

PROPERTIES OF TRANSFORMATIONS

Geometry

Chapter 4

Geometry 4

- 
- A decorative graphic consisting of a long, horizontal arrow pointing to the right. The arrow has a dark grey body with a lighter grey outline and a yellow-orange gradient at the tail.
- This Slideshow was developed to accompany the textbook
 - *Big Ideas Geometry*
 - *By Larson and Boswell*
 - *2022 K12 (National Geographic/Cengage)*
 - Some examples and diagrams are taken from the textbook.

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After this lesson...

- I can translate figures.
- I can write a translation rule for a given translation.
- I can explain what a rigid motion is.
- I can perform a composition of translations on a figure.

4.1 TRANSLATIONS

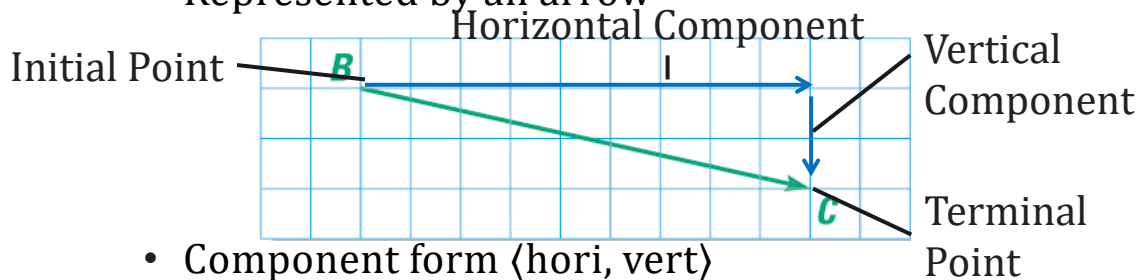


4.1 TRANSLATIONS



4.1 TRANSLATIONS

- Vector $(\overrightarrow{BC})$
 - Measurement with Direction and Magnitude (size)
 - Represented by an arrow



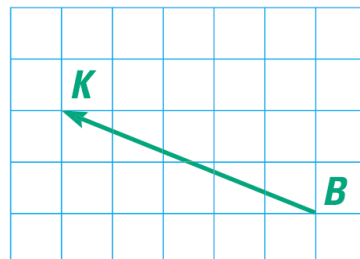
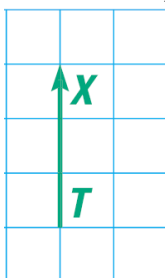
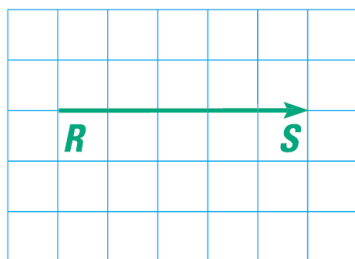
- Component form $\langle \text{hori, vert} \rangle$

- $\overrightarrow{BC} = \langle 9, -2 \rangle$

Can be used to describe translations

4.1 TRANSLATIONS

- Name the vector and write its component form



$$\begin{aligned}\overrightarrow{RS} &= \langle 5, 0 \rangle \\ \overrightarrow{TX} &= \langle 0, 3 \rangle \\ \overrightarrow{BK} &= \langle -5, 2 \rangle\end{aligned}$$

4.1 TRANSLATIONS

- Transformation
 - Moves or changes a figure
 - Original called preimage (i.e. ΔABC)
 - New called image (i.e. $\Delta A'B'C'$)
- Translation
 - Moves every point the same distance in the same direction
 - $(x, y) \rightarrow (x + a, y + b)$
 - where the $\langle a, b \rangle$ is the translation vector

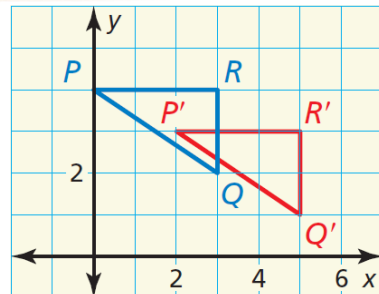
4.1 TRANSLATIONS

- The vertices of $\triangle LMN$ are $L(2, 2)$, $M(5, 3)$, $N(9, 1)$. Translate $\triangle LMN$ using vector $\langle -2, 6 \rangle$.

Translation is $(x, y) \rightarrow (x-2, y+6)$
 $L'(0, 8)$, $M'(3, 9)$, $N'(7, 7)$

4.1 TRANSLATIONS

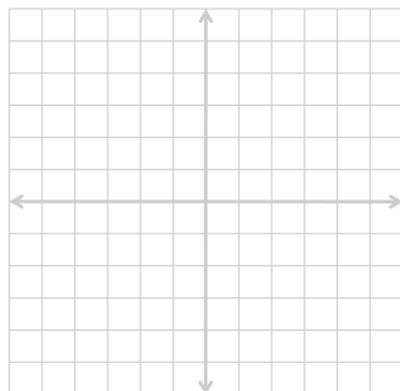
- Write a rule for the translation of $\triangle PQR$ to $\triangle P'Q'R'$.



$$(x, y) \rightarrow (x + 2, y - 1)$$

4.1 TRANSLATIONS

- Draw $\triangle RST$ with vertices $R(2, 2)$, $S(5, 2)$, and $T(3, -2)$. Find the image of each vertex after the translation $(x, y) \rightarrow (x + 1, y + 2)$. Graph the image using prime notation.



