Parallel and Perpendicular Lines

Geometry
Chapter 3

Geometry 3

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Geometry*
  - *By Larson and Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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3.1 Pairs of Lines and Angles

Objectives: By the end of the lesson,

- I can identify lines and planes.
- I can identify parallel and perpendicular lines.
- I can identify pairs of angles formed by transversals.

3.1 Pairs of Lines and Angles

Parallel Lines ||

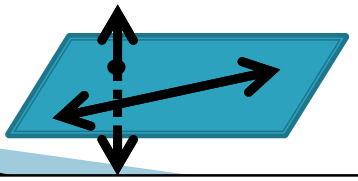
Lines that do NOT intersect and are coplanar

Lines go in the same direction

Skew Lines

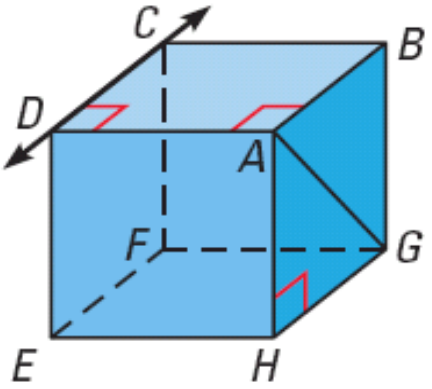
Lines that do NOT intersect and are on different planes

Lines go in different directions



3.1 Pairs of Lines and Angles

- Name the lines through point H that appear skew to $\overleftrightarrow{CD}$
- Name the lines containing point H that appear parallel to $\overleftrightarrow{CD}$
- Name a plane that is parallel to plane CDE and contains point H



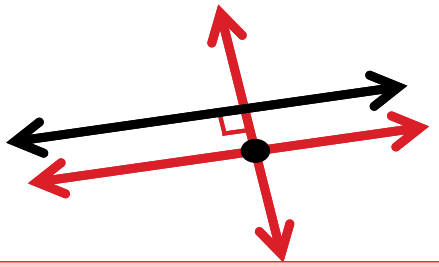
AH, EH

GH

BGH

3.1 Pairs of Lines and Angles

- In a plane, two lines are either
  - Parallel
  - Intersect



Parallel Postulate

If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

3.1 Pairs of Lines and Angles

Transversal

Line that intersects two coplanar lines

Interior ∠

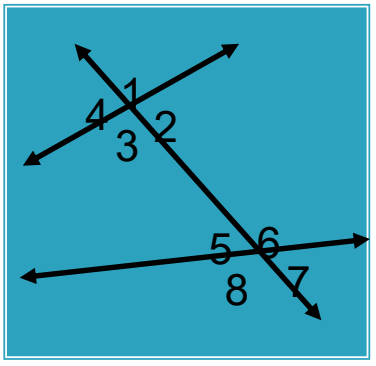
angles that are between the lines

∠2, ∠3, ∠5, ∠6

Exterior ∠

angles that are outside of the lines

∠1, ∠4, ∠7, ∠8



3.1 Pairs of Lines and Angles

Alternate interior angles

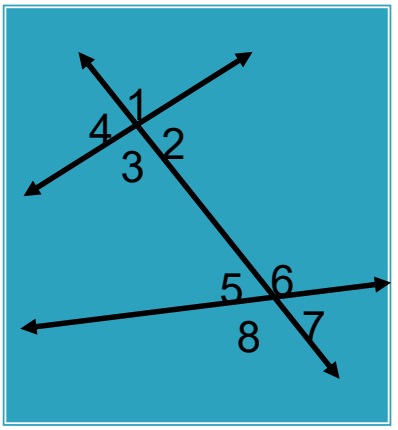
interior angles on opposite sides of the transversal

