Algebra 2
Chapter 7

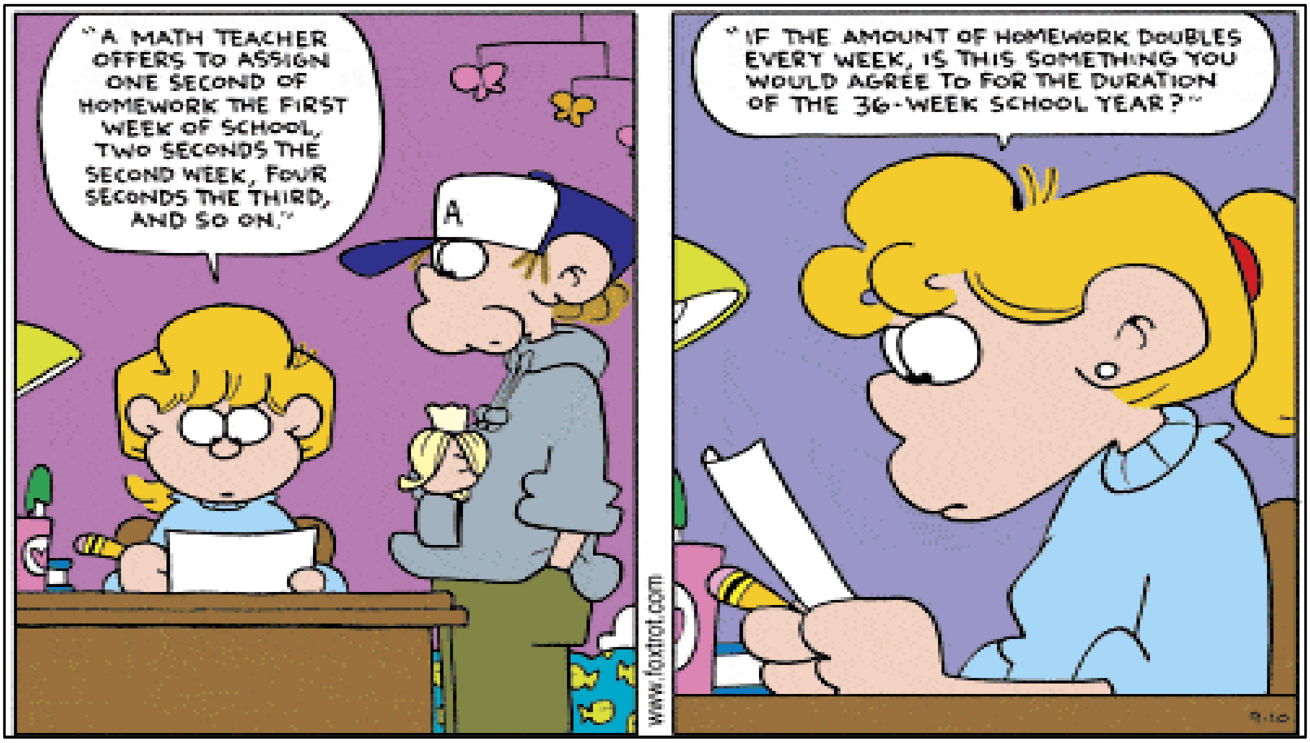
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Algebra II 7

- This Slideshow was developed to accompany the textbook
  - *Larson Algebra 2*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