$R'(3, 4)$, $S'(6, 4)$, $T'(4, 0)$

4.1 TRANSLATIONS

- Rigid Motion
 - Transformation that preserves length and angle measure.
 - A congruence transformation

Translation Theorem

A translation is a rigid motion.

4.1 TRANSLATIONS

- Composition of Transformations
 - Two or more transformations combined into a single transformation

Composition Theorem

The composition of two (or more) rigid motions is a rigid motion.

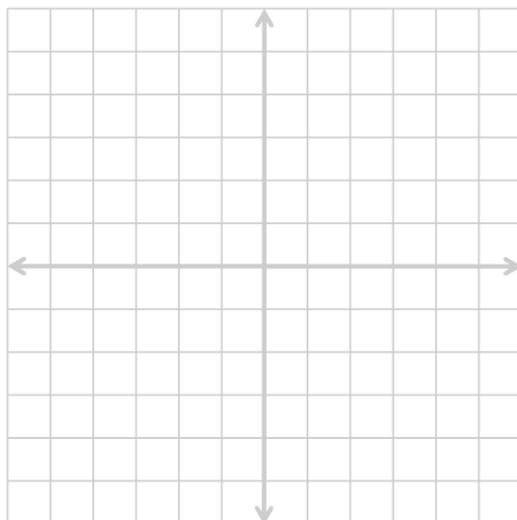
4.1 TRANSLATIONS

- Graph $\overline{RS}$ with endpoints $R(-8, 5)$ and $S(-6, 8)$. Graph its image after the composition.

Translation: $(x, y) \rightarrow (x - 1, y + 4)$

Translation: $(x, y) \rightarrow (x + 4, y - 6)$

- 172 #2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 32, 42, 43, 44, 48, 52



Translation 1: $(x, y) \rightarrow (x - 1, y + 4)$

$R(-8, 5) \rightarrow R'(-8 - 1, 5 + 4) = R'(-9, 9)$

$S(-6, 8) \rightarrow S'(-6 - 1, 8 + 4) = S'(-7, 12)$

Translation 2: $(x, y) \rightarrow (x + 4, y - 6)$

$R'(-9, 9) \rightarrow R''(-9 + 4, 9 - 6) = R''(-5, 3)$

$S'(-7, 12) \rightarrow S''(-7 + 4, 12 - 6) = S''(-3, 6)$

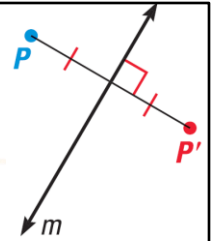
After this lesson...

- I can reflect figures.
- I can perform compositions with reflections.
- I can identify line symmetry in polygons.

4.2 REFLECTIONS



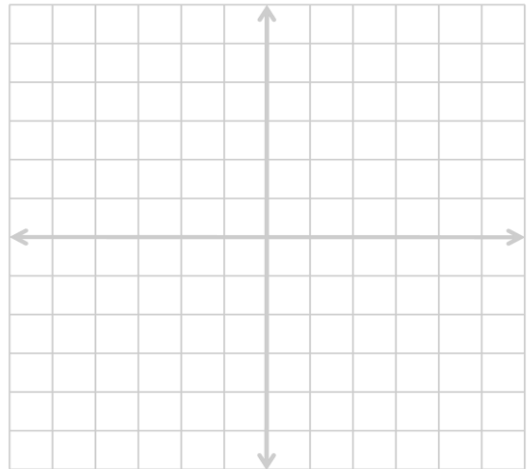
4.2 REFLECTIONS



- Reflection
 - Transformation that uses a line like a mirror to reflect an image.
 - That line is called **Line of Reflection**
 - P and P' are the same distance from the line of reflection
 - The line connecting P and P' is perpendicular to the line of reflection

4.2 REFLECTIONS

- Graph a reflection of $\triangle ABC$ where $A(1, 3)$, $B(5, 2)$, and $C(2, 1)$ in the line $x = 2$.



New points are $A(3, 3)$, $B(-1, 2)$, $C(2, 1)$

4.2 REFLECTIONS

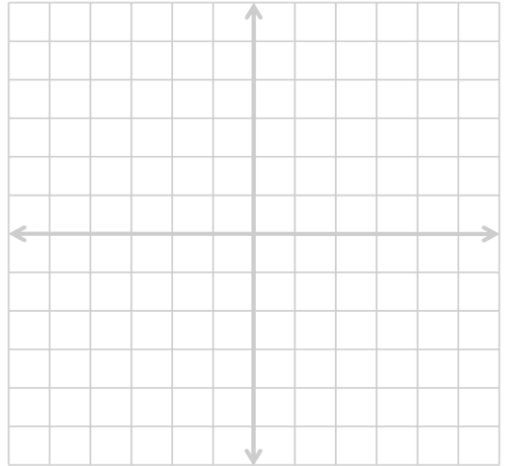
- Coordinate Rules for Reflections
 - Reflected in x -axis: $(a, b) \rightarrow (a, -b)$
 - Reflected in y -axis: $(a, b) \rightarrow (-a, b)$
 - Reflected in $y = x$: $(a, b) \rightarrow (b, a)$
 - Reflected in $y = -x$: $(a, b) \rightarrow (-b, -a)$

Reflection Theorem

A reflection is an rigid motion.

