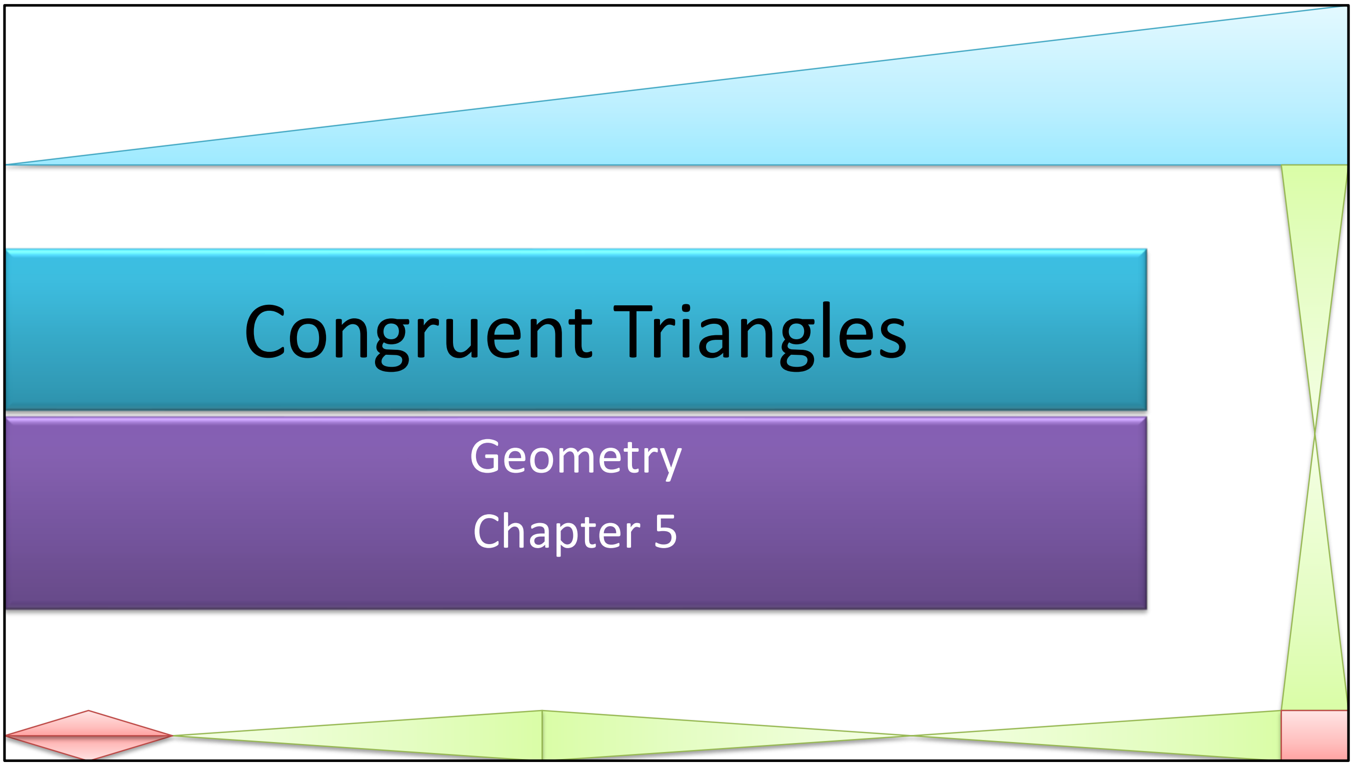

Geometry 5

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Geometry*
  - *By Larson and Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

Slides created by
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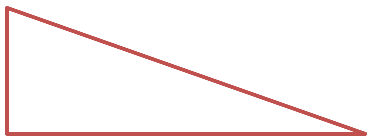
After this lesson…

- I can classify triangles by sides and by angles.
- I can prove theorems about angles of triangles.
- I can find interior and exterior angle measures of triangles.

