Parallel and Perpendicular Lines

Geometry
Chapter 3

Geometry 3

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Geometry*
  - *By Larson and Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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3.1 Pairs of Lines and Angles

- Objectives: By the end of the lesson,
  - I can identify lines and planes.
  - I can identify parallel and perpendicular lines.
  - I can identify pairs of angles formed by transversals.

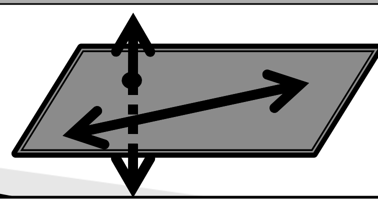
3.1 Pairs of Lines and Angles

Parallel Lines ||

- Lines that do NOT intersect and are coplanar
- Lines go in the same direction

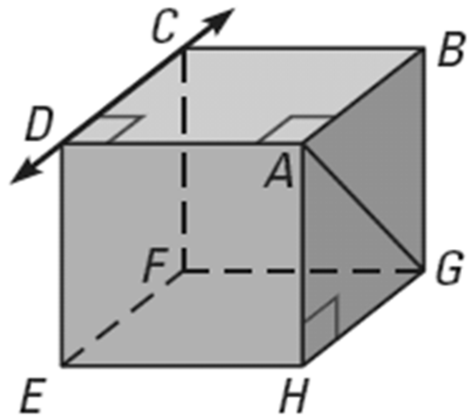
Skew Lines

- Lines that do NOT intersect and are on different planes
- Lines go in different directions



3.1 Pairs of Lines and Angles

- Name the lines through point H that appear skew to $\overleftrightarrow{CD}$
- Name the lines containing point H that appear parallel to $\overleftrightarrow{CD}$
- Name a plane that is parallel to plane CDE and contains point H