∠2 and ∠5, ∠3 and ∠6

Alternate exterior angles

exterior angles on opposite sides of the transversal

∠1 and ∠8, ∠4 and ∠7



3.1 Pairs of Lines and Angles

Consecutive interior angles

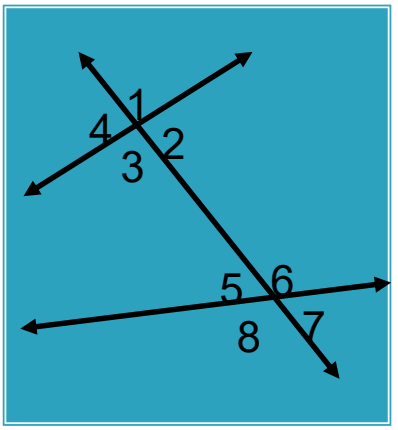
interior angles on the same side of the transversal

∠2 and ∠6, ∠3 and ∠5

Corresponding angles

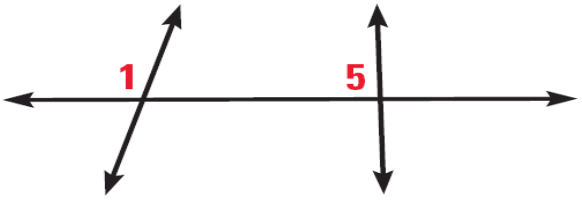
angles on the same location relative to the transversal

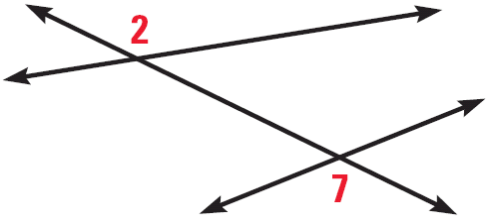
∠1 and ∠6, ∠2 and ∠7,
∠3 and ∠8, ∠4 and ∠5

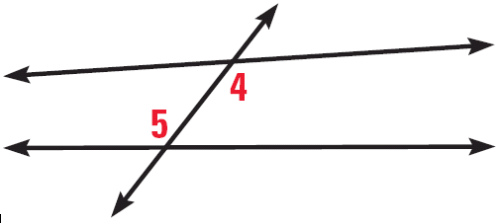


3.1 Pairs of Lines and Angles

- Classify the pair of numbered angles







- *, 12, 14, 15, 16, 20, 21, 22, 24, 28, 32, 33, 35, 36 = 20 total*

Corresponding
Alternate Exterior
Alternate Interior

3.2 Parallel Lines and Transversals

- Objectives: By the end of the lesson,
  - I can use properties of parallel lines to find angle measures.
  - I can prove theorems about parallel lines.

3.2 Parallel Lines and Transversals

- Draw parallel lines on a piece of notebook paper, then draw a transversal.
- Use the protractor to measure all the angles.
- What types of angles are congruent?
  - (*corresponding, alt interior, alt exterior*)
- How are consecutive interior angles related?
  - (*supplementary*)

3.2 Parallel Lines and Transversals

Corresponding Angles Postulate

If 2 || lines are cut by trans., then the corrs ∠ are ≅

Alternate Interior Angles Theorem

If 2 || lines are cut by trans., then the alt int ∠ are ≅

Alternate Exterior Angles Theorem

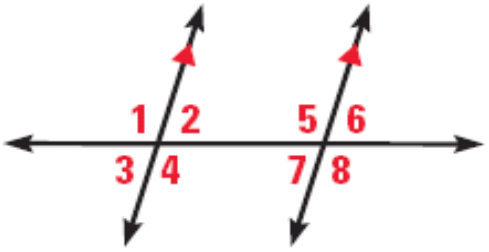
If 2 || lines are cut by trans., then the alt ext ∠ are ≅

Consecutive Interior Angles Theorem

If 2 || lines are cut by trans., then the cons int ∠ are supp.

