Quadrilaterals and Other Polygons

Geometry Chapter 7

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Geometry*
  - *By Larson and Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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7.1 Angles of Polygons

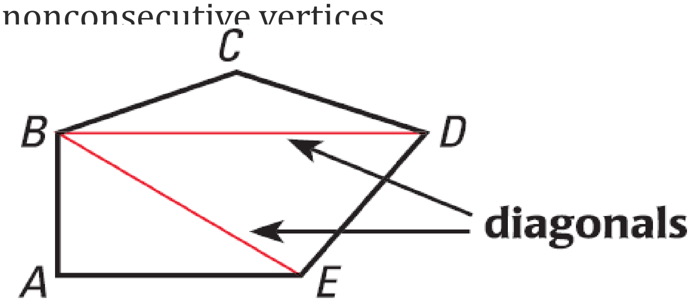
After this lesson…

- I can find the sum of the interior angle measures of a polygon.
- I can find interior angle measures of polygons.
- I can find exterior angle measures of polygons.

7.1 Angles of Polygons

- Polygon
  - Closed figure made of straight segments

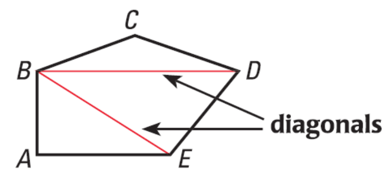
- Diagonal
  - Segment that joins nonconsecutive vertices



Notice that the pentagon is made into 3 triangles.

7.1 Angles of Polygons

- All polygons can be separated into triangles
- The sum of the angles of a triangle is 180°
- For the pentagon, multiply that by 3



Polygon Interior Angles Theorem

Sum of the measures of the interior angles of a n-gon is $(n-2)180°$

$$S = (n-2) \cdot 180°$$

Sum of the measures of the interior angles of a quadrilateral is 360°

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7.1 Angles of Polygons

- The coin is a regular 11-gon. Find the sum of the measures of the interior angles.

- The sum of the measures of the interior angles of a convex polygon is 1440°. Classify the polygon by the number of sides.

- Try #4, 6

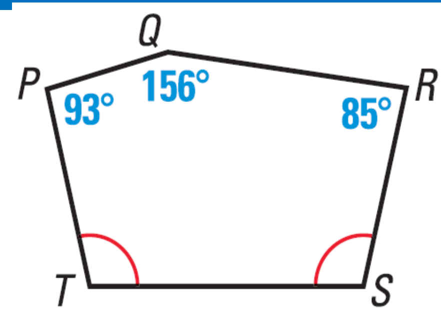
6

$S = (n-2)180°$
$S = (11-2)180° = 1620°$

$1440° = (n-2)180°$
$8 = n-2$
$n = 10$

7.1 Angles of Polygons

- Find $m\angle T$



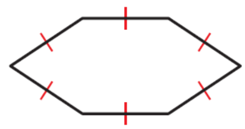
- Try #10

7

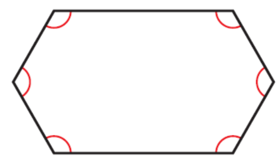
$93° + 156° + 85° + x + x = 540°$
$334 + 2x = 540$
$2x = 206$
$x = 103$

7.1 Angles of Polygons

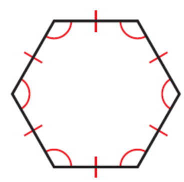
- Equilateral Polygon
  - All sides congruent



- Equiangular Polygon
  - All angles congruent



- Regular Polygon
  - All sides and angles congruent

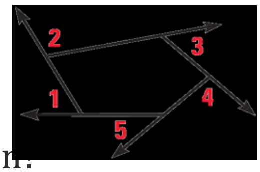


7.1 Angles of Polygons

Polygon Exterior Angles Theorem

Sum of the measures of the exterior angles of a convex polygon 360°

- What is the measure of an exterior angle of a regular pentagon?



- What is the measure of an interior angle of a regular pentagon?

- Try #34

$$\frac{360}{5} = 72°$$

$$180 = x + 72$$
$$x = 108$$

Or

$$S = (5 - 2) \cdot 180$$
$$S = 540$$
$$int\ angle = \frac{540}{5} = 108$$

7.2 Properties of Parallelograms

After this lesson…

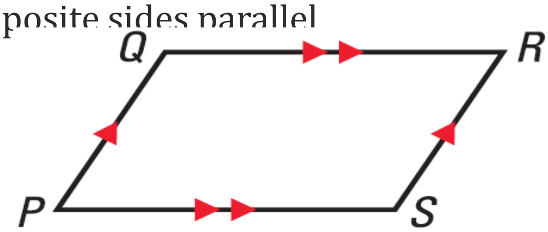
- I can prove properties of parallelograms.
- I can use properties of parallelograms.
- I can solve problems involving parallelograms in the coordinate plane.