Slides created by
Richard Wright, Andrews Academy
rwright@andrews.edu

7.1 Graph Exponential Growth Functions

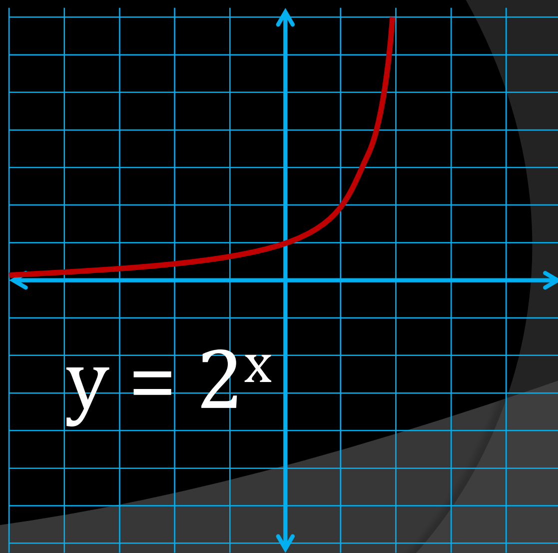


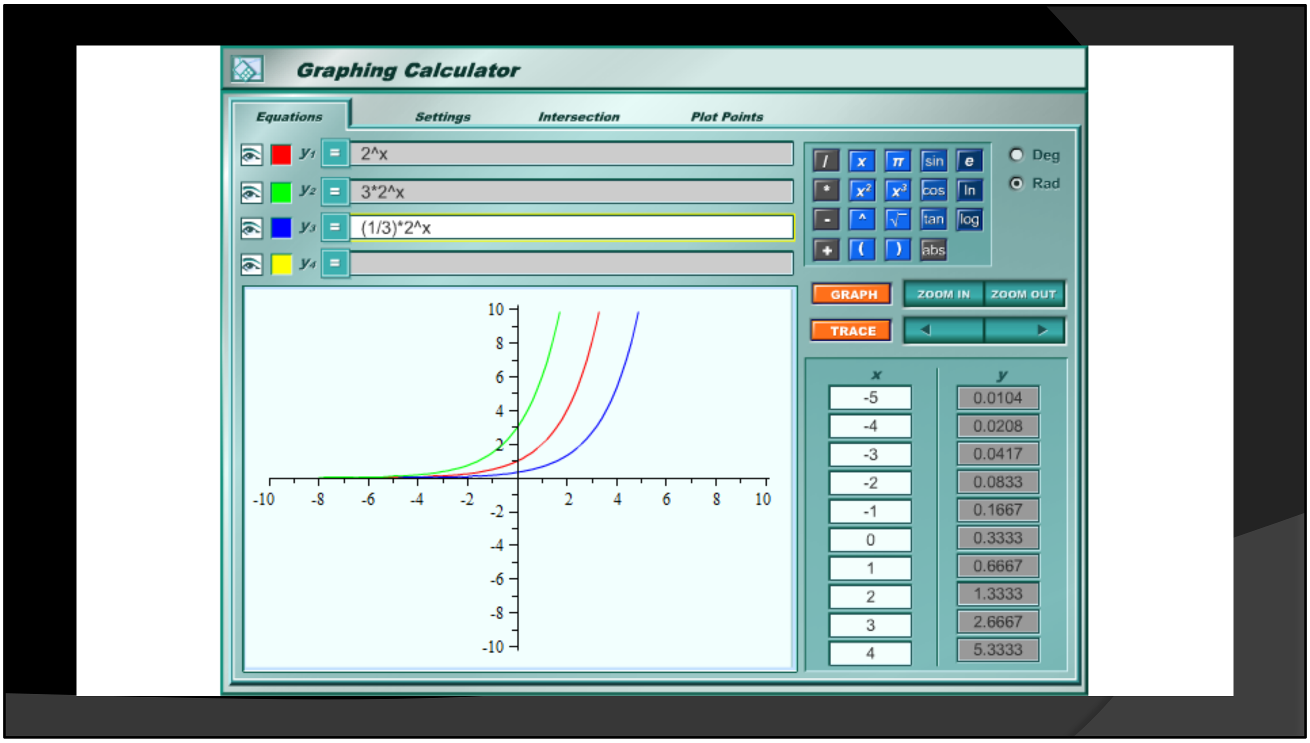
How much work will be done the last week of school?
Formula is 2^{n-1}
Plug in 36: $2^{36-1} = 3.436\text{x}10^{10}$ seconds → 9544371.769 hours → 397682.157 days → 1088.8 years


I'M A LITTLE WORRIED ABOUT ANSWERING THIS ONE WRONG.
IS THAT MR. WRIGHT ? HE'S EVIL.