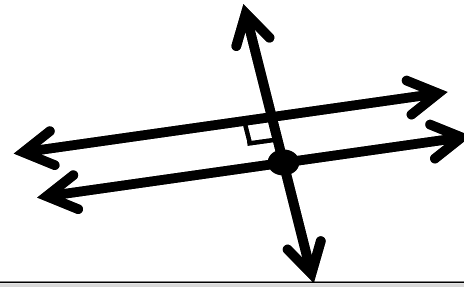
AH, EH

GH

BGH

3.1 Pairs of Lines and Angles

- In a plane, two lines are either
  - Parallel
  - Intersect



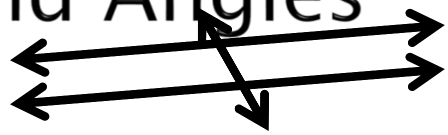
Parallel Postulate

If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

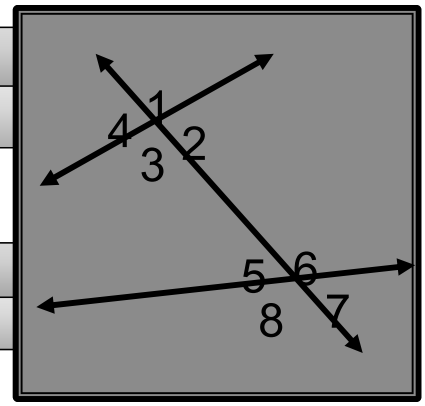
3.1 Pairs of Lines and Angles



Transversal
- Line that intersects two coplanar lines

Interior ∠
- angles that are between the lines
- ∠2, ∠3, ∠5, ∠6

Exterior ∠
- angles that are outside of the lines
- ∠1, ∠4, ∠7, ∠8

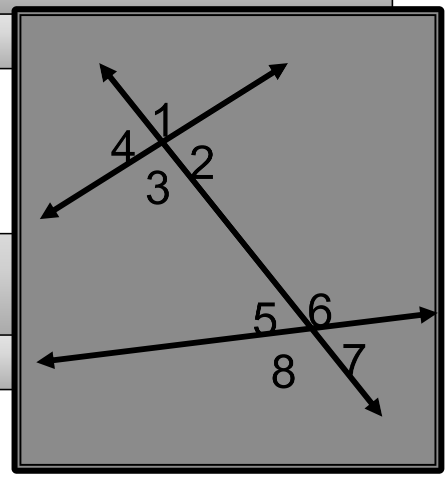


3.1 Pairs of Lines and Angles

Alternate interior angles

interior angles on opposite sides of the transversal

∠2 and ∠5, ∠3 and ∠6



Alternate exterior angles

exterior angles on opposite sides of the transversal

∠1 and ∠8, ∠4 and ∠7

3.1 Pairs of Lines and Angles

Consecutive interior angles

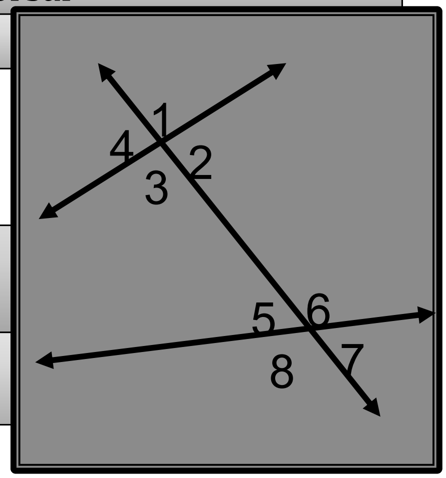
interior angles on the same side of the transversal

∠2 and ∠6, ∠3 and ∠5

Corresponding angles

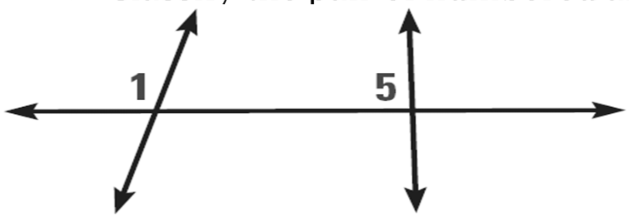
angles on the same location relative to the transversal

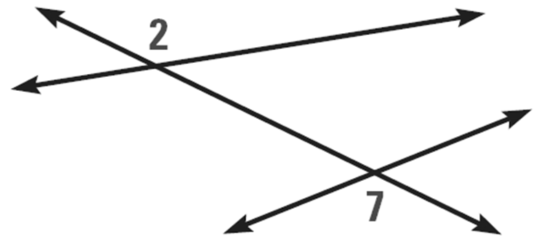
∠1 and ∠6, ∠2 and ∠7,
∠3 and ∠8, ∠4 and ∠5

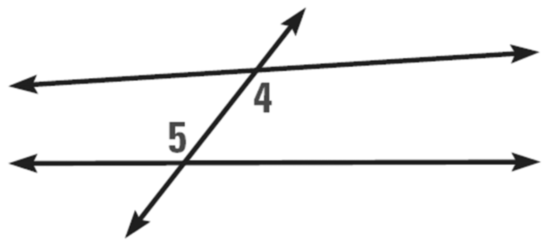


3.1 Pairs of Lines and Angles

- Classify the pair of numbered angles







- *125 #2, 4, 6, 8, 9, 10, 11, 12, 14, 15, 16, 20, 21, 22, 24, 28, 32, 33, 35, 36 = 20 total*

Corresponding
Alternate Exterior
Alternate Interior

3.2 Parallel Lines and Transversals

- Objectives: By the end of the lesson,
  - I can use properties of parallel lines to find angle measures.
  - I can prove theorems about parallel lines.