4.2 REFLECTIONS

- Graph $\triangle ABC$ with vertices $A(1, 3)$, $B(4, 4)$, and $C(3, 1)$. Reflect $\triangle ABC$ in the lines $y = -x$ and $y = x$.



$y = -x$: new points $A(-3, -1)$, $B(-4, -4)$, $C(-1, -3)$

$y = x$: new points $A(3, 1)$, $B(4, 4)$, $C(1, 3)$

4.2 REFLECTIONS

- The vertices of $\triangle LMN$ are $L(-3, 3)$, $M(1, 2)$, and $N(-2, 1)$. Find the reflection of $\triangle LMN$ in the y -axis.

$$(a, b) \rightarrow (-a, b)$$

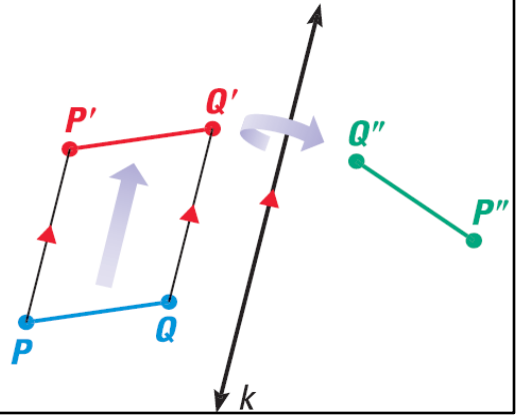
$$L(-3, 3) \rightarrow L'(3, 3)$$

$$M(1, 2) \rightarrow M'(-1, 2)$$

$$N(-2, 1) \rightarrow N'(2, 1)$$

4.2 REFLECTIONS

- Composition of Transformations
 - Two or more transformations combined into a single transformation
- Glide Reflection
 - Translation followed by reflection over a line parallel to the translation

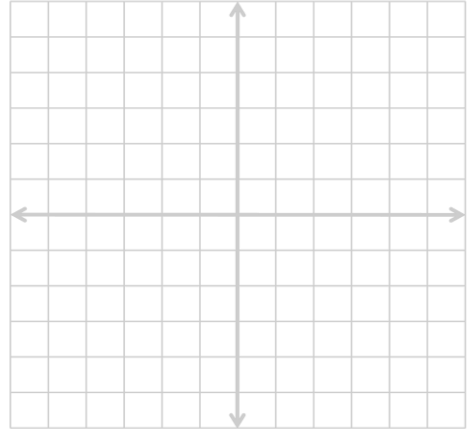


4.2 REFLECTIONS

- The vertices of $\triangle ABC$ are $A(3, 2)$, $B(-1, 3)$, and $C(1, 1)$. Find the image of $\triangle ABC$ after the glide reflection.

Translation: $(x, y) \rightarrow (x, y - 4)$

Reflection: Over y -axis

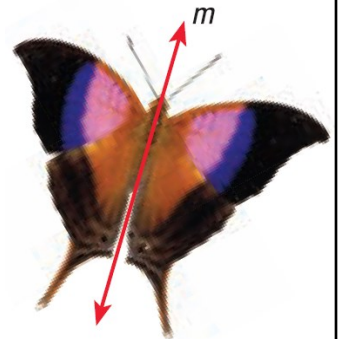


New points after translation: $A'(3, -2)$, $B'(-1, -1)$, $C'(1, -3)$

New points after reflection: $A''(-3, -2)$, $B''(1, -1)$, $C''(-1, -3)$

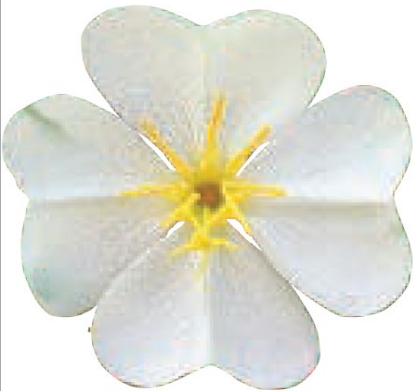
4.2 REFLECTIONS

- Line symmetry
 - The figure can be mapped to itself by a reflection
 - The line of reflection is called **Line of Symmetry**
- Humans tend to think that symmetry is beautiful



4.2 REFLECTIONS

- How many lines of symmetry does the object appear to have?



- 180 #2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 45, 49, 51, 54, 55

Flower: 4 lines of symmetry
Sea Star: 5 lines of symmetry
Goat: 1 line of symmetry

After this lesson...

