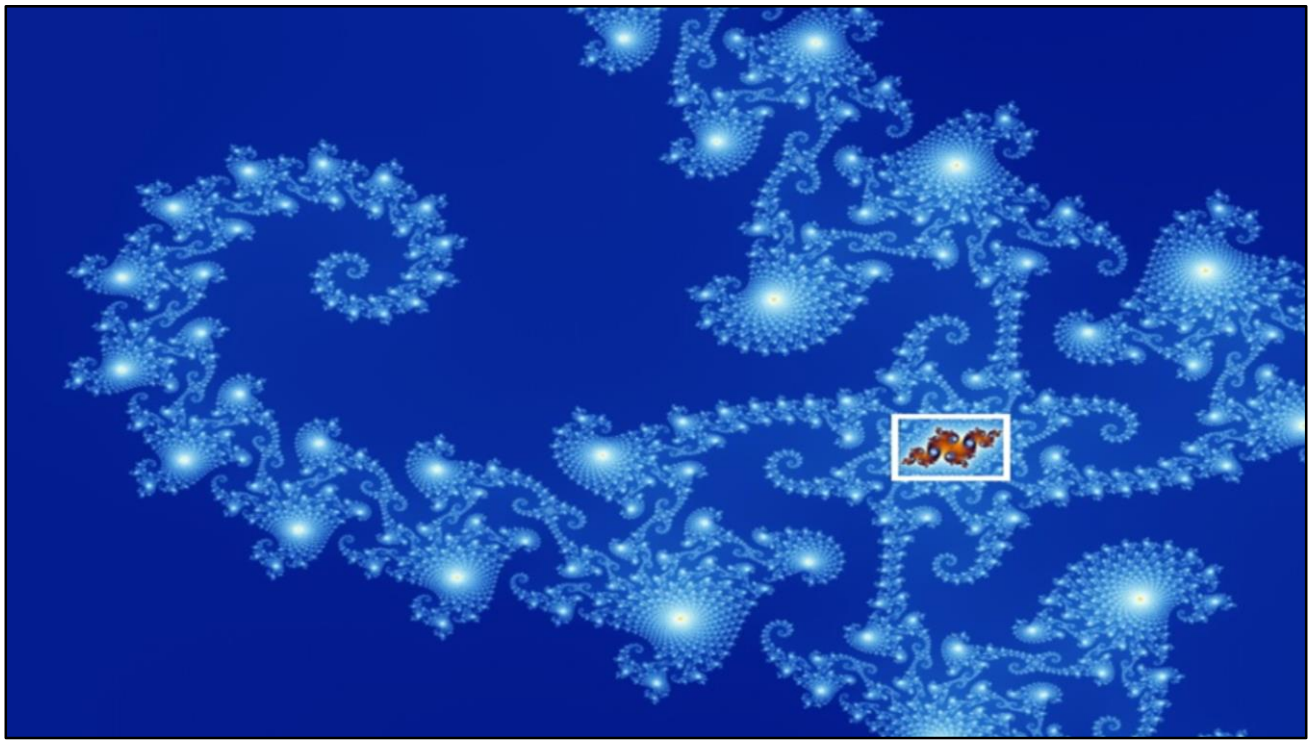


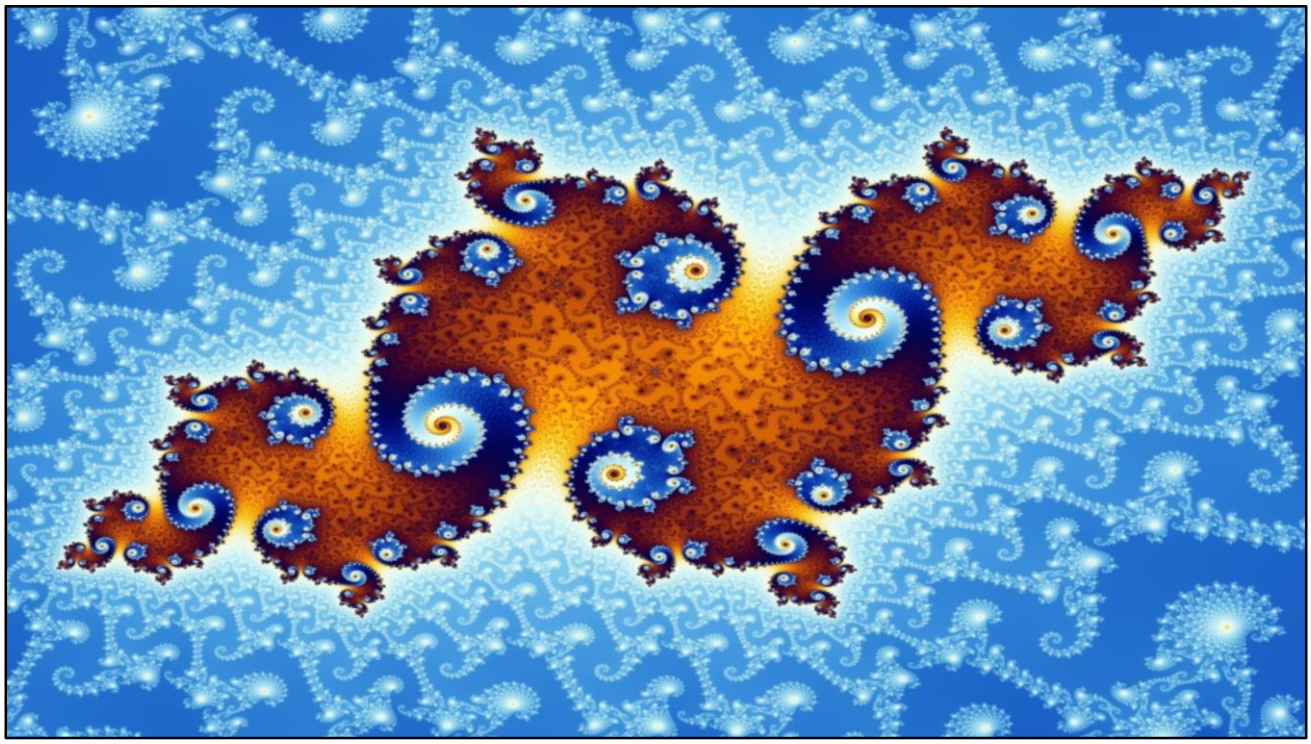












4.6 Perform Operations with Complex Numbers

- Solve the quadratic equation
  - $x^2 - 8 = -36$

- *279 #1, 3-75 every other odd + 5 choice = 25*

$x^2 - 8 = -36$
$x^2 = -28$
$x = \pm\sqrt{-28} = \pm i\sqrt{28} = \pm i\sqrt{4}\sqrt{7} = \pm 2i\sqrt{7}$

Homework Quiz

- 4.6 Homework Quiz

4.7 Complete the Square

- The Perfect Square
- $(x + 3)^2$

- $(x + k)^2 = x^2 + 2kx + k^2$

$=(x + 3)(x + 3)$
$= x^2 + 2(3x) + 32$
$= x^2 + 6x + 9$

4.7 Complete the Square

- What should c be to make a perfect square
  - $x^2 + 8x + c$

you have to add $(8/2)^2 = 42 = 16$ to get a perfect square or $(x + 4)^2$

4.7 Complete the Square

- Rewrite the quadratic so *x* terms on one side and constant on other.
- If the leading coefficient is not 1, divide everything by it.
- Complete the square: add $\left(\frac{b}{2}\right)^2$ to both sides.
- Rewrite the left hand side as a square (factor)
- Square root both sides
- Solve

- $x^2 + 6x = 16$
- It already is
- $x^2 + 6x + \left(\frac{6}{2}\right)^2 = 16 + \left(\frac{6}{2}\right)^2$
- $x^2 + 6x + 3^2 = 16 + 9$
- $(x + 3)^2 = 25$
- $x + 3 = \pm 5$
- $x = -3 \pm 5$
- $x = 2, -8$

4.7 Complete the Square

Solve $2x^2 - 11x + 12 = 0$

ANS: $2x^2 - 11x = -12$

$x^2 - 11/2\ x = -6$

$x^2 - 11/2\ x + (-11/2/2)^2 = -6 + (-11/2/2)^2 \rightarrow (-11/2/2)^2 = (-11/4)^2 = 121/16$