3.2 Parallel Lines and Transversals

- Draw parallel lines on a piece of notebook paper, then draw a transversal.
- Use the protractor to measure all the angles.
- What types of angles are congruent?
  - (*corresponding, alt interior, alt exterior*)
- How are consecutive interior angles related?
  - (*supplementary*)

3.2 Parallel Lines and Transversals

Corresponding Angles Postulate

If 2 || lines are cut by transversal, then the corrs ∠ are ≅

Alternate Interior Angles Theorem

If 2 || lines are cut by transversal, then the alt int ∠ are ≅

Alternate Exterior Angles Theorem

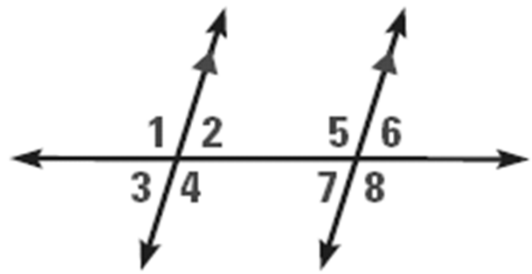
If 2 || lines are cut by transversal, then the alt ext ∠ are ≅

Consecutive Interior Angles Theorem

If 2 || lines are cut by transversal, then the cons int ∠ are supp.

3.2 Parallel Lines and Transversals

- If m∠1 = 105°, find m∠4, m∠5, and m∠8. Tell which postulate or theorem you use in each case



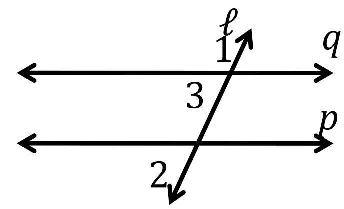
- If m∠3 = 68° and m∠8 = (2x + 4)°, what is the value of x?

m∠4 = 105; vertical angles are congruent
m∠5 = 105; corresponding angles postulate
m∠8 = 105; alt ext angles theorem

m∠3 = m∠2
m∠8 = m∠5
∠2 and ∠5 are cons int angles and are supp
m∠2 + m∠5 = 180
m∠3 + m∠8 = 180
68 + 2x + 4 = 180
2x + 72 = 180
2x = 108
x = 54

3.2 Parallel Lines and Transversals

- Prove that if 2 || lines are cut by a transversal, then the ext angles on the same side of the transversal are supp.
- Given: *p* || *q*
- Prove: ∠1 and ∠2 are supp.



Statements	Reasons

p || q	(given)
m∠1 + m∠3 = 180	(linear pair post)
∠2 ≅ ∠3	(corrs angles post)
m∠2 = m∠3	(def ≅)
m∠1 + m∠2 = 180	(substitution)
∠1 and ∠2 are supp	(def supp)

3.2 Parallel Lines and Transversals

- *131 #2, 4, 5, 6, 8, 10, 12, 14, 15, 18, 20, 22, 23, 24, 26, 29, 30, 32, 33, 38 = 20 total*

3.3 Proofs with Parallel Lines

- Objectives: By the end of the lesson,
  - I can use theorems to identify parallel lines.
  - I can prove theorems about identifying parallel lines.

3.3 Prove Lines are Parallel

Corresponding Angles Converse

If 2 lines are cut by transversal so the corrs ∠ are ≅, then the lines are ||.

Alternate Interior Angles Converse

If 2 lines are cut by transversal so the alt int ∠ are ≅, then the lines are ||.

Alternate Exterior Angles Converse

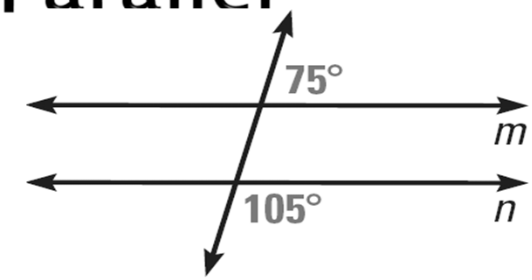
If 2 lines are cut by transversal so the alt ext ∠ are ≅, then the lines are ||.