3.2 Parallel Lines and Transversals

- If m∠1 = 105°, find m∠4, m∠5, and m∠8. Tell which postulate or theorem you use in each case



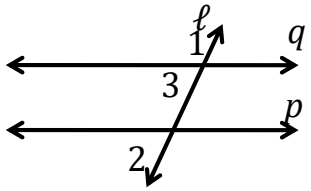
- If m∠3 = 68° and m∠8 = (2*x* + 4)°, what is the value of *x*?

m∠4 = 105; vertical angles are congruent
m∠5 = 105; corresponding angles postulate
m∠8 = 105; alt ext angles theorem

m∠3 = m∠2
m∠8 = m∠5
∠2 and ∠5 are cons int angles and are supp
m∠2 + m∠5 = 180
m∠3 + m∠8 = 180
68 + 2x + 4 = 180
2x + 72 = 180
2x = 108
x = 54

3.2 Parallel Lines and Transversals

- Prove that if 2 || lines are cut by a trans, then the ext angles on the same side of the trans are supp.
- Given: $p \parallel q$
- Prove: $\angle 1$ and $\angle 2$ are supp.



Statements	Reasons
p || q	(given)
m∠1 + m∠3 = 180	(linear pair post)
∠2 ≅ ∠3	(corrs angles post)
m∠2 = m∠3	(def ≅)
m∠1 + m∠2 = 180	(substitution)
∠1 and ∠2 are supp	(def supp)

3.2 Parallel Lines and Transversals

- *131 #2, 4, 5, 6, 8, 10, 12, 14, 15, 18, 20, 22, 23, 24, 26, 29, 30, 32, 33, 38 = 20 total*

3.3 Proofs with Parallel Lines

Objectives: By the end of the lesson,

- I can use theorems to identify parallel lines.
- I can prove theorems about identifying parallel lines.

3.3 Prove Lines are Paralle

Corresponding Angles Converse

If 2 lines are cut by trans. so the corrs ∠ are ≅, then the lines are ||.

Alternate Interior Angles Converse

If 2 lines are cut by trans. so the alt int ∠ are ≅, then the lines are ||.

Alternate Exterior Angles Converse

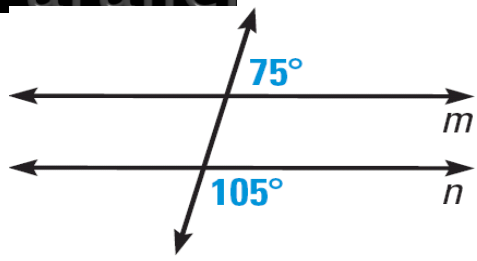
If 2 lines are cut by trans. so the alt ext ∠ are ≅, then the lines are ||.

Consecutive Interior Angles Converse

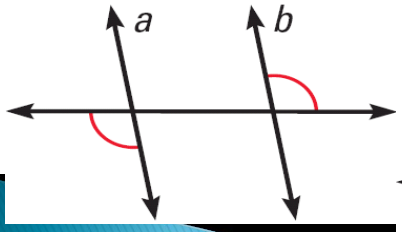
If 2 lines are cut by trans. so the cons int ∠ are supp., then the lines are ||.

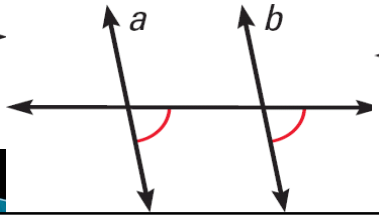
3.3 Prove Lines are Parallel

- Is there enough information to conclude that $m \parallel n$?

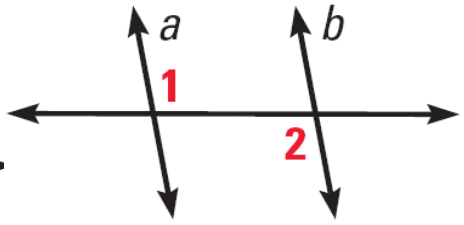


- Can you prove that the lines are parallel? Explain.





$m\angle 1 + m\angle 2 = 180°$



Yes, corresponding angles will both be 75°

Yes, alt ext angles converse
Yes, corres angles converse
No, should be $\angle 1 \cong \angle 2$ by alt int angles converse

3.3 Prove Lines are Paralle

Transitive Property of Parallel Lines

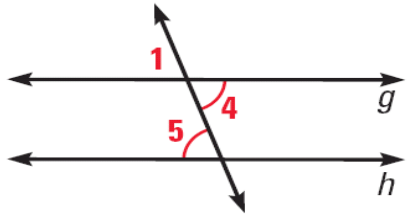
If two lines are parallel to the same line, then they are parallel to each other.