7.2 Properties of Parallelograms

- On scrap paper draw two sets of parallel lines that intersect each other.
- Measure opposite sides. How are opposite sides related?
- Measure opposite angles. How are opposite angles related?

7.2 Properties of Parallelograms

- Definition of parallelogram
  - Quadrilateral with opposite sides parallel



Opposite sides of parallelogram are congruent

Opposite angles of a parallelogram are congruent

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Theorems were demonstrated in the focus

7.2 Properties of Parallelograms

Consecutive angles in a parallelogram are supplementary

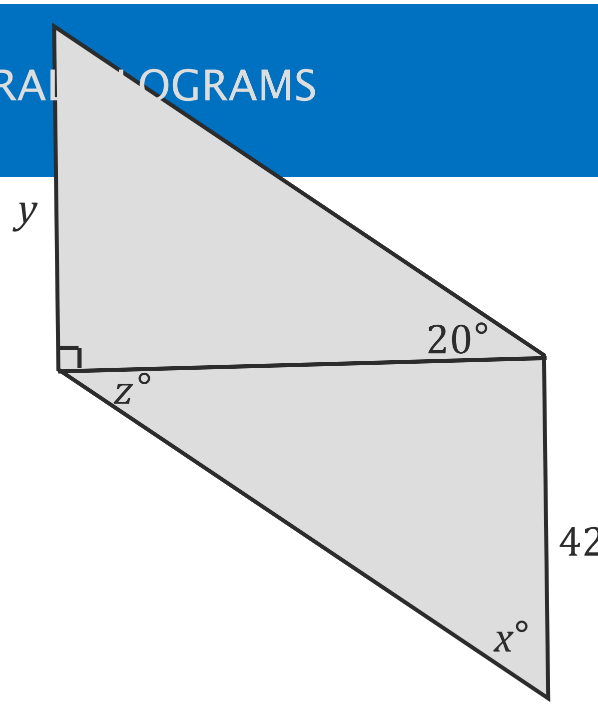
- Remember from parallel lines (chapter 3) that consecutive interior angles are supplementary

Diagonals of a parallelogram bisect each other

- Draw diagonals on your parallelogram
- Measure each part of the diagonals to see if they bisect each other.

7.2 Properties of Paral[l]ograms

- Find x, y, and z if the figure is a parallelogram.



- Try #2

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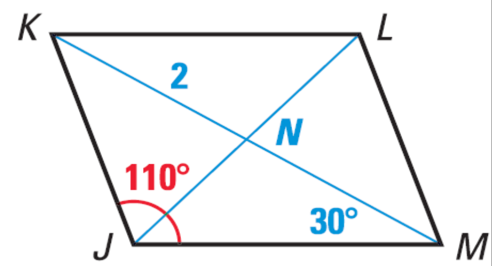
x = 70 Opposite angles of ▱ ≅
y = 42 Opposite sides of ▱ ≅
z = 20 Alternate interior angles thrm

7.2 Properties of Parallelograms

- Find NM
- Find $m\angle JML$
- Find $m\angle KML$
- Try #12



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MN = NK = 2

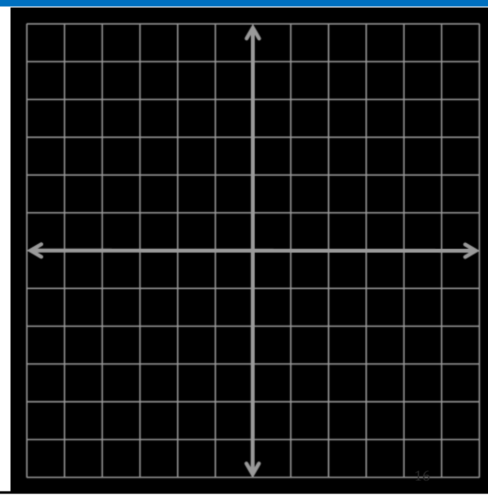
$m\angle JML + 110° = 180° \rightarrow m\angle JML = 70°$

$30° + m\angle KML = 70° \rightarrow m\angle KML = 40°$

7.2 Properties of Parallelograms

- Three vertices of $\square DEFG$ are $D(-1, 4)$, $E(2, 3)$, and $F(4, -2)$. Find the coordinates of vertex G.