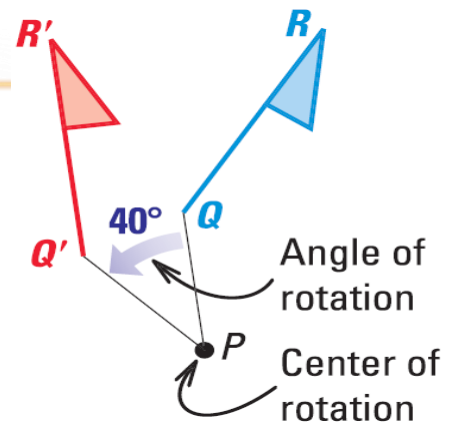
- I can rotate figures.
- I can perform compositions with rotations.
- I can identify rotational symmetry in polygons.

4.3 ROTATIONS



4.3 ROTATIONS

- Rotation
 - Figure is turned about a point called **center of rotation**
 - The amount of turning is **angle of rotation**



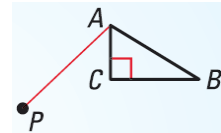
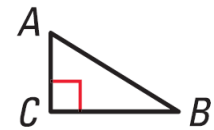
Rotation Theorem

A rotation is a rigid motion.

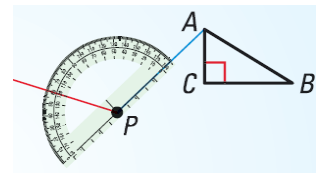
4.3 ROTATIONS

- Draw a 120° rotation of $\triangle ABC$ about P . 

1. Draw a segment from A to P .

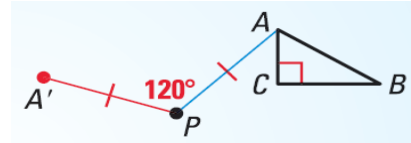


2. Draw a ray to form a 120° angle with $\overline{PA}$

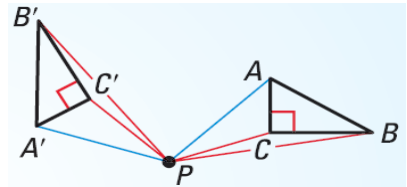


4.3 ROTATIONS

3. Draw A' so that $PA' = PA$

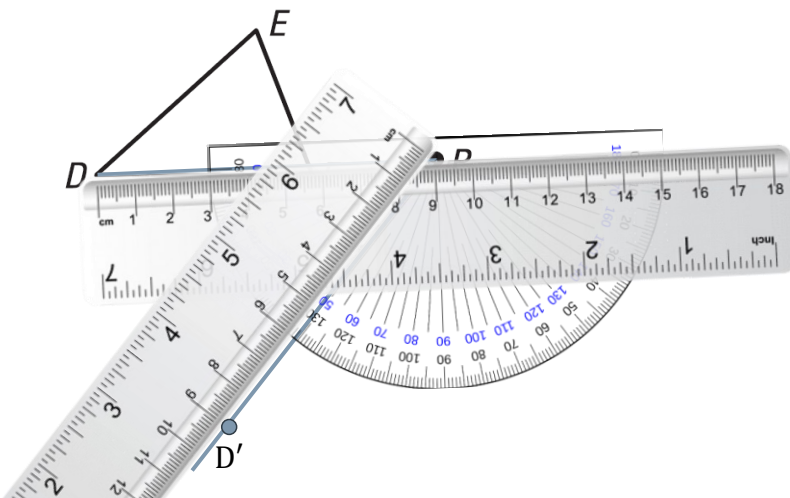


4. Repeat steps 1-3 for each vertex. Draw $\triangle A'B'C'$.



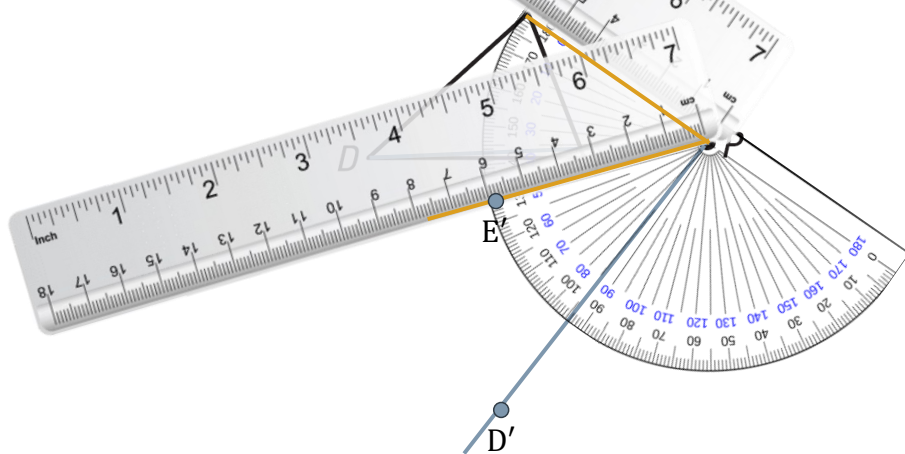
4.3 ROTATIONS

- Draw a 50° counterclockwise rotation of $\triangle DEF$ about P .



