Counting Methods and Probability

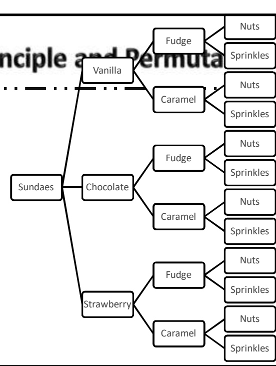
Algebra 2
Chapter 10

10.1 Apply the Counting Principle and Permutations

- Let's say you stop to get an ice cream sundae
- You pick one each of
  - Flavors: vanilla, chocolate, or strawberry
  - Syrups: fudge or caramel
  - Toppings: nuts or sprinkles
- How many different sundaes can you choose?

10.1 Apply the Counting Principle and Permutations

- Each sundae can have 3 flavors
- Each flavor can have 2 syrups
- Each syrup can have 2 toppings
- $3 \cdot 2 \cdot 2 = 12$ sundaes



10.1 Apply the Counting Principle and Permutations

- Fundamental Counting Principle
  - If there are multiple events, multiply the number of ways each event happens to get the total number of ways all the events can happen.

10.1 Apply the Counting Principle and Permutations

- A restaurant offers 8 entrees, 2 salads, 12 drinks, and 6 desserts. How many meals if you choose 1 of each?
- How many different 7 digit phone numbers if the first digit cannot be 0 or 1?

10.1 Apply the Counting Principle and Permutations

- Permutation
  - How many ways to order objects
    - A, B, C →
    - ABC, ACB, BAC, BCA, CAB, CBA → 6 ways
- Number of Permutations of *n* objects taken *r* at a time

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

- Factorial (!) – that number times all whole numbers less than it
  - $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

10.1 Apply the Counting Principle and Permutations

- You have 5 different homework assignments.
  - How many different orders can you complete them all?
  - How many different orders can you complete the first two?

10.1 Apply the Counting Principle and Permutations

- There are 12 books to read over summer.
  - How many orders to read 4 of them?
  - How many orders to read all 12 books?

10.1 Apply the Counting Principle and Permutations

- Permutations with Repetition

$$\frac{n!}{q_1! \cdot q_2! \cdot q_3! \cdots}$$

  - Where n is the number of objects and q is how many times each is repeated.
- How many ways to rearrange WATERFALL?

10.2 Use Combinations and Binomial Theorem

- Combination
  - Arranging of objects without order

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

- Using a standard 52-card deck
  - How many 7-card hands?
  - How many 7-card flushes?

10.2 Use Combinations and Binomial Theorem

- On vacation you can visit up to 5 cities and 7 attractions.
  - How many combinations of 3 cities and 4 attractions?
  - How many combinations to visit at least 8 locations?

10.2 Use Combinations and Binomial Theorem

- A restaurant offers 6 salad toppings. On a deluxe salad, you can have up to 4 toppings. How many combinations?

10.2 Use Combinations and Binomial Theorem

- Binomial Theorem
- $(x + y)^0$ 1
- $(x + y)^1$ 1x 1y
- $(x + y)^2$ $1x^2$ 2xy $1y^2$
- $(x + y)^3$ $1x^3$ $3x^2y$ $3xy^2$ $1y^3$
- $(x + y)^4$ $1x^4$ $4x^3y$ $6x^2y^2$ $4xy^3$ $1y^4$

10.2 Use Combinations and Binomial Theorem

- Binomial Theorem
  - ♥ $(a+b)^n = {}_nC_0a^{n-0}b^0 + {}_nC_1a^{n-1}b^1 + \ldots + {}_nC_ra^{n-r}b^r$

$$= \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

10.2 Use Combinations and Binomial Theorem

- Expand $(a + 3)^5$

10.2 Use Combinations and Binomial Theorem

- Expand $(x + 2y^3)^4$

10.2 Use Combinations and Binomial Theorem

- Find the coefficient of the x^7 term in
  - $(2 - 3x)^{10}$

10.3 Define and Use Probability

- Probability
  - A number between 0 and 1 to indicate how likely something is to happen
  - 0 = cannot happen
  - 1 = always happens
- Theoretical Probability

$$P(A) = \frac{\text{Number of ways A happens}}{\text{Total number of possible outcomes}}$$

10.3 Define and Use Probability

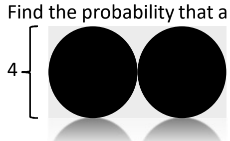
- A spinner with 8 equal sections are numbered 1 to 8. Find
  - P(6)
  - P(n > 5)
- There are 9 students on a team. Names are drawn to determine order of play. What is the probability that 3 of the 5 seniors will be chosen last?

10.3 Define and Use Probability

- Experimental Probability
  - Found by performing an experiment or survey
- Geometric Probability
  - Probabilities found from picking random points from areas or lines

10.3 Define and Use Probability

- Find the probability that a random dart will hit the shaded area.