7.1 Graph Exponential Growth Functions

- Exponential Function
  - $y = b^x$
  - Base (b) is a positive number other than 1





Graph

$y=2^x$

$y=1/3 * 2^x$

$y=3*2^x$

$y=-3*2^x$

7.1 Graph Exponential Growth Functions

- $y = a \cdot 2^x$
  - y-intercept = a
  - x-axis is the asymptote of graph

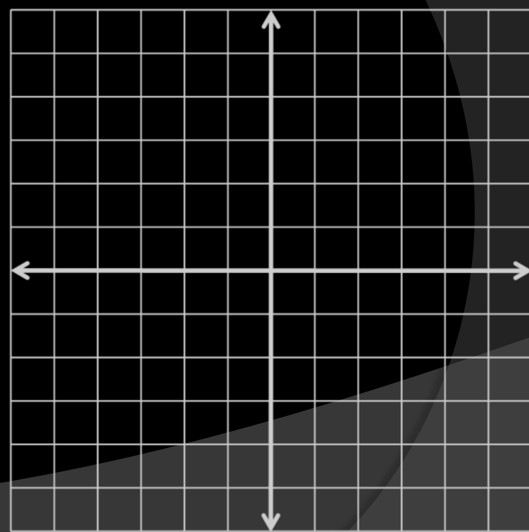
7.1 Graph Exponential Growth Functions

- Exponential Growth Function
  - $y = a \cdot b^{x-h} + k$
- To graph
  - Start with $y = b^x$
  - Multiply y-coordinates by a
  - Move up k and right h
  - (or make table of values)

- Properties of the graph
- y-intercept = a (if h and k=0)
- $y = k$ is asymptote
- Domain is all real numbers
- Range
  - $y > k$ if $a > 0$
  - $y < k$ if $a < 0$

7.1 Graph Exponential Growth Functions

- Graph
- $y = 3 \cdot 2^{x-3} - 2$



7.1 Graph Exponential Growth Functions

- Exponential Growth Model (word problems)
  - $y = a(1 + r)^t$
    - y = current amount
    - a = initial amount
    - r = growth percent
    - 1 + r = growth factor
    - t = time

7.1 Graph Exponential Growth Functions

- Compound Interest
- $A = P\left(1 + \frac{r}{n}\right)^{nt}$
  - A = current amount
  - P = principle (initial amount)
  - r = percentage rate
  - n = number of times compounded per year
  - t = time in years

7.1 Graph Exponential Growth Functions

- If you put $200 into a CD (Certificate of Deposit) that earns 4% interest, how much money will you have after 2 years if you compound the interest monthly? daily?

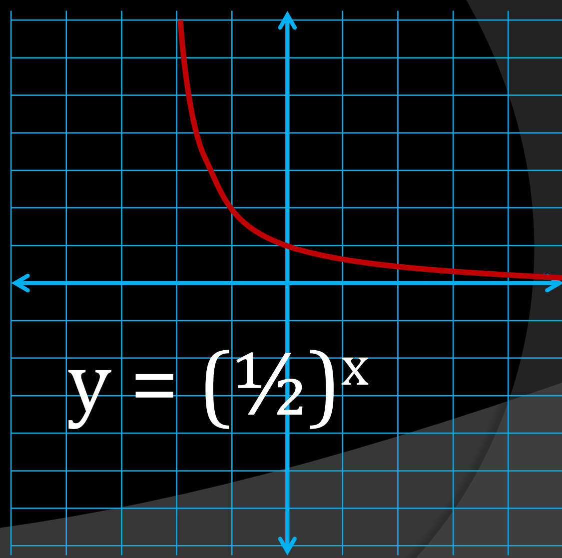
Monthly: $200(1+.04/12)^{12*2}$ = $216.63
Daily: $200(1+.04/365)^{365*2}$ = $216.66

