Geometry 12
This Slideshow was developed to accompany the textbook

*Larson Geometry*

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Some examples and diagrams are taken from the textbook.

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12.1 EXPLORE SOLIDS

- Polyhedron
  - Solid with polygonal sides
  - Flat sides
- Face
  - Side
- Edge
  - Line segment
- Vertex
  - Corner
12.1 Explore Solids

- **Prism**
  - Polyhedron with two congruent surfaces on parallel planes (the 2 ends (bases) are the same)
  - Named by bases (i.e. rectangular prism, triangular prism)

- **Cylinder**
  - Solid with congruent circular bases on parallel planes (not a polyhedron)
12.1 EXPLORE SOLIDS

- Pyramid → polyhedron with all but one face intersecting in one point
- Cone → circular base with the other surface meeting in a point (kind of like a pyramid)
- Sphere → all the points that are a given distance from the center
Euler’s Theorem

The number of faces \( F \), vertices \( V \), and edges \( E \) of a polyhedron are related by

\[ F + V = E + 2 \]

- Convex
  - Any two points can be connected with a segment completely inside the polyhedron
- Concave
  - Not convex
  - Has a “cave”
Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges and describe as convex or concave.

Polyhedron; Square Pyramid; 5 faces, 5 vertices, 8 edges; convex

Not a Polyhedron

Polyhedron; Triangular Prism; 5 faces, 6 vertices, 9 edges; convex
Regular polyhedron
- Polyhedron with congruent regular polygonal faces

Only 5 types (Platonic solids)
- Tetrahedron \(\rightarrow\) 4 faces (triangular pyramid)
- Hexahedron \(\rightarrow\) 6 faces (cube)
- Octahedron \(\rightarrow\) 8 faces (2 square pyramids put together)
- Dodecahedron \(\rightarrow\) 12 faces (made with pentagons)
- Icosahedron \(\rightarrow\) 20 faces (made with triangles)
Cross Section

- Imagine slicing a very thin slice of the solid
- The cross section is the 2-D shape of the thin slice
12.1 Explore Solids

- Find the number of faces, vertices, and edges of a regular dodecahedron. Check with Euler’s Theorem.

- Describe the cross section.

- 798 #2-40 even, 44-60 even = 29

Triangle
Circle
Hexagon
12.1 Answers

12.1 Homework Quiz
Some sports relieve on having very little friction. In biking for example, the smaller the surface area of the tires, the less friction there is. And thus the faster the rider can go.

→ Draw the top triangle first (for some triangles you may have to count a horizontal space as 2)
12.2 Surface Area of Prisms and Cylinders

Nets
- Imagine cutting the three-dimensional figure along the edges and folding it out.
- Start by drawing one surface, then visualize unfolding the solid.
- To find the surface area, add up the area of each of the surfaces of the net.
12.2 SURFACE AREA OF PRISMS AND CYLINDERS

Parts of a right prism

- Bases → parallel congruent surfaces (the ends)
- Lateral faces → the other faces (they are parallelograms)
- Lateral edges → intersections of the lateral faces (they are parallel)
- Altitude → segment perpendicular planes containing the two bases with an endpoint on each plane
- Height → length of the altitude
12.2 Surface Area of Prisms and Cylinders

- Right prism
  - Prism where the lateral edges are altitudes
- Oblique prism
  - Prism that isn’t a right prism
You can find the surface area by adding up the areas of each surface, but if you could use a formula, it would be quicker.

All the lateral surfaces are rectangles.

Area = bh

Add up the areas $L = ah + bh + ch + ... + dh$

$L = (a + b + c + ... + d)h$

Perimeter of base = $a + b + c + ... + d$

$L = Ph$
12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Base Area (B)
  - In a prism, both bases are congruent, so you only need to find the area of one base and multiply by two

<table>
<thead>
<tr>
<th>Surface Area of a Right Prism</th>
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<tbody>
<tr>
<td>$S = 2B + Ph$</td>
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Where $S$ = surface area, $B$ = base area, $P$ = perimeter of base, $h$ = height of prism
Draw a net for a triangular prism.

Find the lateral area and surface area of a right rectangular prism with height 7 inches, length 3 inches, and width 4 inches.

\[ P = 2(3) + 2(4) = 14 \]
\[ L = (14)(7) = 98 \]
\[ B = 3 \cdot 4 = 12 \]
\[ A = 2(12) + 14(7) = 122 \]
Cylinders are the same as prisms except the bases are circles

Lateral Area = \( L = 2\pi rh \)

Surface Area of a Right Cylinder

\[ S = 2\pi r^2 + 2\pi rh \]

Where \( S \) = surface area, \( r \) = radius of base, \( h \) = height of prism
The surface area of a right cylinder is 100 cm². If the height is 5 cm, find the radius of the base.