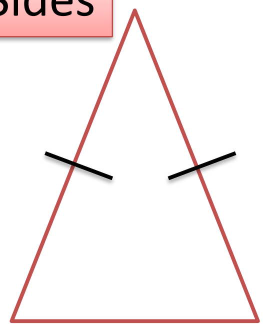
5.1 ANGLES OF TRIANGLES

5.1 Angles of Triangles

Classify Triangles by Sides

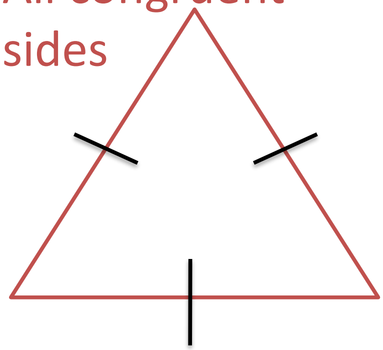


Scalene Triangle
No congruent sides



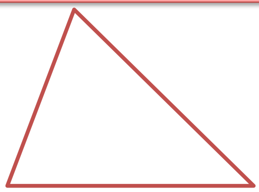
Isosceles Triangle
Two congruent sides

Equilateral Triangle
All congruent sides



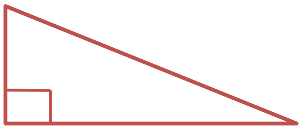
5.1 Angles of Triangles

Classify Triangles by Angles



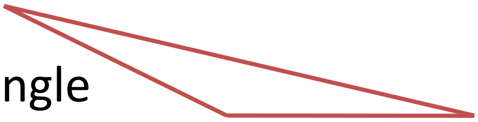
Acute Triangle

3 acute angles



Right Triangle

1 right angle

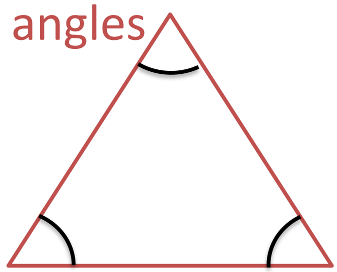


Obtuse Triangle

1 obtuse angle

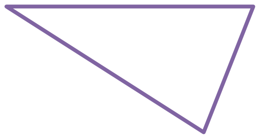
Equiangular Triangle

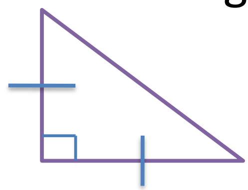
All congruent angles



5.1 Angles of Triangles

- Classify the following triangle by sides and angles





Scalene, Acute
Isosceles, Right

5.1 Angles of Triangles

- ΔABC has vertices A(0, 0), B(3, 3), and C(-3, 3). Classify it by is sides. Then determine if it is a right triangle.

Find length of sides using distance formula

$AB = \sqrt{(3 - 0)^2 + (3 - 0)^2} = \sqrt{9 + 9} = \sqrt{18} \approx 4.24$

$BC = \sqrt{(-3 - 3)^2 + (3 - 3)^2} = \sqrt{(-6)^2 + 0} = \sqrt{36} = 6$

$AC = \sqrt{(-3 - 0)^2 + (3 - 0)^2} = \sqrt{9 + 9} = \sqrt{18} \approx 4.24$

Isosceles

Check slopes to find right angles (perpendicular)

$m_{AB} = (3 - 0)/(3 - 0) = 1$

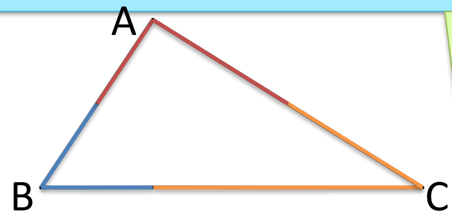
$m_{BC} = (3 - 3)/(-3 - 3) = 0$

$m_{AC} = (3 - 0)/(-3 - 0) = -1$

$AB \perp AC$ so it is a right triangle

5.1 Angles of Triangles

- Take a triangle and tear off two of the angles.
- Move the angles to the 3rd angle.
- What shape do all three angles form?



Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

$m\angle A + m\angle B + m\angle C = 180°$

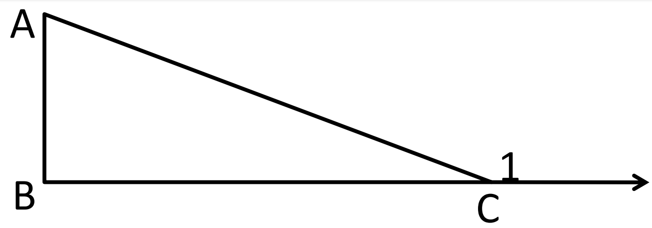
Straight line

5.1 Angles of Triangles

Exterior Angle Theorem

The measure of an exterior angle of a triangle = the sum of the 2 nonadjacent interior angles.

$m\angle 1 = m\angle A + m\angle B$



Proof:

$m\angle A + m\angle B + m\angle ACB = 180°$ (triangle sum theorem)

$m\angle 1 + m\angle ACB = 180°$ (linear pair theorem)

$m\angle 1 + m\angle ACB = m\angle A + m\angle B + m\angle ACB$ (substitution)

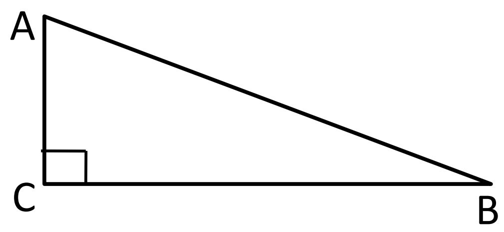
$m\angle 1 = m\angle A + m\angle B$ (subtraction)

5.1 Angles of Triangles

Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

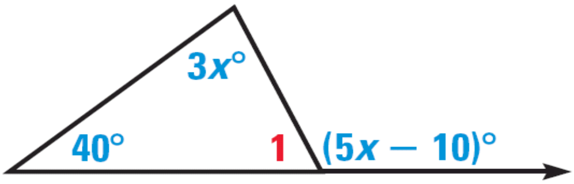
$m\angle A + m\angle B = 90°$



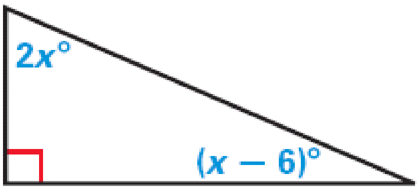
The proof involves saying that all three angles = 180. Since $m\angle C$ is 90, $m\angle A + m\angle B = 90$.