Consecutive Interior Angles Converse

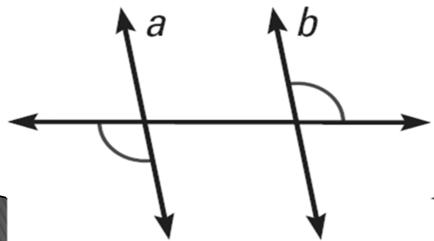
If 2 lines are cut by transversal so the cons int ∠ are supp., then the lines are ||.

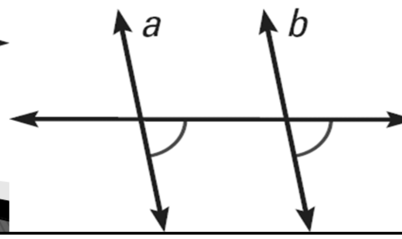
3.3 Prove Lines are Parallel

- Is there enough information to conclude that $m \parallel n$?

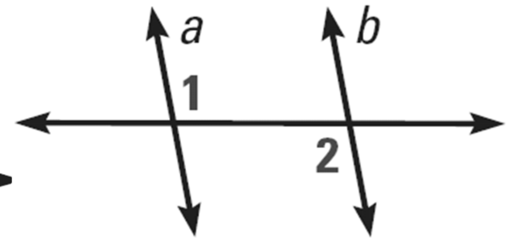


- Can you prove that the lines are parallel? Explain.





$m\angle 1 + m\angle 2 = 180°$



Yes, corresponding angles will both be 75°

Yes, alt ext angles converse
Yes, corres angles converse
No, should be $\angle 1 \cong \angle 2$ by alt int angles converse

3.3 Prove Lines are Parallel

Transitive Property of Parallel Lines

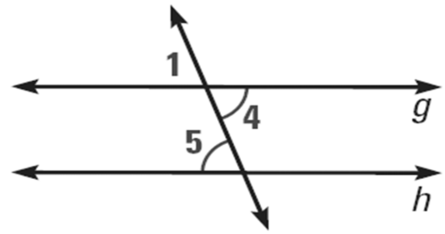
If two lines are parallel to the same line, then they are parallel to each other.

- Paragraph proofs
  - The proof is written in sentences.
  - Still need to have the statements and reasons.

3.3 Prove Lines are Parallel

- Write a paragraph proof to prove that if 2 lines are cut by a transversal so that the alt int ∠s are ≅, then the lines are ||.

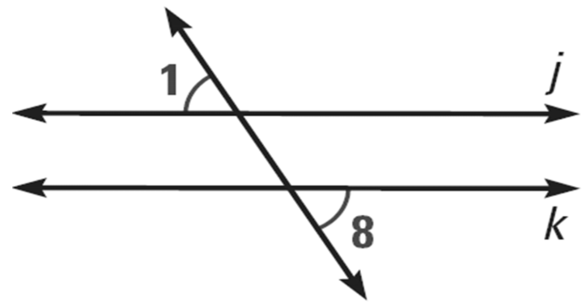
- Given: $\angle 4 \cong \angle 5$
- Prove: $g \parallel h$



It is given that ∠4 ≅ ∠5. By the vertical angle congruence theorem, ∠1 ≅ ∠4. Then by the Transitive Property of Congruence, ∠1 ≅ ∠5. So, by the Corresponding Angles Converse, g || h.

3.3 Prove Lines are Parallel

- If you use the diagram at the right to prove the Alternate Exterior Angles Converse, what GIVEN and PROVE statements would you use?



- *138 #2, 4, 6, 10, 12, 14, 16, 20, 22, 24, 26, 28, 30, 32, 35, 39, 41, 44, 45, 49 = 20 total*

Given: $\angle 1 \cong \angle 8$
Prove: j || k

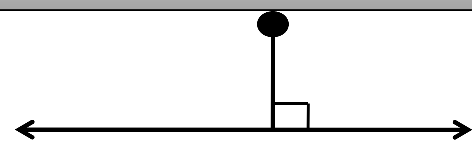
3.4 Proofs with Perpendicular Lines

- Objectives: By the end of the lesson,
  - I can find the distance from a point to a line.
  - I can prove theorems about perpendicular lines.