Quiz

- 7.1 Homework Quiz

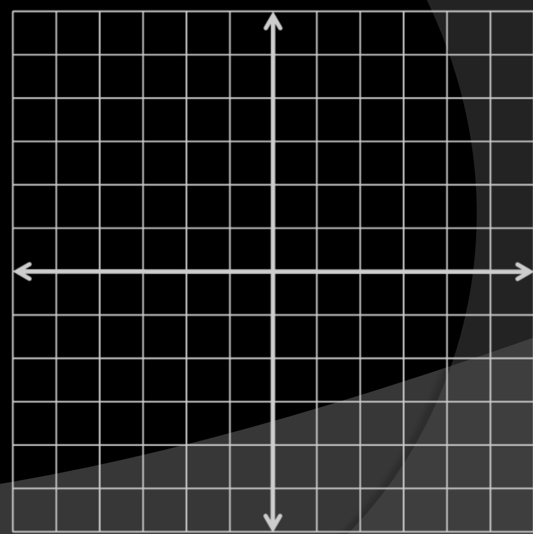
7.2 Graph Exponential Decay Functions

- Exponential Decay
  - $y = a \cdot b^x$
  - $a > 0$
  - $0 < b < 1$
- Follows same rules as growth
  - y-intercept = a
  - y = k is asymptote
  - $y = a \cdot b^{x-h} + k$



7.2 Graph Exponential Decay Functions

- Graph
- $y = 2 \cdot (½)^{x+3} - 2$



7.2 Graph Exponential Decay Functions

- Exponential Decay Model (word problems)
  - $y = a(1 - r)^t$
    - y = current amount
    - a = initial amount
    - r = decay percent
    - 1 – r = decay factor
    - t = time

7.2 Graph Exponential Decay Functions

- A new car cost $23000. The value decreases by 15% each year. Write a model of this decay. How much will the car be worth in 5 years? 10 years?

$y = 23000(1-0.15)^t \rightarrow y = 23000(0.85)^t$

5 years: $y = 23000(0.85)^5 = \$10205.22$

10 years: $y = 23000(0.85)^{10} = \$4528.11$

Quiz

- 7.2 Homework Quiz

7.3 Use Functions Involving *e*

- In math, there are some special numbers like π or i
- Today we will learn about e

7.3 Use Functions Involving *e*

- e
  - Called the natural base
  - Named after Leonard Euler who discovered it
    - (Pronounced "oil-er")
  - Found by putting really big numbers into $\left(1+\frac{1}{n}\right)^n$ = 2.718281828459…
  - Irrational number like π

7.3 Use Functions Involving *e*

- Simplifying natural base expressions
  - Just treat *e* like a regular variable
- $\frac{24e^8}{8e^5}$
- $(2e^{-5x})^{-2}$

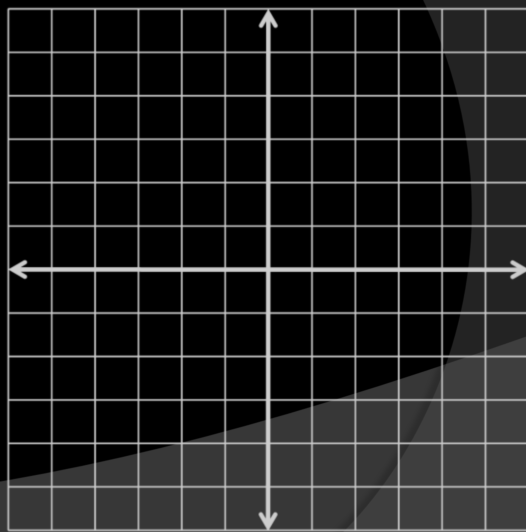
7.3 Use Functions Involving *e*

- Evaluate the natural base expressions using your calculator
- e^3
- $e^{-0.12}$