- Paragraph proofs
  - The proof is written in sentences.
  - Still need to have the statements and reasons.

3.3 Prove Lines are Parallel

- Write a paragraph proof to prove that if 2 lines are cut by a trans. so that the alt int ∠s are ≅, then the lines are ||.

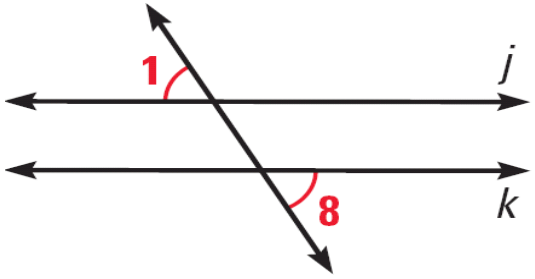
- Given: $\angle 4 \cong \angle 5$
- Prove: $g \parallel h$



It is given that $\angle 4 \cong \angle 5$. By the vertical angle congruence theorem, $\angle 1 \cong \angle 4$. Then by the Transitive Property of Congruence, $\angle 1 \cong \angle 5$. So, by the Corresponding Angles Converse, g || h.

3.3 Prove Lines are Parallel

- If you use the diagram at the right to prove the Alternate Exterior Angles Converse, what GIVEN and PROVE statements would you use?



- *138 #2, 4, 6, 10, 12, 14, 16, 20, 22, 24, 26, 28, 30, 32, 35, 39, 41, 44, 45, 49*

Given: ∠1 ≅ ∠8
Prove: j || k

3.4 Proofs with Perpendicular Lines

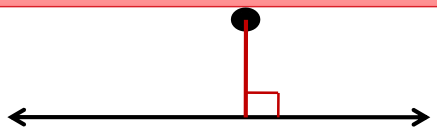
Objectives: By the end of the lesson,

- I can find the distance from a point to a line.
- I can prove theorems about perpendicular lines.

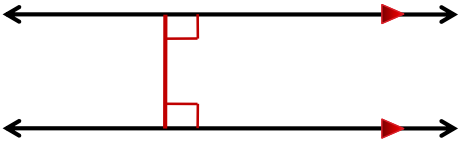
3.4 Proofs with Perpendicular Lines

Distance

From point to line: length of segment from point and ⊥ to line

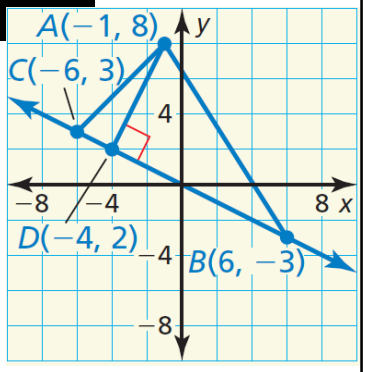


Between two || lines: length of segment ⊥ to both lines



3.4 Proofs with Perpendicular Lines

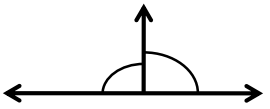
- Find the distance from point A to $\overleftrightarrow{BC}$.



Use the endpoints from the perpendicular segment (−4, 2) and (−1, 8)

Calculate distance $\sqrt{\big(-1-(-4)\big)^2+(8-2)^2} = \sqrt{3^2+(6)^2} = \sqrt{45} = 3\sqrt{5} = 6.7$

3.4 Proofs with Perpendicular Lines



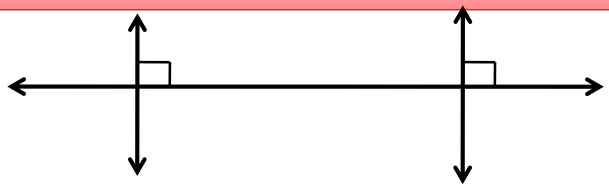
Linear Pair Perpendicular Theorem

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

3.4 Proofs with Perpendicular Lines

Perpendicular Transversal Theorem

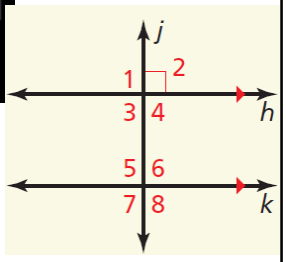
If a trans. is ⊥ to 1 of 2 || lines, then it is ⊥ to the other.