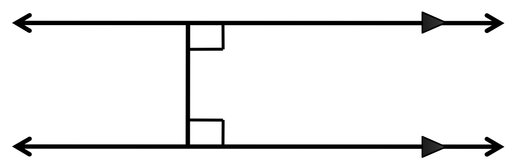
3.4 Proofs with Perpendicular Lines

Distance

From point to line: length of segment from point and ⊥ to line

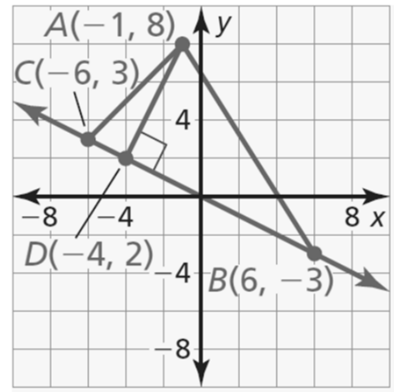


Between two || lines: length of segment ⊥ to both lines



3.4 Proofs with Perpendicular Lines

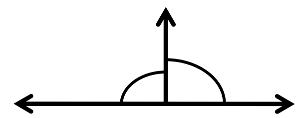
- Find the distance from point A to $\overleftrightarrow{BC}$.



Use the endpoints from the perpendicular segment (−4, 2) and (−1, 8)

Calculate distance $\sqrt{\left(-1-(-4)\right)^2+(8-2)^2} = \sqrt{3^2+(6)^2} = \sqrt{45} = 3\sqrt{5} = 6.7$

3.4 Proofs with Perpendicular Lines



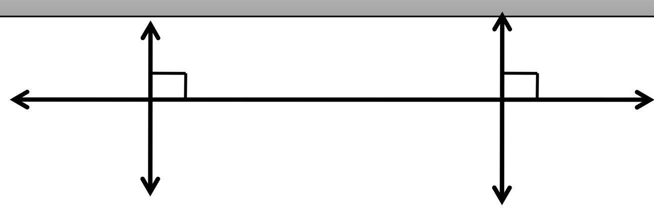
Linear Pair Perpendicular Theorem

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

3.4 Proofs with Perpendicular Lines

Perpendicular Transversal Theorem

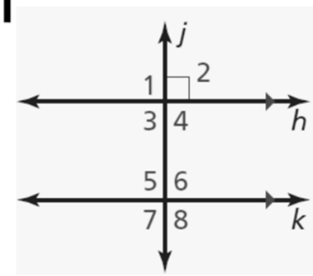
If a transversal is ⊥ to 1 of 2 || lines, then it is ⊥ to the other.



Lines ⊥ to a Transversal Theorem

In a plane, if 2 lines are ⊥ to the same line, then they are || to each other.

3.4 Proofs with Perpendicular Lines



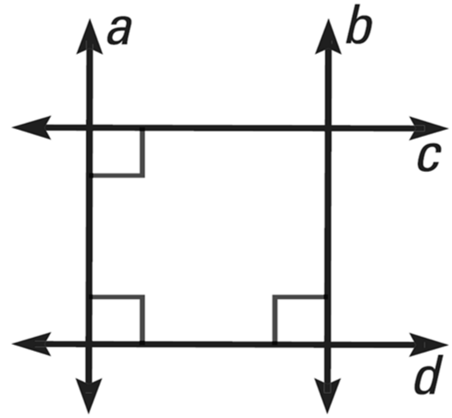
- Prove the Perpendicular Transversal Theorem using the diagram and the Alternate Interior Angles Theorem.
- Given: $h \parallel k, j \perp h$
- Prove: $j \perp k$