5.1 Angles of Triangles

- Find the measure of ∠1 in the diagram.



- Find the measures of the acute angles in the diagram.



40 + 3x = 5x – 10 → 50 = 2x → x = 25
m∠1 + 40 + 3x = 180 → m∠1 + 40 + 3(25) = 180 → m∠1 + 40 + 75 = 180 → m∠1 = 65

2x + x – 6 = 90 → 3x = 96 → x = 32
Top angle: 2x → 2(32) = 64
Angle at right: x - 6 → 32 - 6 = 26

5.1 Angles of Triangles

- 228 #2, 4, 6, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 32, 42, 44, 48, 55, 58, 59 = 20 total

After this lesson…

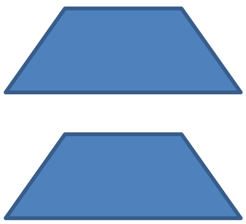
- I can use rigid motions to show that two triangles are congruent.
- I can identify corresponding parts of congruent polygons.
- I can use congruent polygons to solve problems.

5.2 CONGRUENT POLYGONS

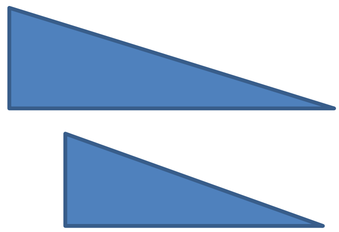
5.2 Congruent Polygons

Congruent $\cong$

Exactly the same shape and size.

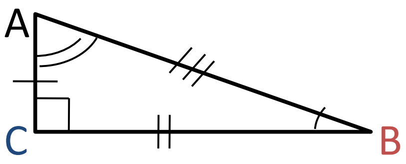


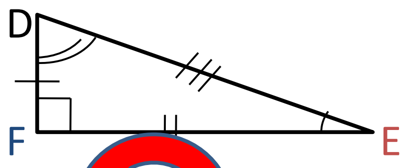
Congruent



Not Congruent

5.2 Congruent Polygons





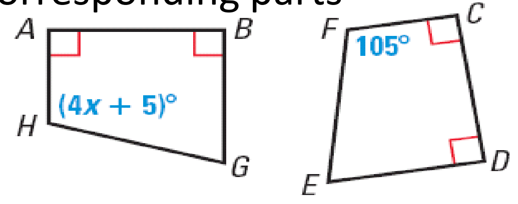
- ΔABC ≅ ΔDEF

ΔABC ≅ ΔEDF

- ∠A ≅ ∠D　　∠B ≅ ∠E　　∠C ≅ ∠F
- $\overline{AB} \cong \overline{DE}$　　$\overline{BC} \cong \overline{EF}$　　$\overline{AC} \cong \overline{DF}$

5.2 Congruent Polygons

- In the diagram, ABGH ≅ CDEF
  - Identify all the pairs of congruent corresponding parts
  - Find the value of x and find m∠H.

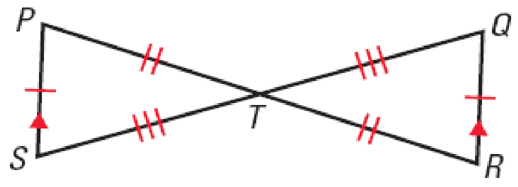


AB ≅ CD, BG ≅ DE, GH ≅ EF, AH ≅ CF
∠A ≅ ∠C, ∠B ≅ ∠D, ∠G ≅ ∠E, ∠H ≅ ∠F

4x + 5 = 105
4x = 100
x = 25
m∠H = 105°

5.2 Congruent Polygons

- Show that ΔPTS ≅ ΔRTQ



All of the corresponding parts of ΔPTS are congruent to those of ΔRTQ by the indicated markings, the Vertical Angle Theorem and the Alternate Interior Angle theorem.

5.2 Congruent Polygons

Third Angle Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

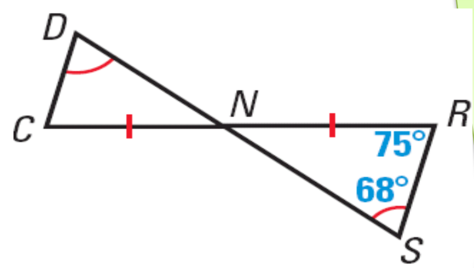
Properties of Congruence of Triangles

Congruence of triangles is Reflexive, Symmetric, and Transitive

75 + 20 + ? = 180
95 + ? = 180
? = 85

5.2 Congruent Polygons

- In the diagram, what is m∠DCN?



- By the definition of congruence, what additional information is needed to know that ΔNDC ≅ ΔNSR?

m∠DCN = 75°; alt int angle theorem (or 3rd angle theorem)

DN ≅ SN, DC ≅ SR

5.2 Congruent Polygons

- 235 #2, 3, 4, 6, 8, 10, 12, 13, 14, 15, 17, 18, 20, 21, 24, 26, 28, 30, 31, 32 = 20 total

After this lesson…

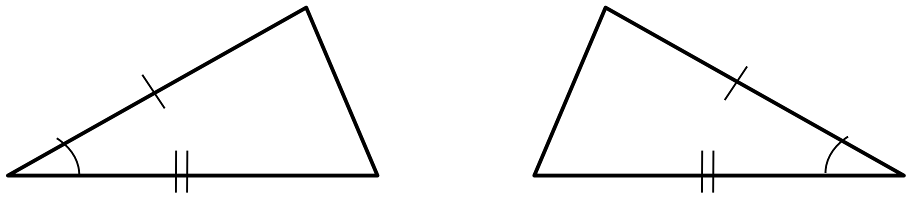
- I can use the SAS Congruence Theorem.