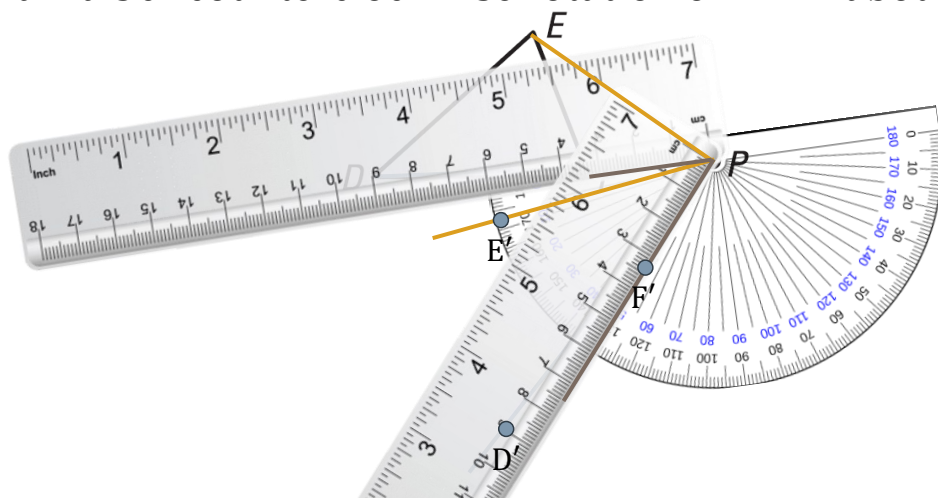
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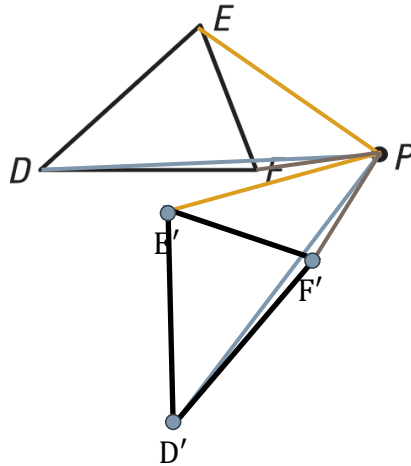
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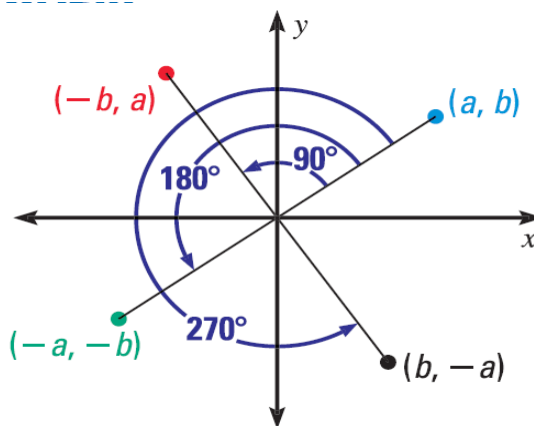
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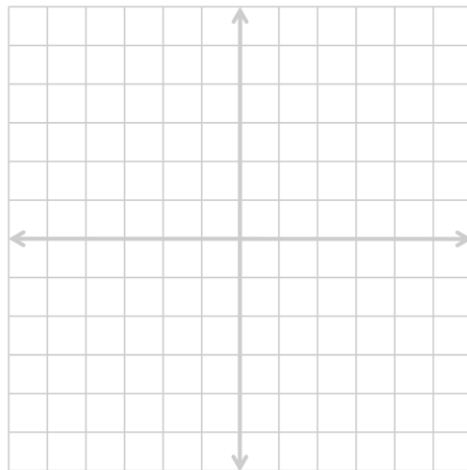
4.3 ROTATIONS

- Coordinate Rules for Counterclockwise Rotations about the Origin
- $90^\circ: (a, b) \rightarrow (-b, a)$
- $180^\circ: (a, b) \rightarrow (-a, -b)$
- $270^\circ: (a, b) \rightarrow (b, -a)$



4.3 ROTATIONS

- If $E(-3, 2)$, $F(-3, 4)$, $G(1, 4)$, and $H(2, 2)$. Find the image for a 270° rotation about the origin.



$$(a, b) \rightarrow (b, -a)$$

$$E(-3, 2) \rightarrow E'(2, 3)$$

$$F(-3, 4) \rightarrow F'(4, 3)$$

$$G(1, 4) \rightarrow G'(4, -1)$$

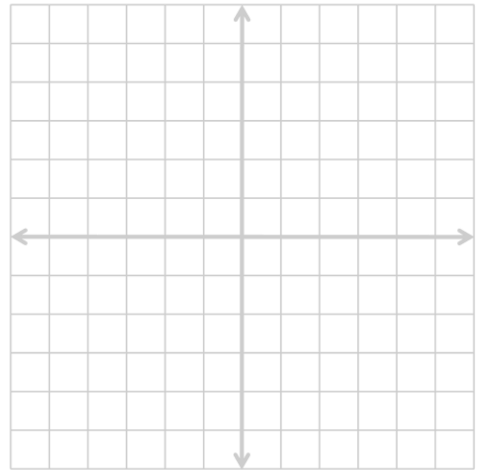
$$H(2, 2) \rightarrow H'(2, -2)$$

4.3 ROTATIONS

- Graph $\overline{RS}$ with endpoints $R(1, -3)$ and $S(2, -6)$ and its image after the composition.