Statements	Reasons

STATEMENTS	**REASONS**
1. $h \parallel k, j \perp h$	**1.** Given
2. $m\angle 2 = 90°$	**2.** Definition of perpendicular lines
3. $\angle 2 \cong \angle 3$	**3.** Vertical Angles Congruence Theorem
4. $\angle 3 \cong \angle 6$	**4.** Alternate Interior Angles Theorem
5. $\angle 2 \cong \angle 6$	**5.** Transitive Property of Angle Congruence
6. $m\angle 2 = m\angle 6$	**6.** Definition of congruent angles
7. $m\angle 6 = 90°$	**7.** Substitution Property of Equality
8. $j \perp k$	**8.** Definition of perpendicular lines

3.4 Proofs with Perpendicular Lines

- Is $b \parallel a$?
- Is $b \perp c$?
- *146 #2, 10, 12, 14, 16, 18, 20, 21, 24, 26, 34, 40, 42, 45, 46 = 15 total*



Yes, lines perpendicular to transversal theorem

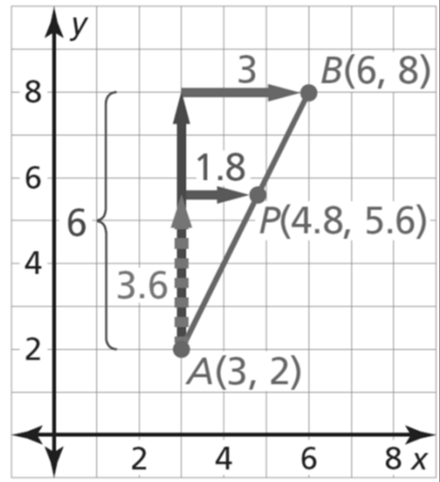
Yes, c || d by the lines ⊥ to trans theorem; b ⊥ c by the ⊥ trans theorem

3.5A Equations of Parallel and Perpendicular Lines

- Objectives: By the end of the lesson,
  - I can partition directed line segments using slope.
  - I can use slopes to identify parallel and perpendicular lines.

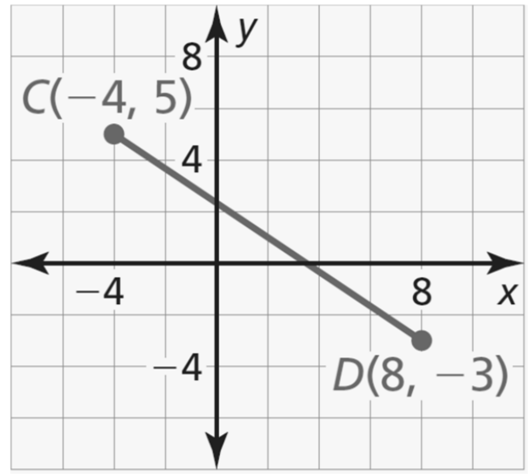
3.5A Equations of Parallel and Perpendicular Lines

- Partitioning a Directed Line Segment
  - Segment from *A* to *B*
  - Want the ratio of *AP* to *PB* to be something like 3 to 2
  - That means there are 3 + 2 = 5 pieces
  - Point *P* is $\frac{3}{5}$ of the way from *A*
  - Find the rise and run
  - Multiply the rise and run by the fraction $\frac{3}{5}$ and add to point A
  - The result is the coordinates of *P*



3.5A Equations of Parallel and Perpendicular Lines

- Find the coordinates of point F along the directed line segment CD so that the ratio of CF to FD is 3 to 5.