5.3 PROVING TRIANGLE CONGRUENCE BY SAS

5.3 Proving Triangle Congruence by SAS

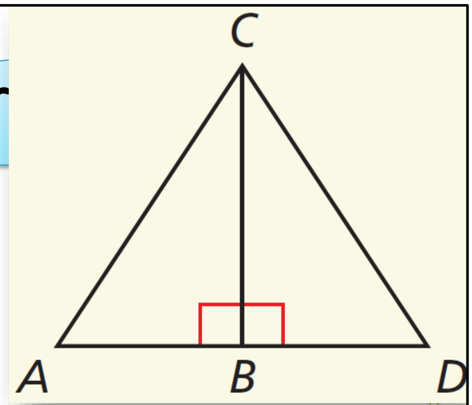
SAS (Side-Angle-Side Congruence Postulate)

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent



The angle must be between the sides!!!

5.3 Proving Triangle Congr

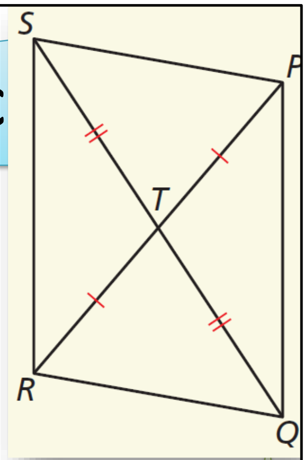


- **Given** B is the midpoint of $\overline{AD}$.
  $\angle ABC$ and $\angle DBC$ are right angles.
- **Prove** $\triangle ABC \cong \triangle DBC$

Statements	Reasons

B is the midpoint of $\overline{AD}$. $\angle ABC$ and $\angle DBC$ are right angles (given)
$\overline{AB} \cong \overline{BD}$ (definition of midpoint)
$\angle ABC \cong \angle DBC$ (rt. Angles are $\cong$)
$\overline{BC} \cong \overline{BC}$ (reflexive)
$\triangle ABC \cong \triangle DBC$ (SAS)

5.3 Proving Triangle Congruenc



- What can you conclude about $\triangle PTS$ and $\triangle RTQ$? Explain.

- 241 #2, 4, 6, 7, 8, 10, 12, 17, 18, 19, 23, 24, 27, 29, 31 = 15 total

PT≅TR and ST≅TQ (given)
$\angle PTS \cong \angle RTQ$ (vertical angles are ≅)
$\triangle PTS \cong \triangle RTQ$ (SAS)

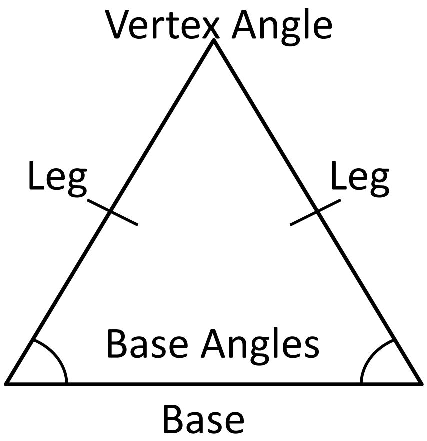
After this lesson…

- I can prove and use theorems about isosceles triangles.
- I can prove and use theorems about equilateral triangles.

5.4 EQUILATERAL AND ISOSCELES TRIANGLES

5.4 Equilateral and Isosceles Triangles

- Parts of an Isosceles Triangle



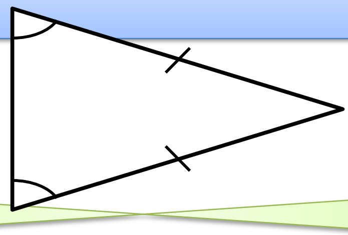
5.4 Equilateral and Isosceles Triangles

Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

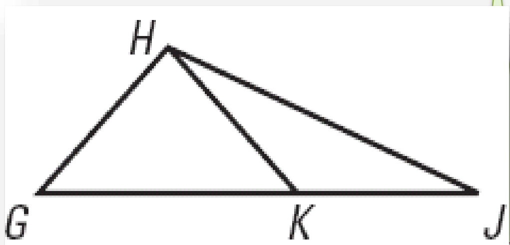
Converse of Base Angles Theorem

If two angles of a triangle are congruent, then the two sides opposite them are congruent.



5.4 Equilateral and Isosceles Triangles

- Complete the statement
  - If $\overline{HG} \cong \overline{HK}$, then ∠__?__ ≅ ∠__?__.
  - If ∠KHJ ≅ ∠KJH, then __?__ ≅ __?__.