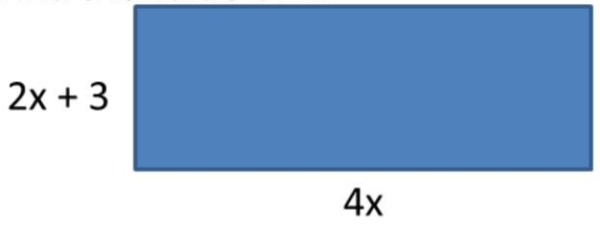
$(x - 11/4)^2 = 25/16$

$x - 11/4 = \pm\ 5/\ 4$

$x = 11/4 \pm 5/\ 4 = (11 \pm 5)/4 = 4, 3/2$

4.7 Complete the Square

The area of the rectangle is 56. Find the value of x.



$4x(2x + 3) = 56$
$8x^2 + 12x = 56$
$x^2 + 3/2\ x = 7$
$x^2 + 3/2\ x + (3/4)^2 = 7 + 9/16$
$(x + ¾)^2 = 121/16$
$x + ¾ = \pm\sqrt{(121/16)}$
$x + ¾ = \pm 11/4$
$x = -3/4 \pm 11/4$
$x = 8/4 =$ **2** and $14/4 =$ **7/2**

4.7 Complete the Square

- Writing quadratic functions in Vertex Form
- $y = a(x - h)^2 + k$
  - (h, k) is the vertex

1. Start with standard form
2. Group the terms with the x
3. Factor out any number in front of the x^2
4. Add $(b/2)^2$ to both sides (inside the group on the right)
5. Rewrite as a perfect square
6. Subtract to get the y by itself

1. $y = 2x^2 + 12x + 16$
2. $y = (2x^2 + 12x) + 16$
3. $y = 2(x^2 + 6x) + 16$
4. $y + 2(9) = 2(x^2 + 6x + 9) + 16$
5. $y + 18 = 2(x + 3)^2 + 16$
6. $y = 2(x + 3)^2 - 2$
   - Vertex is at (-3, -2)
   - -2 is the minimum for this function
   - Find the max or min by completing the square to find the vertex

4.7 Complete the Square

- *288 #3-55 every other odd, 61, 65 + 4 choice = 20*

Homework Quiz

- 4.7 Homework Quiz

4.8 Use the Quadratic Formula and the Discriminant

- Completing the square was a long laborious process. Today we can develop a method to make it quicker.

4.8 Use the Quadratic Formula and the Discriminant

- Solve $ax^2 + bx + c = 0$
- $ax^2 + bx = -c$
- $x^2 + \frac{b}{a}x = -\frac{c}{a}$
- $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$
- $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$
- $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$
- $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$
- $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

$2x^2 + 6x – 4 = 0$

$ax^2 + bx + c = 0$

$x^2 + 3x – 2 = 0$ Divide by 2 to get a = 1 $x^2 + (b/a)x + (c/a) = 0$

$x^2 + 3x = 2$ Add two to get x's by self $x^2 + (b/a)x = -(c/a)$

$x^2 + 3x + (3/2)^2 = 2 + (3/2)^2$ Add the square of half of middle to get perfect square $x^2 + (b/a)x + (b/2a)^2 = -(c/a) + (b/2a)^2$

$(x + 3/2)^2 = 2 + 9/4$ Factor left $(x + (b/2a))^2 = -c/a + b^2/4a^2$

$x + 3/2 = \pm\sqrt{(17/4)}$ Square root $x + (b/2a) = \pm\sqrt{((-4ac+b^2)/(4a^2))}$

$x = -3/2 \pm\sqrt{(17/4)}$ Subtract $x = -b/2a \pm\sqrt{((b^2-4ac)/(4a^2))}$

$x = (-3\pm\sqrt{17})/2$ Simplify $x = (-b\pm\sqrt{(b^2-4ac))/2a}$

4.8 Use the Quadratic Formula and the Discriminant

- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- *This is called the quadratic formula and always works for quadratic equations.*
- *It even finds the complex solutions.*
- *The part under the square root, **discriminant**, tells you what kind of solutions you are going to have.*
  - $b^2 - 4ac > 0$ → two distinct real solutions
  - $b^2 - 4ac = 0$ → exactly one real solution (a double solution)
  - $b^2 - 4ac < 0$ → two distinct imaginary solutions