3 + 5 = 8
Fraction of line from C to F is 3/8
Rise = -8
Run = 12

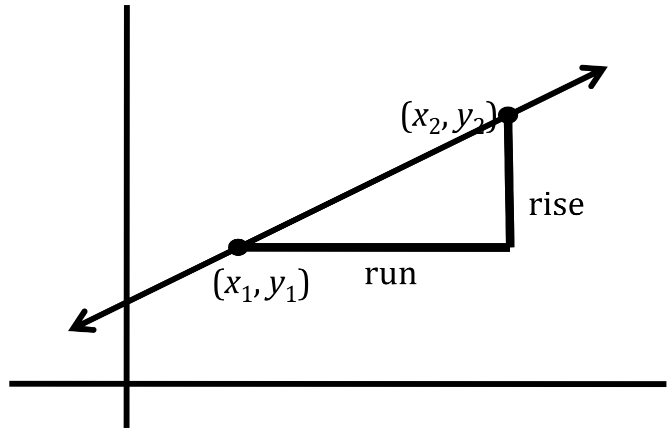
$$x = x_1 + fraction \cdot run = -4 + \frac{3}{8}(12) = 0.5$$
$$y = y_1 + fraction \cdot rise = 5 + \frac{3}{8}(-8) = 2$$

(0.5, 2)

3.5A Equations of Parallel and Perpendicular Lines

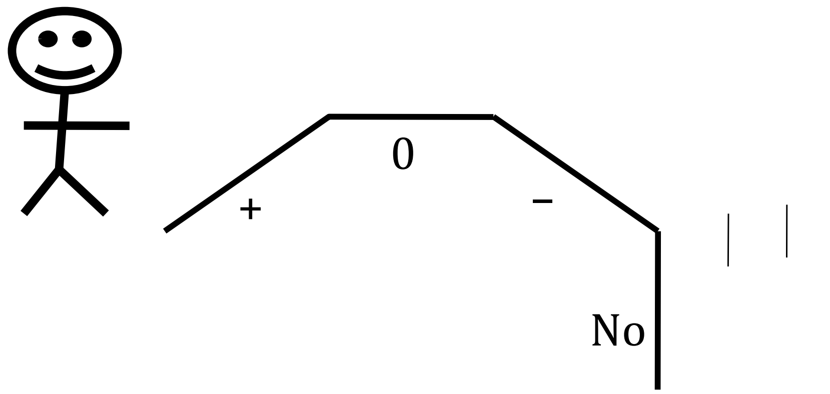
- Slope = $\frac{\text{rise}}{\text{run}}$

- $m = \frac{y_2 - y_1}{x_2 - x_1}$



3.5A Equations of Parallel and Perpendicular Lines

- Positive Slope
  - Rises
- Zero Slope
  - Horizontal
- Negative Slope
  - Falls
- No Slope (Undefined)
  - Vertical



There's **No Slope** to stand on.

3.5A Equations of Parallel and Perpendicular Lines

Slopes of Parallel Lines

- In a coordinate plane, 2 nonvertical lines are parallel iff they have the same slope.
- And, any 2 vertical lines are parallel.
- $m_1 = 2$; $m_2 = 2$

Slopes of Perpendicular Lines

- In a coordinate plane, 2 nonvertical lines are perpendicular iff the products of their slopes is -1.
- Or, Slopes are negative reciprocals.
- And, horizontal lines are perpendicular to vertical lines
- $m_1 = 2$; $m_2 = -\frac{1}{2}$

3.5A Equations of Parallel and Perpendicular Lines

- Tell whether the lines are *parallel*, *perpendicular*, or *neither*.
  - Line 1: through (–2, 8) and (2, –4)
  - Line 2: through (–5, 1) and (–2, 2)

- *154 #1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 53, 54, 57 = 13 total*

Line 1: (-4 – 8)/(2 – (-2)) → -12/4 → -3
Line 2: (2 – 1)/(-2 – (-5)) → 1/3
Perpendicular

3.5B Equations of Parallel and Perpendicular Lines

- Objectives: By the end of the lesson,
  - I can write equations of parallel and perpendicular lines.
  - I can find the distance from a point to a line.