∠HKG ≅ ∠HGK

KJ ≅ KH

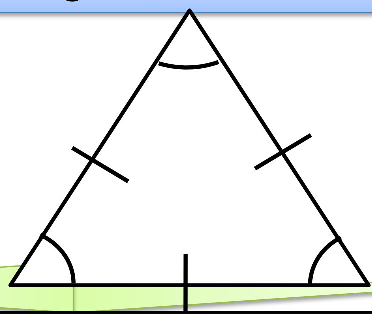
5.4 Equilateral and Isosceles Triangles

Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.

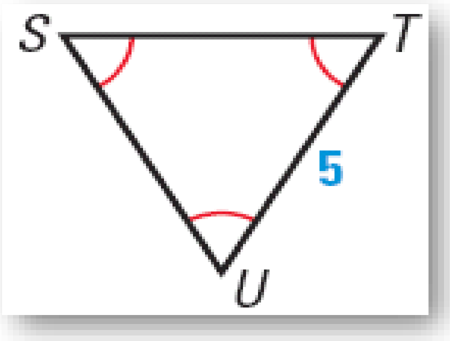
Corollary to the Converse of Base Angles Theorem

If a triangle is equiangular, then it is equilateral.



5.4 Equilateral and Isosceles Triangles

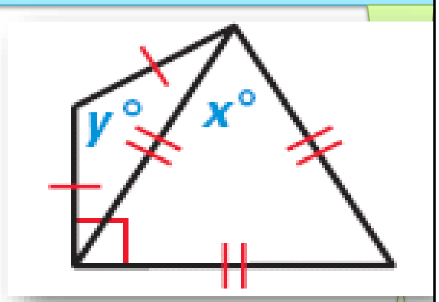
- Find ST
- Find m∠T



ST = 5
m∠T = 60° (all angles in equilateral/equiangular triangles are 60°)

5.4 Equilateral and Isosceles Triangles

- Find the values of x and y



- 248 #2, 4, 6, 8, 12, 14, 16, 18, 20, 21, 22, 24, 27, 28, 30, 36, 38, 39, 40, 43 = 20 total

x = 60; equilateral triangle
Each base angle by y; 60 + ? = 90 → ? = 30
Angle sum theorem: 30 + 30 + y = 180 → y = 120

After this lesson…

- I can use the SSS Congruence Theorem.
- I can use the Hypotenuse-Leg Congruence Theorem.

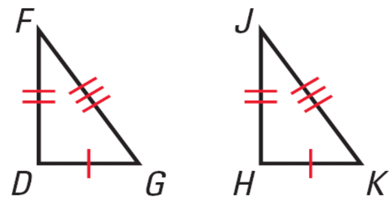
5.5 PROVING TRIANGLE CONGRUENCE BY SSS

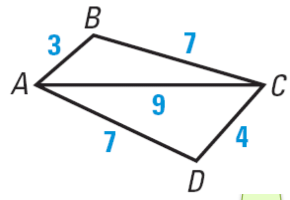
5.5 Proving Triangle Congruence by SSS

SSS (Side-Side-Side Congruence Postulate)

If three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent

- True or False
  - ΔDFG ≅ ΔHJK





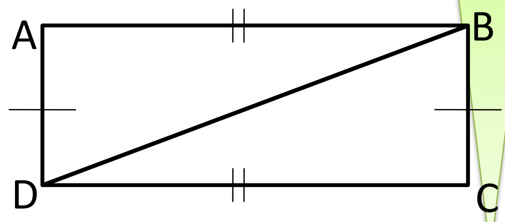
  - ΔACB ≅ ΔCAD

True

False

5.5 Proving Triangle Congruence by SSS

- Given: $\overline{AB} \cong \overline{DC}; \overline{AD} \cong \overline{BC}$
- Prove: ΔABD ≅ ΔCDB

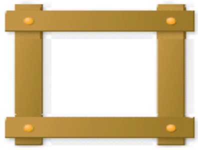


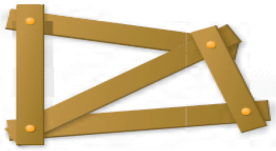
Statements	Reasons

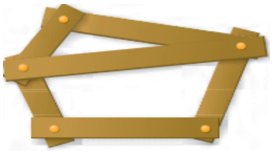
AB ≅ DC; AD ≅ CB (given)
BD ≅ BD (reflexive)
ΔABD ≅ ΔCDB (SSS)

5.5 Proving Triangle Congruence by SSS

- Stable structures are made out of triangles
- Determine whether the figure is stable.