- Try #26



Graph points. Use rise and run of DE starting at F to find G. Use rise and run of EF to start at D to verify G.
(1, −1)

7.3 Proving That a Quadrilateral Is a Parallelogram

After this lesson…

- I can identify features of a parallelogram.
- I can prove that a quadrilateral is a parallelogram.
- I can find missing lengths that make a quadrilateral a parallelogram.
- I can show that a quadrilateral in the coordinate plane is a parallelogram.

7.3 Proving That a Quadrilateral Is a Parallelogram

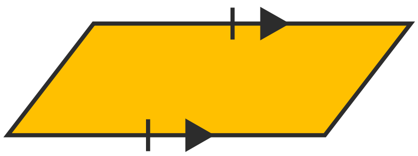
- Review
- What are the properties of parallelograms?
  - Opposite sides parallel
  - Opposite sides are congruent
  - Opposite angles are congruent
  - Diagonals bisect each other

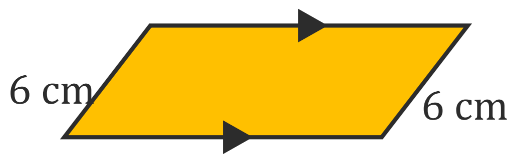
7.3 Proving That a Quadrilateral Is a Parallelogram

- If we can show any of these things in a quadrilateral, then it is a parallelogram.
  - If both pairs of opposite sides of a quad are parallel, then it is a parallelogram (definition of parallelogram)
  - If both pairs of opposite sides of a quad are congruent, then it is a parallelogram.
  - If both pairs of opposite angles of a quad are congruent, then it is a parallelogram.
  - If the diagonals of a quad bisect each other, then it is a parallelogram.
  - If one pair of opposite sides of a quad is both parallel and congruent, then it is a parallelogram.

7.3 Proving That a Quadrilateral Is a Parallelogram

- Is it a parallelogram?





- Try #2

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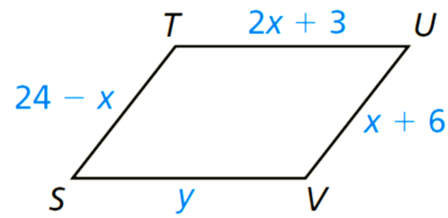
Yes; 1 pair of opposite sides parallel and congruent

No, congruent is not same as parallel

7.3 Proving That a Quadrilateral Is a Parallelogram

- For what values of x and y is quadrilateral $STUV$ a parallelogram?



- Try #8

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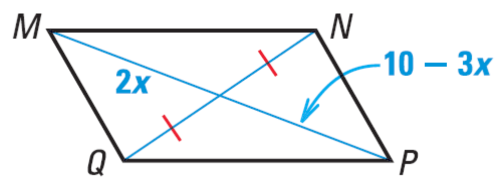
$24 – x = x + 6$
$24 = 2x + 6$
$18 = 2x$
$x = 9$

$y = 2x + 3$
$y = 2(9) + 3$
$y = 21$

7.3 Proving That a Quadrilateral Is a Parallelogram

- Find x so that $MNPQ$ is a parallelogram.



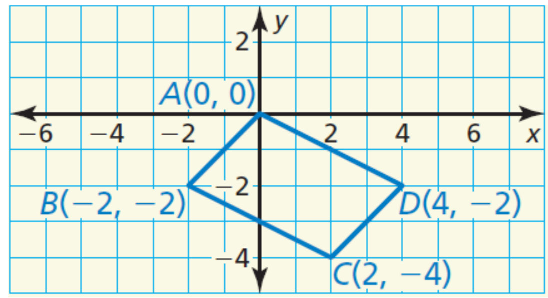
- Try #14

Diagonals bisect each other

$$2x = 10 - 3x$$
$$5x = 10$$
$$x = 2$$

7.3 Proving That a Quadrilateral Is a Parallelogram

- Show that quadrilateral *ABCD* is a parallelogram.



- Try #16

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Show the diagonals have the same midpoint (bisect each other)
Or show the opposite sides have the same slope (parallel)

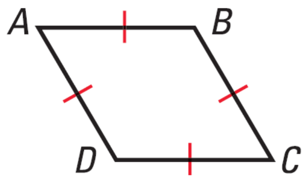
7.4 Properties of Special Parallelograms

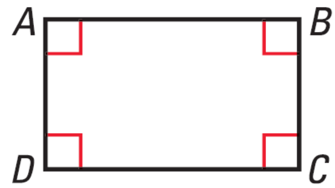
After this lesson…

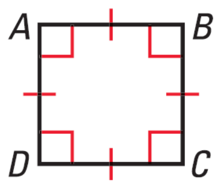
- I can identify special quadrilaterals.
- I can explain how special parallelograms are related.
- I can find missing measures of special parallelograms.
- I can identify special parallelograms in a coordinate plane.