3.5B Equations of Parallel and Perpendicular Lines

- Slope-intercept form of a line
  - $y = mx + b$
    - m = slope
    - b = y-intercept
- To write equations of lines using slope-intercept form
  - Find the slope
  - Find the y-intercept
    - It is given or,
    - Plug the slope and a point into $y = mx + b$ and solve for b
  - Write the equation of the line by plugging in m and b into $y = mx + b$

3.5B Equations of Parallel and Perpendicular Lines

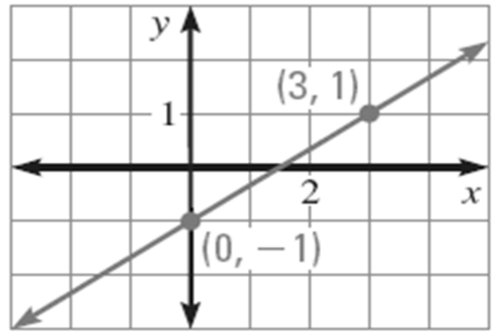
- Write an equation of the line that passes through (1, 5) and is parallel to the line with the equation $y = 3x - 5$.

$m = 3$ (parallel same slope)

$$y = mx + b$$
$$5 = 3(1) + b$$
$$b = 2$$
$$y = 3x + 2$$

3.5B Equations of Parallel and Perpendicular Lines

- Write an equation of the line perpendicular to the line in the graph and passing through (3, 1).



$$m_{given} = \frac{1-(-1)}{3-0} = \frac{2}{3}$$

$$m_{\perp} = -\frac{3}{2}$$

$$y = mx + b$$

$$1 = \left(-\frac{3}{2}\right)3 + b$$

$$1 = -\frac{9}{2} + b$$

$$\frac{11}{2} = b$$

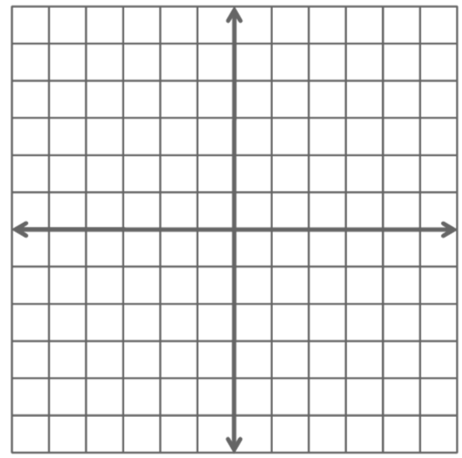
$$y = -\frac{3}{2}x + \frac{11}{2}$$

3.5B Equations of Parallel and Perpendicular Lines

- Find the distance from a point to a line
  - Find the equation of the line perpendicular to the given line and passing through the point.
  - Use a graph or system of equations to find where the lines intersect.
  - Find the distance between the given point and the point of intersection.

3.5B Equations of Parallel and Perpendicular Lines

- Find the distance from the point (6, −2) to the line $y = 2x - 4$.



- *154 #12, 14, 16, 18, 20, 22, 24, 36, 38, 46, 62, 64 = 12 total*

Equation of Perpendicular line

$$m_\perp = -\frac{1}{2}$$
$$y = mx + b$$
$$-2 = -\frac{1}{2}(6) + b$$
$$b = 1$$
$$y = -\frac{1}{2}x + 1$$

Find intersection of two lines (substitution)

$$\begin{cases} y = 2x - 4 \\ y = -\frac{1}{2}x + 1 \end{cases}$$
$$2x - 4 = -\frac{1}{2}x + 1$$
$$\frac{5}{2}x = 5$$
$$x = 2$$
$$y = 2(2) - 4 = 0$$

(2, 0)
Find distance between (6, −2) and (2, 0)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(2-6)^2 + \left(0 - (-2)\right)^2}$$
$$d = \sqrt{16 + 4} = \sqrt{20}$$
$$d = 2\sqrt{5} \approx 4.5$$