Lines ⊥ to a Transversal Theorem

In a plane, if 2 lines are ⊥ to the same line, then they are || to each other.

3.4 Proofs with Perpendicular Lines



- Prove the Perpendicular Transversal Theorem using the diagram and the Alternate Interior Angles Theorem.
- Given: $h \parallel k, j \perp h$
- Prove: $j \perp k$

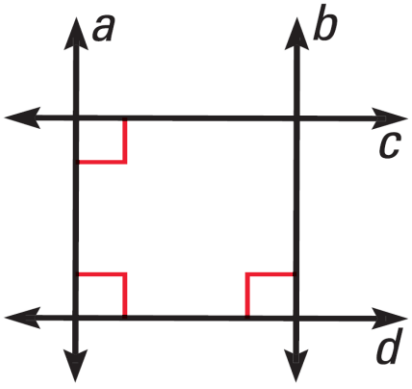
Statements	Reasons

STATEMENTS **REASONS**

1. $h \parallel k, j \perp h$	**1.** Given
2. $m\angle 2 = 90°$	**2.** Definition of perpendicular lines
3. $\angle 2 \cong \angle 3$	**3.** Vertical Angles Congruence Theorem
4. $\angle 3 \cong \angle 6$	**4.** Alternate Interior Angles Theorem
5. $\angle 2 \cong \angle 6$	**5.** Transitive Property of Angle Congruence
6. $m\angle 2 = m\angle 6$	**6.** Definition of congruent angles
7. $m\angle 6 = 90°$	**7.** Substitution Property of Equality
8. $j \perp k$	**8.** Definition of perpendicular lines

3.4 Proofs with Perpendicular Lines

- Is $b \parallel a$?



- Is $b \perp c$?

- *146 #2, 10, 12, 14, 16, 18, 20, 21, 24, 26, 34, 40, 42, 45, 46 = 15 total*

Yes, lines perpendicular to transversal theorem

Yes, c || d by the lines ⊥ to trans theorem; b ⊥ c by the ⊥ trans theorem

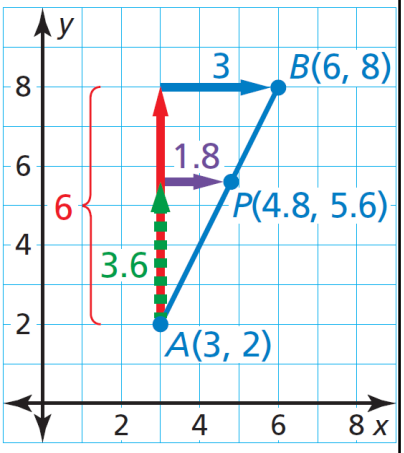
3.5A Equations of Parallel and Perpendicular Lines

Objectives: By the end of the lesson,

- I can partition directed line segments using slope.
- I can use slopes to identify parallel and perpendicular lines.