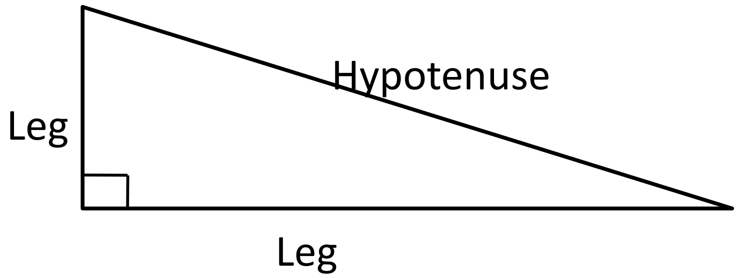
Not stable

Stable since has triangular construction

Not stable, lower section does not have triangular construction

5.5 Proving Triangle Congruence by SSS

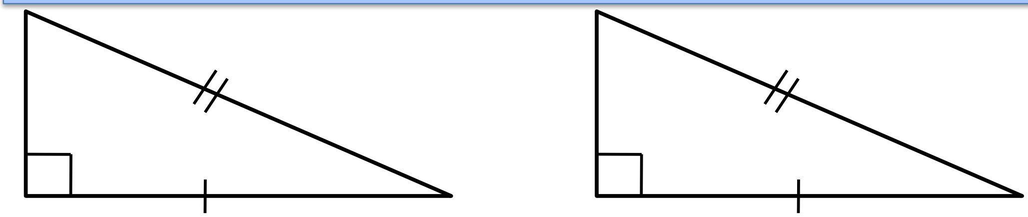
- Right triangles are special
  - If we know two sides are congruent we can use the Pythagorean Theorem (ch 7) to show that the third sides are congruent



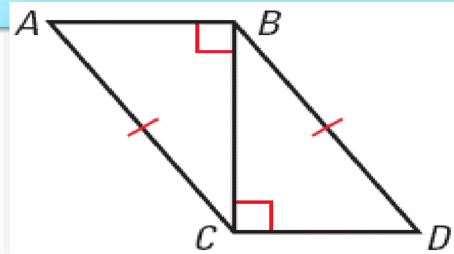
5.5 Proving Triangle Congruence by SSS

HL (Hypotenuse-Leg Congruence Theorem)

If the hypotenuse and a leg of a **right** triangle are congruent to the hypotenuse and a leg of another **right** triangle, then the two triangles are congruent



5.5 Proving Triangle Congruence by SSS



- Given: ∠ABC and ∠BCD are rt ∠s; $\overline{AC} \cong \overline{BD}$
- Prove: ΔACB ≅ ΔDBC

Statements	Reasons

∠ABC and ∠BCD are rt ∠s; AC ≅ BD (given)
ΔACB and ΔDBC are rt Δ (def rt Δ)
BC ≅ CB (reflexive)
ΔACB ≅ ΔDBC (HL)

5.5 Proving Triangle Congruence by SSS

- 256 #1, 2, 3, 4, 6, 7, 8, 10, 12, 14, 18, 20, 22, 26, 28, 31, 32, 34, 35, 36 = 20 total

After this lesson…

- I can prove the AAS Congruence Theorem.
- I can use the ASA and AAS Congruence Theorems.

5.6 PROVING TRIANGLE CONGRUENCE BY ASA AND AAS

5.6 Proving Triangle Congruence by ASA and AAS

- Use a ruler to draw a line of 5 cm.
- On one end of the line use a protractor to draw a 30° angle.
- On the other end of the line draw a 60° angle.
- Extend the other sides of the angles until they meet.
- Compare your triangle to your neighbor’s.
- This illustrates ASA.

Everyone’s triangle should be congruent

5.6 Proving Triangle Congruence by ASA and AAS

ASA (Angle-Side-Angle Congruence Postulate)

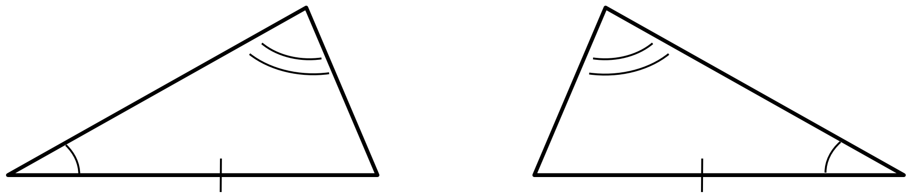
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent

The side must be between the angles!

5.6 Proving Triangle Congruence by ASA and AAS

AAS (Angle-Angle-Side Congruence Theorem)

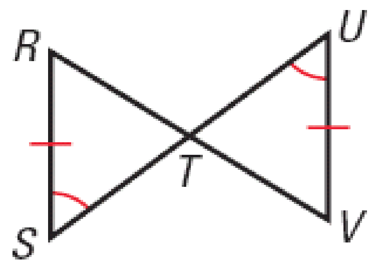
If two angles and a non-included side of one triangle are congruent to two angles and a non-included side of another triangle, then the two triangles are congruent



The side is NOT between the angles!

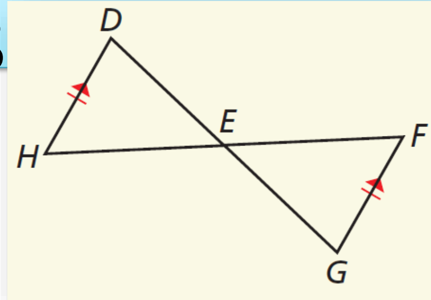
5.6 Proving Triangle Congruence by ASA and AAS

- In the diagram, what postulate or theorem can you use to prove that ΔRST ≅ ΔVUT?