$e^3 = 20.085537$
$e^{-0.12} = 0.88692044$

7.3 Use Functions Involving *e*

- To graph make a table of values
- $f(x) = a \cdot e^{rx}$
  - $a > 0$
  - If $r > 0$ → growth
  - If $r < 0$ → decay
- Graph $y = 2e^{0.5x}$



7.3 Use Functions Involving *e*

- Compound Interest
- $A = P\left(1 + \frac{r}{n}\right)^{nt}$
    - A = current amount
    - P = principle (initial amount)
    - r = percentage rate
    - n = number of times compounded per year
    - t = time in years
- Compounded continuously
    - $A = Pe^{rt}$

Quiz

- 7.3 Homework Quiz

7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Definition of Logarithm with Base b
- $\log_b y = x \;\Leftrightarrow\; b^x = y$
- Read as "log base b of y equals x"
- Rewriting logarithmic equations
- $\log_3 9 = 2$ →
- $\log_8 1 = 0$ →
- $\log_5(1 / 25) = -2$ →

$3^2 = 9$
$8^0 = 1$
$5^{-2} = 1/25$

7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Special Logs
  - $\log_b 1 = 0$
  - $\log_b b = 1$
- Evaluate
  - $\log_4 64$
  - $\log_2 \frac{1}{8}$
  - $\log_{1/4} 256$

Rewrite $\log_b 1 = 0 \rightarrow b^0 = 1$
Rewrite $\log_b b = 1 \rightarrow b^1 = b$

Rewrite $\log_4 64 = x \rightarrow 4^x = 64 \rightarrow x = 3$
Rewrite $\log_2 0.125 = x \rightarrow 2^x = 1/8 \rightarrow x = -3$
Rewrite $\log_{1/4} 256 = x \rightarrow (¼)^x = 256 \rightarrow 4^{-x} = 256 \rightarrow 4^{-x} = 4^4 \rightarrow -x = 4 \rightarrow x = -4$

7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Using a calculator
- Common Log (base 10)
  - $\log_{10} x = \log x$
  - Find log 12
- Natural Log (base e)
  - $\log_e x = \ln x$
  - Find ln 2

1.0792
0.6931

7.4 Evaluate Logarithms and Graph Logarithmic Functions

- When the bases are the same, the base and the log cancel
- $5^{\log_5 7} = 7$

- $\log_3 81^x$
- $= \log_3 3^{4x}$
- $= 4x$

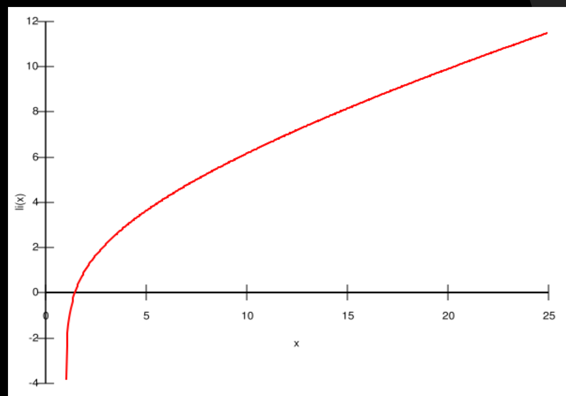
7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Finding Inverses of Logs
- $y = \log_8 x$
- $x = \log_8 y$ Switch x and y
- $y = 8^x$ Rewrite to solve for y

- To graph logs
  - Find the inverse
  - Make a table of values for the inverse
  - Graph the log by switching the x and y coordinates of the inverse.