Example: Draw a net for the cylinder and find its surface area.

\[ 100 = 2\pi r^2 + 2\pi r(5) \]
\[ 100 = 2\pi r^2 + 10\pi r \]
\[ 0 = 2\pi r^2 + 10\pi r - 100 \]
\[ 0 = r^2 + 5r - 15.915 \]
\[ r = \frac{-5 \pm \sqrt{5^2 - 4(1)(-15.915)}}{2(1)} \]
\[ r = \frac{-5 \pm \sqrt{88.662}}{2} \]
\[ r = 2.2, -7.2 \]

Only 2.2 makes sense because the radius must be positive.

\[ S = 2\pi 2^2 + 2\pi (2)(5) \]
\[ S = 8\pi + 20\pi = 28\pi \]
12.2 Answers

12.2 Homework Quiz
Pyramids

- All faces except one intersect at one point called **vertex**
- The **base** is the face that does not intersect at the vertex
- **Lateral faces** → faces that meet in the vertex
- **Lateral edges** → edges that meet in the vertex
- **Altitude** → segment that goes from the vertex and is perpendicular to the base
Lateral area is ½ because the sides are triangles.
Base Area

\[ B = \frac{1}{2} Pa \]

\[ B = \frac{1}{2} (5 \cdot 8)(5.5) = 110 \]

\[ \ell^2 = 5.5^2 + 4.8^2 \]

\[ \ell = 7.3 \]

\[ S = B + \frac{1}{2} P\ell \]

\[ S = 110 + \frac{1}{2} (5 \cdot 8)(7.3) = 256 \]
Cones
- Cones are just like pyramids except the base is a circle.
- Lateral Area = \( \pi r \ell \)

**Surface Area of a Right Cone**

\[
S = \pi r^2 + \pi r \ell
\]

Where \( r \) = base radius, \( \ell \) = slant height.
Example: The So-Good Ice Cream Company makes Cluster Cones. For packaging, they must cover each cone with paper. If the diameter of the top of each cone is 6 cm and its slant height is 15 cm, what is the area of the paper necessary to cover one cone?

814 #2-32 even, 35-39 all = 21
Extra Credit 817 #2, 6 = +2

Looking for lateral area.

\[ L = \pi 3(15) = 141.4 \text{ cm}^2 \]
12.3 Answers

12.3 Homework Quiz
12.4 Volume of Prisms and Cylinders

- Create a right prism using geometry cubes
- Count the lengths of the sides
- Count the number of cubes.
- Remember this to verify the formulas we are learning today.
Volume of a Prism

\[ V = Bh \]
Where \( B \) = base area, \( h \) = height of prism

Volume of a Cylinder

\[ V = \pi r^2 h \]
Where \( r \) = radius, \( h \) = height of cylinder
Cut into two prisms

Top

\[ V = 1(1)(1) = 1 \]

Bottom \[ V = 3(1)(2) = 6 \]

Total \[ V = 1 + 6 = 7 \]
Base Area (front)
Find height of triangle

\[\begin{align*}
5^2 + x^2 &= 10^2 \\
25 + x^2 &= 100 \\
x^2 &= 75 \\
x &= 5\sqrt{3}
\end{align*}\]

Area=triangle - square

\[B = \frac{1}{2} \cdot 10 \cdot (5\sqrt{3}) - 3^2\]
\[B = 25\sqrt{3} - 9 \approx 34.301\]

Volume = Bh

\[V = (25\sqrt{3} - 9)(6) = 150\sqrt{5} - 54 \approx 205.8\]
Find volume of washers without holes:  
$$V = \pi \frac{1}{2} 9 = 7.06858$$

Find volume of hole:  
$$V = \pi (\frac{3}{8})^2 9 = 3.97608$$

Find volume of washers with holes:  
$$7.06858 - 3.97608 = 3.09251$$

Find volume of one washer:  
$$\frac{3.09251}{150} = 0.02 \text{ in}^3$$
Cavalieri’s Principle

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

Find the volume.

\[ B = \frac{1}{2} (9)(5) = 22.5 \, m^2 \]
\[ V = (22.5 \, m^2)(8 \, m) = 180 \, m^3 \]
How much ice cream will fill an ice cream cone?

How could you find out without filling it with ice cream?

What will you measure?
12.5 Volume of Pyramids and Cones

Volume of a Pyramid

\[ V = \frac{1}{3} Bh \]

Where \( B \) = base area, \( h \) = height of pyramid

Volume of a Cone

\[ V = \frac{1}{3} \pi r^2 h \]

Where \( r \) = radius, \( h \) = height of cone
Find the volume.

\[ B = \frac{1}{2} Pa \]
\[ \frac{1}{2} \text{ central angle} = \frac{1}{2} \left( \frac{360}{6} \right) = 30 \]
\[ \tan 30 = \frac{2}{a} \]
\[ a = \frac{2}{\tan 30} = 3.464 \]
\[ B = \frac{1}{2} (4 \cdot 6)(3.464) = 41.569 \]
\[ V = \frac{1}{3} (41.569)(11) = 152.42 \]

\[ \tan 40 = \frac{r}{5.8} \]
\[ r = 5.8 \cdot \tan 40 = 4.8668 \]
\[ V = \frac{1}{3} \pi (4.8668^2 \cdot 5.8) = 143.86 \]
12.5 Answers