4.8 Use the Quadratic Formula and the Discriminant

- *What types of solutions to* $5x^2 + 3x - 4 = 0$*?*
- Solve $5x^2 + 3x = 4$

$3^2 - 4(5)(-4) = 9 + 80 = 89$ → two distinct real roots

Put in standard form → $5x^2 + 3x - 4 = 0$
Quadratic formula → $x = (-3 \pm \sqrt{(3^2-4(5)(-4)))}/(2(5))$
Simplify → $(-3 \pm \sqrt{(89)})/10$

4.8 Use the Quadratic Formula and the Discriminant

- Solve $4x^2 - 6x + 3 = 0$

- *296 #1-73 every other odd + 6 choice = 25*

$$4x^2 - 6x + 3 = 0$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{6 \pm \sqrt{36 - 48}}{8}$$

$$x = \frac{6 \pm \sqrt{-12}}{8}$$

$$x = \frac{6 \pm 2\sqrt{3}i}{8}$$

$$x = \frac{3 \pm \sqrt{3}i}{4} \ (reduce\ top\ and\ bottom\ by\ 2)$$

Homework Quiz

- 4.8 Homework Quiz

4.9 Graph and Solve Quadratic Inequalities

- Often in real-life, what we are looking for is a range instead of just one value.
- An example is how many tickets must be purchased to make at least $500?
- That means real-life often uses inequalities.

4.9 Graph and Solve Quadratic Inequalities

Graph inequalities

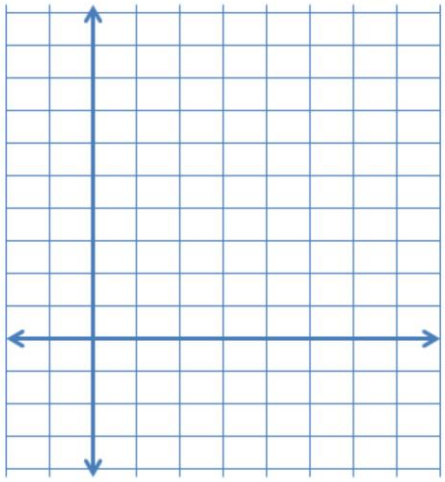
- Graph the quadratic as if it were an equation
  - Find the vertex $\left(-\frac{b}{2a}\right)$
  - Make a table of values choosing points on both sides of the vertex
  - Graph the points and connect the dots

4.9 Graph and Solve Quadratic Inequalities

- Dotted line or solid line
- Shade
  - Pick a test point (not on the line)
  - Try plugging it in the inequality
  - If you get a true statement shade that side of the line
  - If you get a false statement shade the other side of the line.

4.9 Graph and Solve Quadratic Inequalities

- Graph $y \geq x^2 - 10x + 25$



ANS: Vertex at x = 10/2 = 5

Table of values to include x = {1, 2, 3, 4, 5, 6, 7, 8, 9}

Pick (0, 0) as test point → $0 \geq 25$ false shade other side of line.

4.9 Graph and Solve Quadratic Inequalities

Solve inequalities in one variable.

1. Make = 0
2. Factor or use the quadratic formula to find the zeros
3. Graph the zeros on a number line (notice it cuts the line into three parts)
4. Pick a number in each of the three parts as test points
5. Test the points in the original inequality to see true or false
6. Write inequalities for the regions that were true