7.4 Evaluate Logarithms and Graph Logarithmic Functions

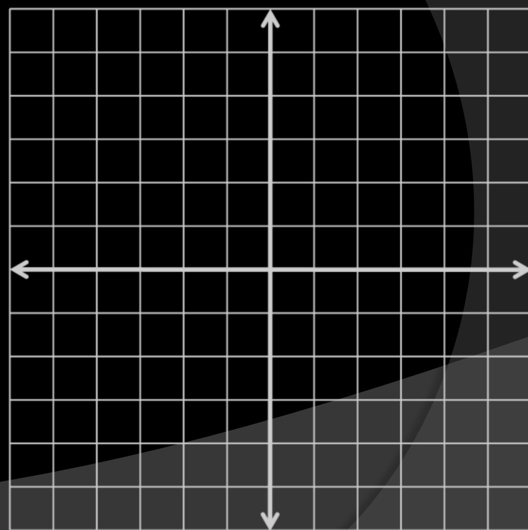
- Properties of graphs of logs
- $y = \log_b (x - h) + k$
  - $x = h$ is vert. asymptote
  - Domain is $x > h$
  - Range is all real numbers
  - If $b > 1$, graph rises
  - If $0 < b < 1$, graph falls



7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Graph
  - $y = \log_2 x$
  - Inverse
  - $x = \log_2 y$
  - $y = 2^x$

x	y
-3	1/8
-2	¼
-1	½
0	1
1	2
2	4
3	8



Graph the points with x and y switched

(1/8, -3)
(1/4, -2)
(1/2, -1)
(1, 0)
(2, 1)
(4, 2)
(8, 3)

Quiz

- 7.4 Homework Quiz

7.5 Apply Properties of Logarithms

- Product Property
  - $\log_b uv = \log_b u + \log_b v$
- Quotient Property
  - $\log_b \frac{u}{v} = \log_b u - \log_b v$
- Power Property
  - $\log_b u^n = n \log_b u$

7.5 Apply Properties of Logarithms

- Use $\log_9 5 = 0.732$ and $\log_9 11 = 1.091$ to find
  - $\log_9 \frac{5}{11}$
  - $\log_9 55$
  - $\log_9 25$

$\log_9 5/11 \rightarrow \log_9 5 - \log_9 11 \rightarrow 0.732 – 1.091 \rightarrow -0.359$
$\log_9 55 \rightarrow \log_9 (5 \cdot 11) \rightarrow \log_9 5 + \log_9 11 \rightarrow 0.732 + 1.091 \rightarrow 1.823$
$\log_9 25 \rightarrow \log_9 5^2 \rightarrow 2 \log_9 5 \rightarrow 2(0.732) \rightarrow 1.464$

7.5 Apply Properties of Logarithms

- Expand: $\log_5 2x^6$
- Condense: $2 \log_3 7 - 5 \log_3 x$

$\log_5 2 + \log_5 x^6 \rightarrow \log_5 2 + 6 \log_5 x$
$\log_3 7^2 - \log_3 x^5 \rightarrow \log_3 (49/x^5)$

7.5 Apply Properties of Logarithms

- Change-of-Base Formula
  - $\log_c u = \frac{\log_b u}{\log_b c}$

- Evaluate $\log_4 8$

$\log_4 8 = (\log 8)/(\log 4) = 1.5$

Quiz

- 7.5 Homework Quiz

7.6 Solve Exponential and Logarithmic Equations

- Solving Exponential Equations
  - Method 1) if the bases are equal, then exponents are equal
  - $2^{4x} = 32^{x-1}$

$2^{4x} = 2^{5(x-1)} \rightarrow 4x = 5(x-1) \rightarrow 4x = 5x - 5 \rightarrow -x = -5 \rightarrow x = 5$

7.6 Solve Exponential and Logarithmic Equations

- Solving Exponential Equations
  - Method 2) take log of both sides
  - $4^x = 15$
- $5^{x+2} + 3 = 25$

$\log 4^x = \log 15 \rightarrow x \log 4 = \log 15 \rightarrow x = \log 15 / \log 4 \rightarrow x = 1.95$

$5^{x+2} = 22 \rightarrow \log 5^{x+2} = \log 22 \rightarrow (x+2) \log 5 = \log 22 \rightarrow x+2 = \log 22 / \log 5 \rightarrow x = -0.079$

7.6 Solve Exponential and Logarithmic Equations

- Solving Logarithmic Equations
  - Method 1) if the bases are equal, then logs are equal
  - $\log_3 (5x - 1) = \log_3 (x + 7)$