∠RTS ≅ ∠UTV by Vert. Angles are Congruent
ΔRST ≅ ΔVUT by AAS

5.6 Proving Triangle Congruence by ASA and AAS

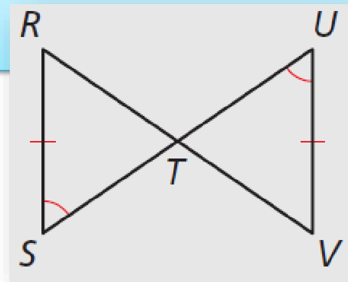


- **Given:** $\overline{DH} \parallel \overline{GF}$, $\overline{DH} \cong \overline{GF}$
- **Prove:** $\triangle DEH \cong \triangle GEF$

Statements	Reasons

$\overline{DH} \parallel \overline{GF}$, $\overline{DH} \cong \overline{GF}$	(given)
$\angle D \cong \angle G$, $\angle H \cong \angle F$	(alt int angles thrm)
$\Delta DEH \cong \Delta GEF$	(ASA)

5.6 Proving Triangle Congruence by ASA and AAS



- **Given:** $\overline{RS} \cong \overline{VU}, \angle S \cong \angle U$
- **Prove:** $\triangle RST \cong \triangle VUT$

Statements	Reasons

$\overline{RS} \cong \overline{VU}, \angle S \cong \angle U$ (given)
$\angle RTS \cong \angle VTU$ (vert angles ≅)
$\Delta RST \cong \Delta VUT$ (AAS)

5.6 Proving Triangle Congruence by ASA and AAS

- 264 #2, 4, 6, 8, 12, 14, 16, 22, 24, 28, 35, 38, 39, 40, 41 = 15 total

After this lesson…

- I can use congruent triangles to prove statements.
- I can use congruent triangles to solve real-life problems.
- I can use congruent triangles to prove constructions.

5.7 USING CONGRUENT TRIANGLES

5.7 Using Congruent Triangles

- By the definition of congruent triangles, we know that the corresponding parts have to be congruent

CPCTC

Corresponding Parts of Congruent Triangles are Congruent

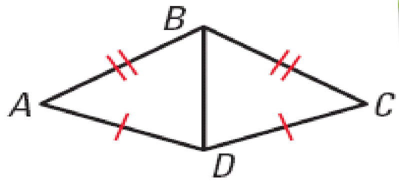
Your book just calls this "definition of congruent triangles"

5.7 Using Congruent Triangles

- To show that parts of triangles are congruent
  - First show that the triangles are congruent using
    - SSS, SAS, ASA, AAS, HL
  - Second say that the corresponding parts are congruent using
    - CPCTC or “def ≅ Δ”

5.7 Using Congruent Triangles

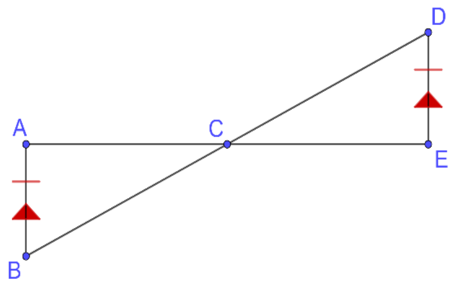
- Write a plan for a proof to show that ∠A ≅ ∠C



- Show that $\overline{BD} \cong \overline{BD}$ by reflexive
- Show that triangles are ≅ by SSS
- Say ∠A ≅ ∠C by def ≅ Δ or CPCTC

5.7 Using Congruent Trian

- Given: $\overline{AB} \cong \overline{DE}, \overline{AB} \parallel \overline{DE}$
- Prove: C is the midpoint of $\overline{AB}$



$\overline{AB} \cong \overline{DE}, \overline{AB} \parallel \overline{DE}$		*(given)*
∠B ≅ ∠D, ∠A ≅ ∠E		*(Alt. Int. ∠ Thrm)*
ΔABC ≅ ΔEDC	*(ASA)*	
$\overline{AC} \cong \overline{CE}$		*(CPCTC)*
C is midpoint of $\overline{AE}$		*(Def midpoint)*

5.7 Using Congruent Triangles

- 271 #2, 3, 4, 6, 8, 10, 13, 17, 19, 20, 23, 25, 26, 27, 28 = 15 total

After this lesson…

- I can place figures in a coordinate plane.
- I can write plans for coordinate proofs.
- I can write coordinate proofs.

5.8 COORDINATE PROOFS

5.8 Coordinate Proofs

Coordinate Proof

- Place geometric figures in a coordinate plane (graph)
- When variables are used for the coordinates, the result is true for all figures of that type
- Use formulas to prove things
  - Midpoint formula
  - Distance formula
  - Slope formula