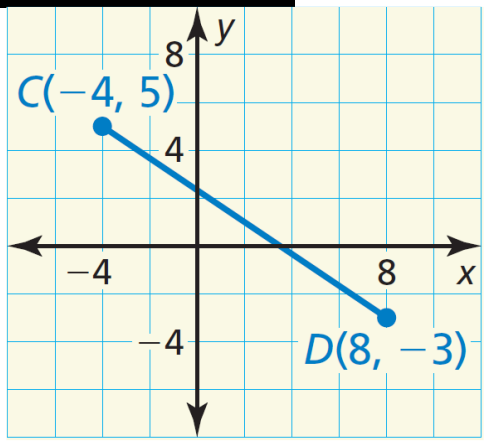
3.5A Equations of Parallel and Perpendicular Lines

- Partitioning a Directed Line Segment
  - Segment from A to B
  - Want the ratio of AP to PB to be something like 3 to 2
  - That means there are 3 + 2 = 5 pieces
  - Point P is $\frac{3}{5}$ of the way from A
  - Find the rise and run
  - Multiply the rise and run by the fraction $\frac{3}{5}$ and add to point A
  - The result is the coordinates of P



3.5A Equations of Parallel and Perpendicular Lines

- Find the coordinates of point F along the directed line segment CD so that the ratio of CF to FD is 3 to 5.



3 + 5 = 8
Fraction of line from C to F is 3/8
Rise = -8
Run = 12

$$x = x_1 + fraction \cdot run = -4 + \frac{3}{8}(12) = 0.5$$

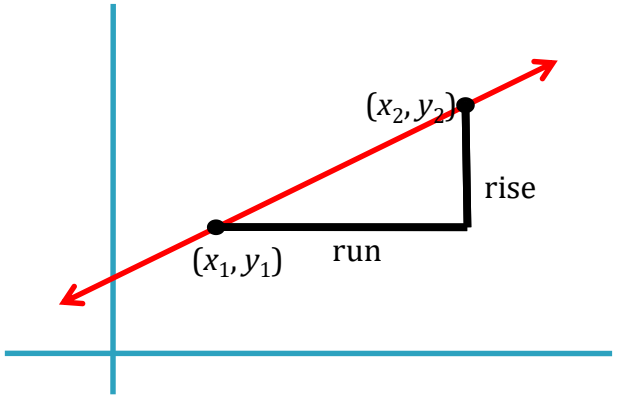
$$y = y_1 + fraction \cdot rise = 5 + \frac{3}{8}(-8) = 2$$

(0.5, 2)

3.5A Equations of Parallel and Perpendicular Lines

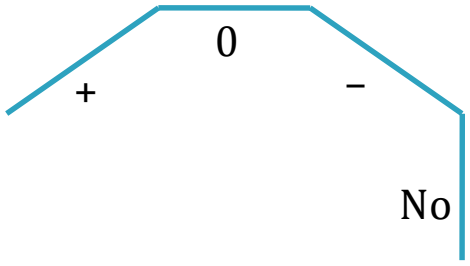
- Slope $= \frac{\text{rise}}{\text{run}}$

- $m = \frac{y_2 - y_1}{x_2 - x_1}$



3.5A Equations of Parallel and Perpendicular Lines

- Positive Slope
  - Rises
- Zero Slope
  - Horizontal
- Negative Slope
  - Falls
- No Slope (Undefined)
  -



There's **No Slope** to stand on.

3.5A Equations of Parallel and Perpendicular Lines

Slopes of Parallel Lines

In a coordinate plane, 2 nonvertical lines are parallel iff they have the same slope.

And, any 2 vertical lines are parallel.

$m_1 = 2$; $m_2 = 2$

Slopes of Perpendicular Lines

In a coordinate plane, 2 nonvertical lines are perpendicular iff the products of their slopes is -1.

Or, Slopes are negative reciprocals.

And, horizontal lines are perpendicular to vertical lines

$m_1 = 2$; $m_2 = -\frac{1}{2}$

3.5A Equations of Parallel and Perpendicular Lines

- Tell whether the lines are *parallel*, *perpendicular*, or *neither*.
  - Line 1: through (–2, 8) and (2, –4)
  - Line 2: through (–5, 1) and (–2, 2)

- *154 #1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 53, 54, 57 = 13 total*

Line 1: (-4 – 8)/(2 – (-2)) → -12/4 → -3
Line 2: (2 – 1)/(-2 – (-5)) → 1/3
Perpendicular

3.5B Equations of Parallel and Perpendicular Lines

Objectives: By the end of the lesson,

- I can write equations of parallel and perpendicular lines.
- I can find the distance from a point to a line.