10.3 Define and Use Probability

- Odds
  - When all outcomes are equally likely, the odds in favor of an event A is

$$\text{Odds in favor of A} = \frac{\text{Number of outcomes in A}}{\text{Number of outcomes not in A}}$$

- You can write odds as a ratio $\frac{a}{b}$ or $a : b$

10.3 Define and Use Probability

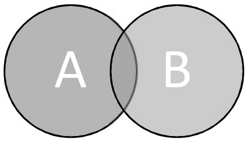
- A card is randomly drawn from a standard deck. Find the indicated odds.
  - In favor of drawing a heart
  - Against drawing a queen

10.4 Find Probability of Disjoint and Overlapping Events (OR)

- Let's say you have 1 event and you want one of two results to happen
  - This is a compound event
- There may be some intersections where one condition satisfies both events so the events are overlapping
- If there is no intersection, then they are disjoint or mutually exclusive

10.4 Find Probability of Disjoint and Overlapping Events (OR)

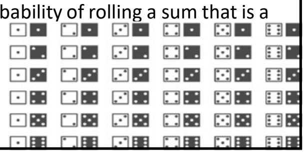
- P(A or B) = P(A) + P(B) – P(A and B)



- If they are disjoint or mutually exclusive
  - P(A and B) = 0

10.4 Find Probability of Disjoint and Overlapping Events (OR)

- One D6 is rolled. What is the probability of rolling a multiple of 3 or 5?
- Two D6 are rolled. What is the probability of rolling a sum that is a multiple of 2 or 3?



10.4 Find Probability of Disjoint and Overlapping Events (OR)

- In a poll of high school Jrs., 6 out of 15 took French and 11 out of 15 took math. 14 out of 15 took French or math. What is the probability that a student took both French and math?

10.4 Find Probability of Disjoint and Overlapping Events (OR)

- Complements ($\overline{A}$)
  - All the outcomes not in A
  - $P(\overline{A}) = 1 - P(A)$
- A card is randomly selected from a standard 52-card deck. Find
  - P(not K)
  - P(not (A or red))

10.5 Find Probabilities of Independent and Dependent Events (AND)

- Independent events
  - 1 event has no effect on another event
- P(A and B) = P(A)·P(B)

10.5 Find Probabilities of Independent and Dependent Events (AND)

- A game machine claims that 1 in every 15 wins. What is the probability that you win twice in a row?
- In a survey 9 out of 11 men and 4 out of 7 women said they were satisfied with a brand of orange juice. If the next 3 customers are 2 women and 1 man, what is the probability that all will be satisfied?

10.5 Find Probabilities of Independent and Dependent Events (AND)

- An auto repair company finds that 1 in 100 cars have to be returned for the same reason. If you take your car in 10 times, what is the probability that you will have the same thing fixed at least once.

10.5 Find Probabilities of Independent and Dependent Events (AND)

- Dependent Events
  - Dependent – 1 event affects the next
- Conditional Probability P(B|A)
  - Probability that B occurs given that A already occurred
- P(A and B) = P(A)·P(B|A)

10.5 Find Probabilities of Independent and Dependent Events (AND)

- You randomly draw 2 cards from a standard 52-card deck. Find the probability that the 1st card is a diamond and the 2nd is red if:
  - You replace
  - You don't replace

10.5 Find Probabilities of Independent and Dependent Events (AND)

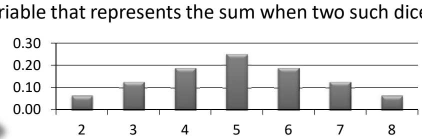
- Three children have a choice of 12 summer camps. If they choose randomly, what is the probability that they choose different camps (it is possible to choose the same camp)?

10.5 Find Probabilities of Independent and Dependent Events (AND)

- In a town, 95% of students graduate HS. A study shows that at age 25, 81% of HS grads held full-time jobs while only 63% of those who did not graduate held full-time jobs. What is the probability that a randomly selected student will have a full-time job?

10.6 Construct and Interpret Binomial Distributions

- Construct Probability Distributions
  - Make a table of all possible values of X and P(X)
  - Use that data to draw a bar graph (histogram)
- A tetrahedral die has four sides numbered 1 through 4. Let X be a random variable that represents the sum when two such dice are rolled.



10.6 Construct and Interpret Binomial Distributions

- Binomial Distributions
  - Two outcomes: Success or failure
  - Independent trials (n)
  - Probability for success is the same for each trial (p)

- P(k successes) = $_{n}C_{k}\, p^{k}\, (1-p)^{n-k}$

10.6 Construct and Interpret Binomial Distributions

- At college, 53% of students receive financial aid. In a random group of 9 students, what is the probability that exactly 5 of them receive financial aid?

10.6 Construct and Interpret Binomial Distributions

- Draw a histogram of binomial distribution of students in example 1 and find the probability of fewer than 3 students receiving financial aid.