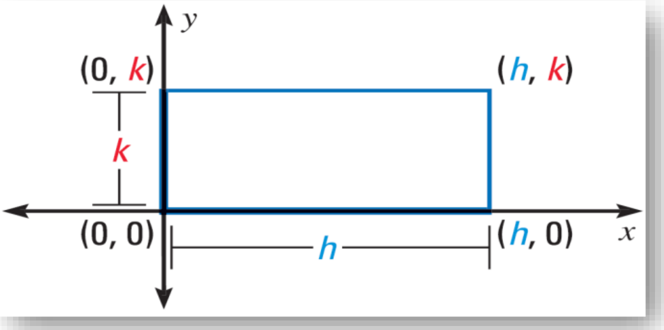
5.8 Coordinate Proofs

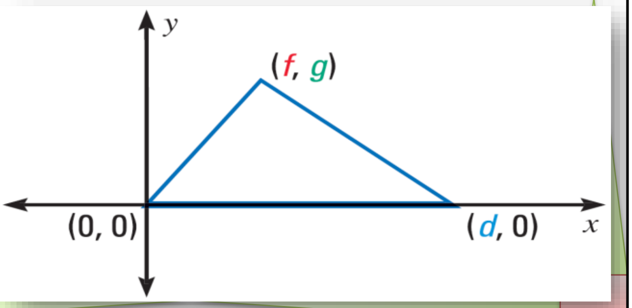
- Place figures for Coordinate Proof

1. Use the origin as a vertex or center.
2. Place at least one side of the polygon on an axis.
3. Usually keep the figure within the first quadrant.
4. Use coordinates that make computations as simple as possible.

- You will prove things by calculating things like slope, distance, and midpoints

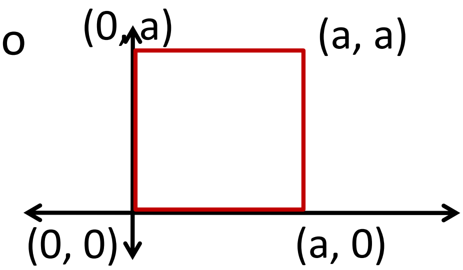
5.8 Coordinate Proofs



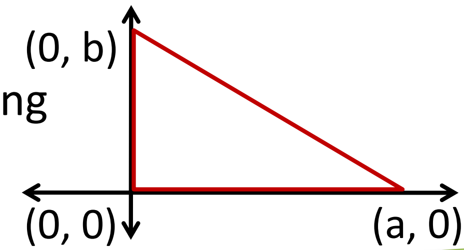


5.8 Coordinate Proofs

- Place a **square** in a coordinate plane so that it is convenient for finding side lengths. Assign coordinates.

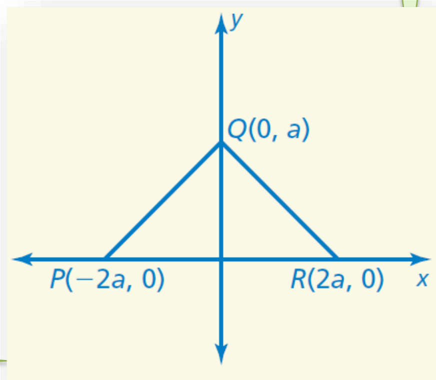


- Place a **right triangle** in a coordinate plane so that it is convenient for finding side lengths. Assign coordinates.



5.8 Coordinate Proofs

- Place an isosceles triangle in a coordinate plane with vertices *P*(−2*a*, 0), *Q*(0, *a*), and *R*(2*a*, 0). Then find the side lengths and the coordinates of the midpoint of each side.



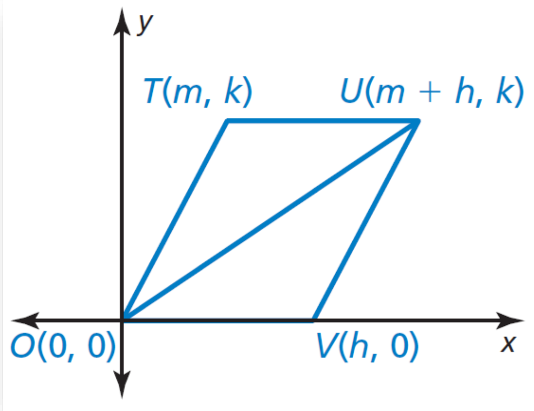
$$PQ=\sqrt{\left(0-(-2a)\right)^2+(a-0)^2}=\sqrt{4a^2+a^2}=\sqrt{5a^2}=a\sqrt{5};M_{\overline{PQ}}\left(\frac{-2a+0}{2},\frac{0+a}{2}\right)$$
$$=\left(-a,\frac{a}{2}\right)$$

$$QR=\sqrt{(0-2a)^2+(a-0)^2}=\sqrt{4a^2+a^2}=\sqrt{5a^2}=a\sqrt{5};M_{\overline{QR}}\left(\frac{2a+0}{2},\frac{0+a}{2}\right)$$
$$=\left(a,\frac{a}{2}\right)$$

$$PR=\sqrt{\left(2a-(-2a)\right)^2+(0-0)^2}=\sqrt{16a^2}=4a;M_{\overline{PR}}\left(\frac{2a+(-2a)}{2},\frac{0+0}{2}\right)=(0,0)$$

5.8 Coordinate Proofs

- **Given** Coordinates of vertices of quadrilateral *OTUV*
- **Prove** $\angle TOU \cong \angle VUO$



- 277 #2, 4, 6, 8, 11, 12, 15, 16, 22, 23, 25, 26, 29, 32, 33 = 15 total

The slope of $\overline{TO}$ is $\frac{k-0}{m-0} = \frac{k}{m}$.
The slope of $\overline{UV}$ is $\frac{k-0}{(m+h)-h} = \frac{k}{m}$.
Because they have the same slope, $\overline{TO} \parallel \overline{UV}$.
By the Alternate Interior Angles Theorem, $\angle TOU \cong \angle VUO$.