12.5 Homework Quiz
12.6 SURFACE AREA AND VOLUME OF SPHERES

Terms

- **Sphere** → all points equidistant from center
- **Radius** → segment from center to surface
- **Chord** → segment that connects two points on the sphere
- **Diameter** → chord contains the center of the sphere
- **Tangent** → line that intersects the sphere in exactly one place
12.6 Surface Area and Volume of Spheres

- Intersections of plane and sphere
  - Point $\rightarrow$ plane tangent to sphere
  - Circle $\rightarrow$ plane not tangent to sphere
  - Great Circle $\rightarrow$ plane goes through center of sphere (like equator)
    - Shortest distance between two points on sphere
    - Cuts sphere into two hemispheres
You can think about cutting a sphere into many small regular square pyramids. 

\[ V = \frac{1}{3} Bh \]  
the area of all the bases is \( 4\pi r^2 \) and \( h = r \)
Find the volume of the empty space in a box containing three golf balls. The diameter of each is about 1.5 inches. The box is 4.5 inches by 1.5 inches by 1.5 inches.

842 #2-36 even, 40-44 even = 21

Volume of box: $4.5(1.5)(1.5) = 10.125$
Volume of each ball: $\frac{4}{3}\pi \cdot 0.75^3 = 1.767$
Volume of empty space: $10.125 - 3(1.767) = 4.824$
12.6 Answers

12.6 Homework Quiz
Russian Matryoshka dolls nest inside each other. Each doll is the same shape, only smaller. The dolls are similar solids.
12.7 Explore Similar Solids

- Similar Solids
  - Solids with same shape but not necessarily the same size
  - The lengths of sides are proportional
  - The ratios of lengths is called the scale factor
12.7 EXPLORE SIMILAR SOLIDS

- Congruent Solids
  ◇ Similar solids with scale factor of 1:1
- Following four conditions must be true
  ◇ Corresponding angles are congruent
  ◇ Corresponding edges are congruent
  ◇ Areas of corresponding faces are equal
  ◇ The volumes are equal
Determine if the following pair of shapes are similar, congruent or neither.

Cone A: $r = 4.3$, $h = 12$, slant height = 14.3
Cone B: $r = 8.6$, $h = 25$, slant height = 26.4

- Ratios: $\frac{8.6}{4.3} = 2$, $\frac{25}{12} = 2.08$. Not proportional so neither

Right Cylinder A: $r = 5.5$, height = 7.3
Right Cylinder B: $r = 5.5$, height = 7.3

1:1 ratio so congruent.
Similar Solids Theorem

If 2 solids are similar with a scale factor of \(a:b\), then the areas have a ratio of \(a^2:b^2\) and the volumes have a ratio of \(a^3:b^3\).
Cube C has a surface area of 216 square units and Cube D has a surface area of 600 square units. Find the scale factor of C to D.

Find the edge length of C.

Use the scale factor to find the volume of D.

\[ \text{850 #2-26 even, 30-48 even = 23} \]
\[ \text{Extra Credit 854 #2, 4 = +2} \]

Areas:
\[ \frac{216}{600} = \frac{9}{25} = \frac{c^2}{d^2} \]
\[ \frac{c}{d} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5} \]

Cube surface area: \( S = 6c^2 \)
\[ 216 = 6c^2 \]
\[ 36 = c^2 \]
\[ c = 6 \]

Volumes:
\[ \frac{c^3}{d^3} = \frac{3^3}{5^3} \]
\[ \frac{6^3}{d^3} = \frac{3^3}{5^3} \]
\[ \frac{216}{d^3} = \frac{27}{125} \]
\[ 27d^3 = 216(125) \]
\[ d^3 = 1000 \]
volume of D is 1000
ANSWERS AND QUIZ

- 12.7 Answers
- 12.7 Homework Quiz
861 #1-17 all = 17

12. CHAPTER TEST

Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.

1. 2. 3.

Find the surface area of the solid. The prism, pyramid, cylinder, and cone are right. Round your answer to two decimal places, if necessary.

4. 5. 6.

Find the volume of the right prism or right cylinder. Round your answer to two decimal places, if necessary.

7. 8. 9. 10.

In Exercises 13–15, solve for x.

13. Volume = 324 in.³
14. Volume = \( \frac{32}{5} \) ft³
15. Volume = 180 cm³

MARBLE: The diameter of the marble shown is 30 millimeters. Find the surface area and volume of the marble.

16. PACKAGING: Two similar cylindrical cans have a scale factor of 2 : 3. The smaller can has surface area 500 square inches and volume 750 cubic inches. Find the surface area and volume of the larger can.