Rotation: 180° about the origin

Reflection: in the y -axis



Rotation: $(a, b) \rightarrow (-a, -b)$

$R(1, -3) \rightarrow R'(-1, 3)$

$S(2, -6) \rightarrow S'(-2, 6)$

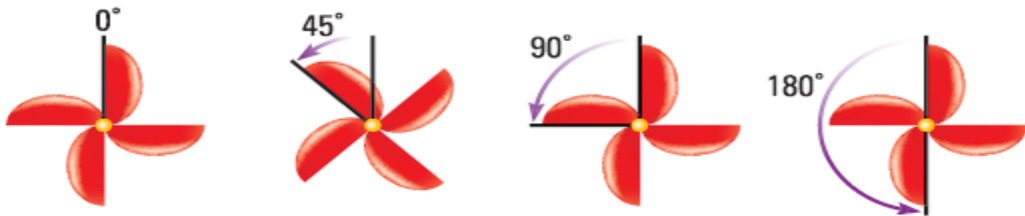
Reflection: $(a, b) \rightarrow (-a, b)$

$R'(-1, 3) \rightarrow R''(1, 3)$

$S'(-2, 6) \rightarrow S''(2, 6)$

4.3 ROTATIONS

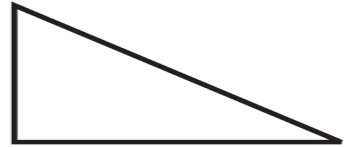
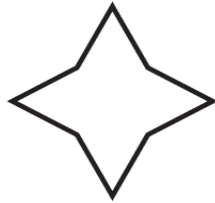
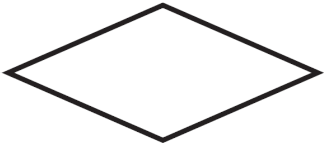
- Rotational Symmetry
 - The figure can be mapped to itself by a rotation of 180° or less about the center of the figure
 - The center of rotation is called the **Center of Symmetry**



Note: the 45° is not a symmetry

4.3 ROTATIONS

- Does the figure have rotational symmetry? What angles?



- 188 #2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 40, 42, 43, 46, 47

Rhombus: 180°

Octagon: 90° , 180°

Triangle: none

After this lesson...

- I can identify congruent figures.
- I can describe congruence transformations.
- I can use congruence transformations to solve problems.

4.4 CONGRUENCE AND TRANSFORMATIONS

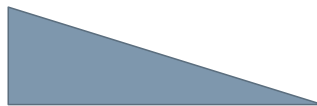
4.4 CONGRUENCE AND TRANSFORMATIONS

Congruent $\cong$

Exactly the same shape and size.



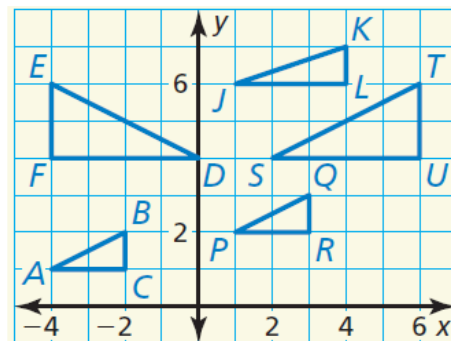
Congruent



Not Congruent

4.4 CONGRUENCE AND TRANSFORMATIONS

- Identify any congruent figures in the coordinate plane. Explain.



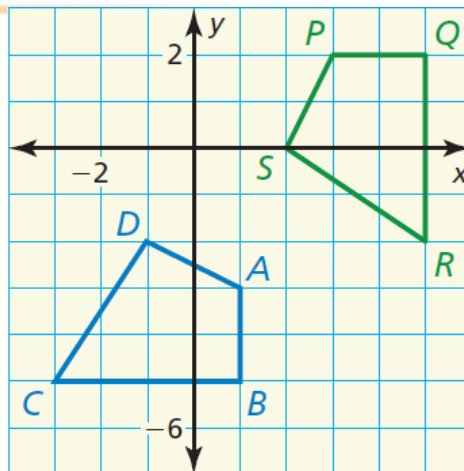
$\triangle ABC \cong \triangle PQR$, $\triangle DEF \cong \triangle STU$; $\triangle PQR$ is a translation 5 units right and 1 unit up of $\triangle ABC$. $\triangle STU$ is a reflection of $\triangle DEF$ in the line $x = 1$.

4.4 CONGRUENCE AND TRANSFORMATIONS

- Congruence Transformation
 - Transformation with rigid motion
 - Preimage $\cong$ image

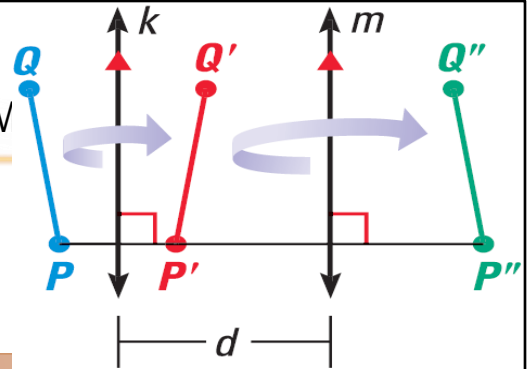
4.4 CONGRUENCE AND TRANSFORMATIONS

- Describe a congruence transformation that maps quadrilateral $ABCD$ to quadrilateral $PQRS$.