3.5B Equations of Parallel and Perpendicular Lines

- Slope-intercept form of a line
  - $y = mx + b$
    - m = slope
    - b = y-intercept
- To write equations of lines using slope-intercept form
  - Find the slope
  - Find the y-intercept
    - It is given or,
    - Plug the slope and a point into $y = mx + b$ and solve for b
  - f the line by plugging in m and b into $y = mx + b$

3.5B Equations of Parallel and Perpendicular Lines

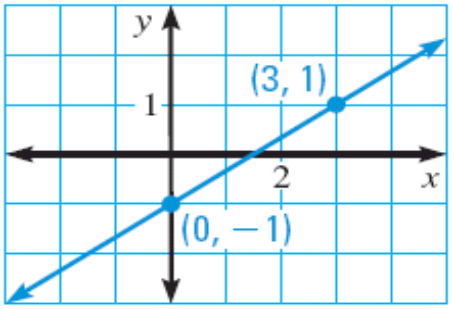
- Write an equation of the line that passes through (1, 5) and is parallel to the line with the equation $y = 3x - 5$.

$m = 3$ (parallel same slope)

$$y = mx + b$$
$$5 = 3(1) + b$$
$$b = 2$$
$$y = 3x + 2$$

3.5B Equations of Parallel and Perpendicular Lines

- Write an equation of the line perpendicular to the line in the graph and passing through (3, 1).



$$m_{given} = \frac{1-(-1)}{3-0} = \frac{2}{3}$$

$$m_{\perp} = -\frac{3}{2}$$

$$y = mx + b$$

$$1 = \left(-\frac{3}{2}\right)3 + b$$

$$1 = -\frac{9}{2} + b$$

$$\frac{11}{2} = b$$

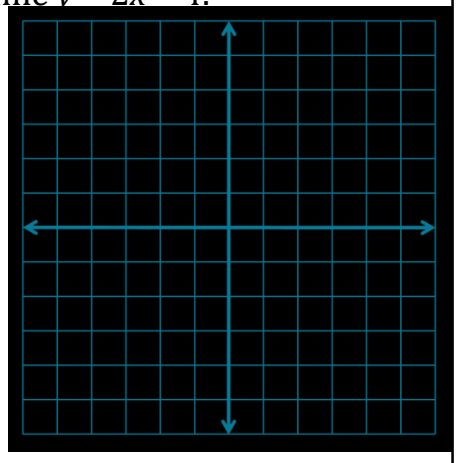
$$y = -\frac{3}{2}x + \frac{11}{2}$$

3.5B Equations of Parallel and Perpendicular Lines

- Find the distance from a point to a line
  - Find the equation of the line perpendicular to the given line and passing through the point.
  - Use a graph or system of equations to find where the lines intersect.
  - Find the distance between the given point and the point of intersection.

3.5B Equations of Parallel and Perpendicular Lines

- Find the distance from the point (6, −2) to the line $y = 2x - 4$.



- *154 #12, 14, 1[illegible] 22, 24, 36, 38, 46, 62, 64 = 12 total*

Equation of Perpendicular line

$$m_{\perp} = -\frac{1}{2}$$
$$y = mx + b$$
$$-2 = -\frac{1}{2}(6) + b$$
$$b = 1$$
$$y = -\frac{1}{2}x + 1$$

Find intersection of two lines (substitution)

$$\begin{cases} y = 2x - 4 \\ y = -\frac{1}{2}x + 1 \end{cases}$$
$$2x - 4 = -\frac{1}{2}x + 1$$
$$\frac{5}{2}x = 5$$
$$x = 2$$
$$y = 2(2) - 4 = 0$$

(2, 0)

Find distance between (6, −2) and (2, 0)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(2 - 6)^2 + \big(0 - (-2)\big)^2}$$
$$d = \sqrt{16 + 4} = \sqrt{20}$$
$$d = 2\sqrt{5} \approx 4.5$$