❑ $p^2 - 4p \leq 5$

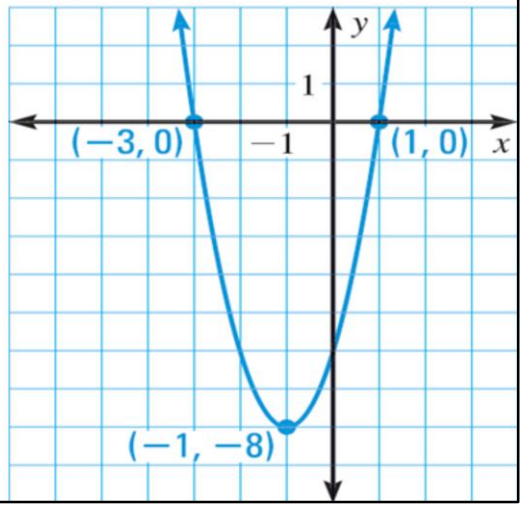
1. $p^2 - 4p - 5 \leq 0$
2. $(p - 5)(p + 1) \leq 0$
   - Zeros are 5, -1

4. test points = -2, 0, 6
   - $-2 \rightarrow 4 - 4(-2) \leq 5 \rightarrow 12 \leq 5$ false
   - $0 \rightarrow 0 - 4(0) \leq 5 \rightarrow 0 \leq 5$ true
   - $6 \rightarrow 36 - 4(6) \leq 5 \rightarrow 12 \leq 5$ false

6. middle region true so $-1 \leq p \leq 5$

4.9 Graph and Solve Quadratic Inequalities

- Or You could also solve the quadratic inequality in one variable by graphing the quadratic
  - Make the equation = 0
  - Graph
  - When the graph is below the x-axis; ≤ 0
  - When the graph is above the x-axis; ≥ 0



$2x^2 + 4x - 6 \leq 0$ when $-3 \leq x \leq 1$
$2x^2 + 4x - 6 \geq 0$ when $x \leq -3$ or $x \geq 1$

4.9 Graph and Solve Quadratic Inequalities

- *304 #3-75 every other odd + 6 choice = 25*

Homework Quiz

- 4.9 Homework Quiz

4.10 Write Quadratic Functions and Models

- We can easily find the equations of lines ($y = mx + b$), but curves are a little more tricky.

- Today we will learn how to fit a quadratic function to some given points.

4.10 Write Quadratic Functions and Models

- Vertex Form → given vertex and one other point
  - Fill in the vertex $y = a(x - h)^2 + k$
  - Plug in your other point and find a
  - Write the equation
- Example: vertex at (-2, 1), point at (1, -1)

ANS: $y = a(x + 2)^2 + 1 \rightarrow -1 = a(1 + 2)^2 + 1 \rightarrow -1 = 9a + 1 \rightarrow -2 = 9a \rightarrow a = -2/9 \rightarrow y = -2/9 (x + 2)^2 + 1$

4.10 Write Quadratic Functions and Models

- Intercept Form → given the x-intercepts and one other point
  - Fill in **y = a(x – p)(x – q)** with the intercepts (p and q)
  - Plug in your other point and find a
  - Write the equation
- Example: x-ints of 1 and 4, point at (2, -6)

ANS: $y = a(x – 1)(x – 4) \rightarrow -6 = a(2 – 1)(2 – 4) \rightarrow -6 = a(1)(-2) \rightarrow a = 3 \rightarrow y = 3(x – 1)(x – 4)$

4.10 Write Quadratic Functions and Models

- Standard Form → any 3 points
  - Use one of the above two and simplify OR
  - Fill in the x and y of $\mathbf{ax^2 + bx + c = y}$ with each of the three points creating a system of three equations with the variables of a, b, and c
  - Solve the system
  - Write your equation

4.10 Write Quadratic Functions and Models

- Example: points (-2, -1), (1, 11), (2, 27)

Use Cramer's rule $y = 3x^2 + 7x + 1$

4.10 Write Quadratic Functions and Models

- Best-Fitting Quadratic Regression
  - (Graphing Calculators)
  - Gives the best-fitting quadratic model for a given set of at least 3 data points
- Steps
  - Push STAT
  - Select Edit…
  - Enter x values in L1 and y values in L2
  - Push STAT
  - Select CALC → QuadReg
  - Push Enter again

4.10 Write Quadratic Functions and Models

- *312 #3-39 every other odd, 47, 51 + 3 choice = 15*

Homework Quiz

- 4.10 Homework Quiz

4. REVIEW

- *323 # 20 choice = 20*