90° rotation about the origin, followed by a translation 1 unit up

4.4 CONGRUENCE AND TRANSFORM



Reflections in Parallel Lines Theorem

If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a translation.

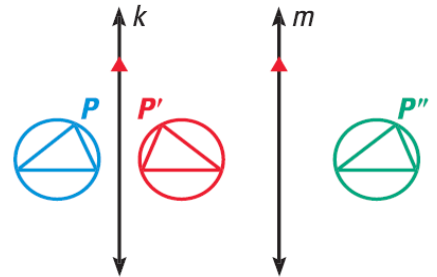
If P'' is the image of P , then

1. $\overline{PP''}$ is $\perp$ to k and m , and
2. $PP'' = 2d$ where d is the distance between k and m

4.4 CONGRUENCE AND TRANSFORMATIONS

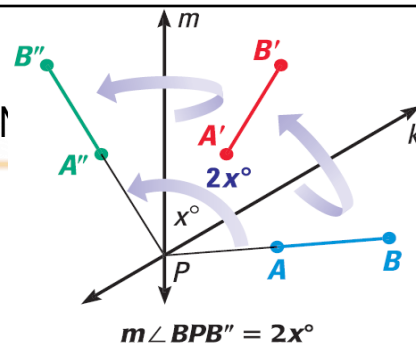
Use the figure below. The distance between line k and line m is 1.6 cm.

1. The preimage is reflected in line k , then in line m . Describe a single transformation that maps the blue figure to the green.
2. What is the distance from P and P'' ?



1. Translation in the x direction
2. $2(1.6 \text{ cm}) = 3.2 \text{ cm}$ (Reflections in Parallel Lines Thrm)

4.4 CONGRUENCE AND TRANSFORMATION



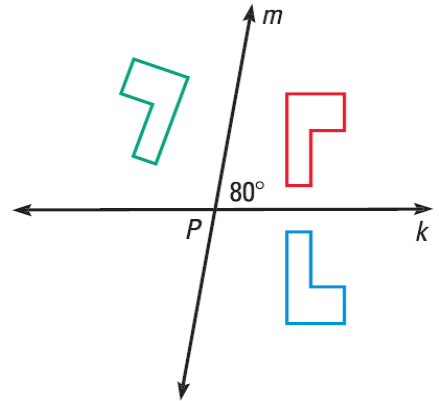
Reflections in Intersecting Lines Theorem

If lines k and m intersect at point P , then a reflection in line k followed by a reflection in line m is the same as a rotation about point P .

The angle of rotation is $2x^\circ$, where x° is the measure of the acute or right angle formed k and m .

4.4 CONGRUENCE AND TRANSFORMATIONS

- In the diagram, the preimage is reflected in line k , then in line m . Describe a single transformation that maps the blue figure to the green.



- 196 #2, 4, 6, 8, 10, 12, 14, 15, 16, 18, 20, 24, 26, 28, 35, 36, 42, 46, 49, 50

Counterclockwise rotation of 160° about point P

After this lesson...

- I can identify dilations.
- I can dilate figures.
- I can solve real-life problems involving scale factors and dilations.

4.5 DILATIONS

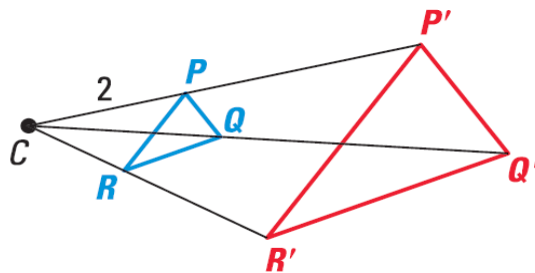


4.5 DILATIONS

- Dilation
 - Enlarge or reduce
 - Image is similar to preimage
 - Scale factor is k
 - If $0 < k < 1$, then reduction
 - If $k > 1$, then enlargement

4.5 DILATIONS

- The image point P' lies on $\overrightarrow{CP}$. The scale factor k is a positive number such that $k = \frac{CP'}{CP}$ and $k \neq 1$



- Scale factor is $\frac{\text{image}}{\text{preimage}}$

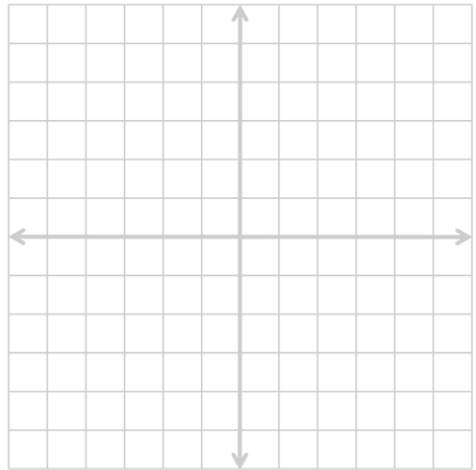
Scale

4.5 DILATIONS

- Coordinate Rule for Dilations
 - $(x, y) \rightarrow (kx, ky)$
 - Where k is the scale factor