$5x-1=x+7 \rightarrow 4x = 8 \rightarrow x = 2$

7.6 Solve Exponential and Logarithmic Equations

- Solving Logarithmic Equations
  - Method 2) exponentiating both sides
    - Make both sides exponents with the base of the log
  - $\log_4 (x + 3) = 2$

$4\text{^}(\log_4 (x+3)) = 4\text{^}2 \rightarrow x+3 = 16 \rightarrow x = 13$

7.6 Solve Exponential and Logarithmic Equations

- $\log_2 2x + \log_2(x-3) = 3$

$$\log_2 2x + \log_2(x-3) = 3$$
$$\log_2\big(2x \cdot (x-3)\big) = 3$$
$$2x(x-3) = 2^3$$
$$2x^2 - 6x - 8 = 0$$
$$x^2 - 3x - 4 = 0$$
$$(x-4)(x+1) = 0$$
$$x - 4 = 0, x + 1 = 0$$
$$x = 4, -1$$
$$-1\ extraneous$$

Solution x=4

Quiz

- 7.6 Homework Quiz

7.7 Write and Apply Exponential and Power Functions

- Just as 2 points determine a line, so 2 points will determine an exponential equation.

7.7 Write and Apply Exponential and Power Functions

- Exponential Function
  - $y = a b^x$

- If given 2 points
  - Fill in both points to get two equations
  - Solve for a and b by substitution

7.7 Write and Apply Exponential and Power Functions

- Find the exponential function that goes through (-1, 0.0625) and (2, 32)

$0.0625 = ab^{-1} \rightarrow 0.0625 = a/b \rightarrow a = 0.0625b$
$32 = ab^2$

Substitute
$32 = (0.0625b)b^2 \rightarrow 32 = 0.0625b^3 \rightarrow 512 = b^3 \rightarrow b = 8$
$a = 0.0625b \rightarrow a = 0.0625(8) = 0.5$
$y = 0.5 * 8^x$

7.7 Write and Apply Exponential and Power Functions

- Steps if given a table of values
  - Find ln y of all points
  - Graph ln y vs x
  - Draw the best fit straight line
  - Pick two points on the line and find equation of line (remember to use ln y instead of just y)
  - Solve for y
- OR use the ExpReg feature on a graphing calculator
  - Enter points in STAT → EDIT
  - Go to STAT → CALC → ExpReg → Enter → Enter

7.7 Write and Apply Exponential and Power Functions

- Writing a Power Function
  - $y = a x^b$

- Steps are the same as for exponential function
  - Fill in both points to get two equations
  - Solve for a and b by substitution

7.7 Write and Apply Exponential and Power Functions

- Write power function through (3, 8) and (9, 12)

$8 = a3^b \rightarrow a = 8 / 3^b$
$12 = a\ 9^b$

Substitute
$12 = (8/3^b)9^b \rightarrow 12 = 8\ (9^b/3^b) \rightarrow 12 = 8\ (9/3)^b \rightarrow 12 = 8\ 3^b \rightarrow 12/8 = 3^b \rightarrow \log 3/2 = \log 3^b \rightarrow \log 3/2 = b \log 3 \rightarrow b = \log 3/2 / \log 3 \rightarrow b = 0.369$
$a = 8/3^b \rightarrow a = 8 / 3^{0.369} \rightarrow a = 16/3$
$y = 16/3\ x^{0.369}$

7.7 Write and Apply Exponential and Power Functions

- Steps if given a table of values
  - Find ln y and ln x of all points
  - Graph ln y vs ln x
  - Draw the best fit straight line
  - Pick two points on the line and find equation of line (remember to use ln y and ln x instead of just y)
  - Solve for y
- OR use the PwrReg feature on a graphing calculator
  - Enter points in STAT → EDIT
  - Go to STAT → CALC → PwrReg → Enter → Enter

Quiz

- 7.7 Homework Quiz