4.5 DILATIONS

- Graph $\triangle PQR$ with vertices $P(4, 6)$, $Q(-4, 2)$, and $R(2, -6)$ and its image after a dilation with a scale factor of 0.5.



$$(x, y) \rightarrow (kx, ky)$$

$$P(4, 6) \rightarrow P'(0.5 \cdot 4, 0.5 \cdot 6) = P'(2, 3)$$

$$Q(-4, 2) \rightarrow Q'(0.5 \cdot -4, 0.5 \cdot 2) = Q'(-2, 1)$$

$$R(2, -6) \rightarrow R'(0.5 \cdot 2, 0.5 \cdot -6) = R'(1, -3)$$

4.5 DILATIONS

- Draw and label $\triangle RST$, then construct a dilation of $\triangle RST$ with R as the center of dilation and a scale factor of 3.
1. Draw $\triangle RST$, then draw rays $\overrightarrow{RS}$ and $\overrightarrow{RT}$
 2. Using a ruler, measure RS . Multiply by the scale factor. Using the ruler mark this length RS' on $\overrightarrow{RS}$. Repeat for the other rays.
 3. Draw $\triangle R'S'T'$

4.5 DILATIONS

- You are using a magnifying glass that shows the image of an object as three times the object's actual size. Determine the actual length of a spider when the image of the spider seen through the magnifying glass is 6.75 centimeters long.

- 204 #2, 4, 6, 8, 10, 14, 16, 18, 20, 22, 24, 26, 28, 34, 38, 50, 52, 55, 56, 59



$$\begin{aligned}k &= 3 = \frac{\text{image}}{\text{preimage}} \\3 &= \frac{6.75}{\text{preimage}} \\3 \cdot \text{preimage} &= 6.75 \\ \text{preimage} &= 2.25 \text{ cm}\end{aligned}$$

After this lesson...

- I can perform similarity transformations.
- I can describe similarity transformations.
- I can prove that figures are similar.

4.6 SIMILARITY AND TRANSFORMATIONS

4.6 SIMILARITY AND TRANSFORMATIONS

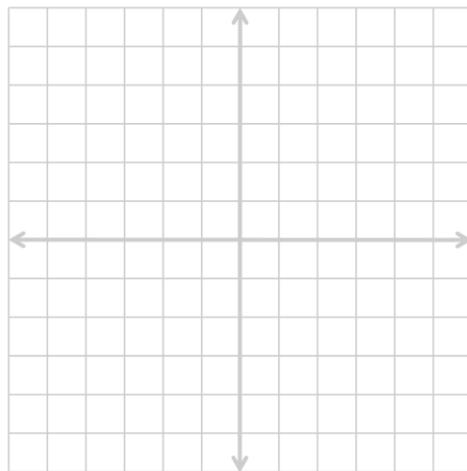
- Similar figures
 - Same shape; different sizes
- Similarity Transformation
 - Dilation or
 - Composition of dilation and another transformation

4.6 SIMILARITY AND TRANSFORMATIONS

- Graph $\triangle ABC$ with vertices $A(12, -6)$, $B(0, -3)$, and $C(3, -9)$ and its image after the similarity transformation.

Reflection: in the y -axis

Dilation: $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$



Reflection: $(x, y) \rightarrow (-x, y)$

$A(12, -6) \rightarrow A'(-12, -6)$

$B(0, -3) \rightarrow B'(0, -3)$

$C(3, -9) \rightarrow C'(-3, -9)$

Dilation: $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$

$$A'(-12, -6) \rightarrow A''\left(\frac{1}{3}(-12), \frac{1}{3}(-6)\right) = A''(-4, -2)$$

$$B'(0, -3) \rightarrow B''\left(\frac{1}{3}(0), \frac{1}{3}(-3)\right) = B''(0, -1)$$

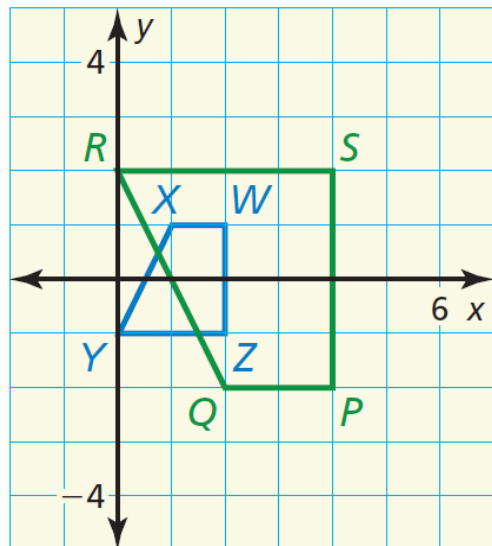
$$C'(-3, -9) \rightarrow C''\left(\frac{1}{3}(-3), \frac{1}{3}(-9)\right) = C''(-1, -3)$$

4.6 SIMILARITY AND TRANSFORMATIONS

- Describe a similarity transformation that maps trapezoid $WXYZ$ to trapezoid $PQRS$.

Skip 12, do 13
instead

- 211 #2, 4, 6, 8, 10, ~~12~~, **13**, 14, 16, 17, 19, 21, 22, 23, 24, 28



Sample answer: reflection in the x -axis, followed by a dilation with a scale factor of 2