7.4 Properties of Special Parallelograms

- All of these are parallelograms
- Rhombus
  - Four ≅ sides
- Rectangle
  - Four right ∠s
- Square
  - Rhombus and Rectangle
  - Four ≅ sides
  - Four right ∠s







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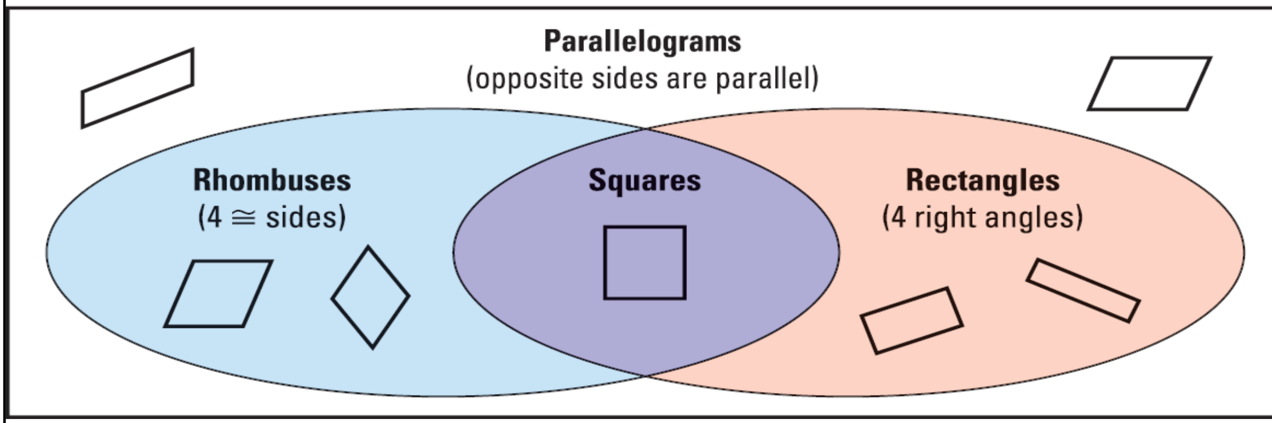
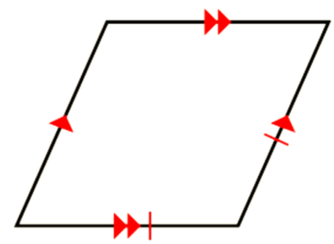
7.4 Properties of Special Parallelograms



7.4 Properties of Special Parallelograms

- For any rectangle $EFGH$, is it *always* or *sometimes* true that $\overline{FG} \cong \overline{GH}$?
- Classify the figure.
- Try #2, 8



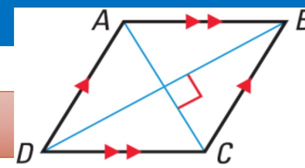
Sometimes, $\overline{FG}$ and $\overline{GH}$ are consecutive sides, not opposite

Rhombus (parallel sides which makes parallelogram; opposite sides are ≅ and adjacent sides are ≅, so all sides are ≅)

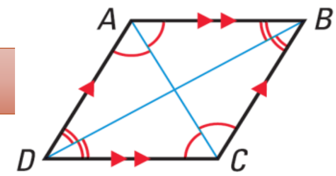
7.4 Properties of Special Parallelograms

- Diagonals

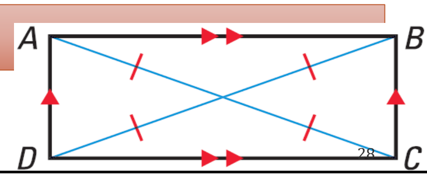
Rhombus: diagonals are perpendicular



Rhombus: diagonals bisect opposite angles



Rectangle: diagonals are congruent



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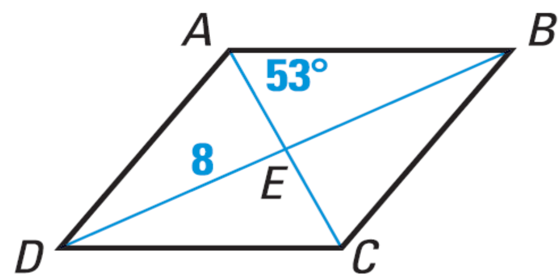
7.4 Properties of Special Parallelograms

- *ABCD* is a rhombus
- Find $m\angle BCE$



- Find $m\angle ABD$
- Find $m\angle AED$
- Try #12

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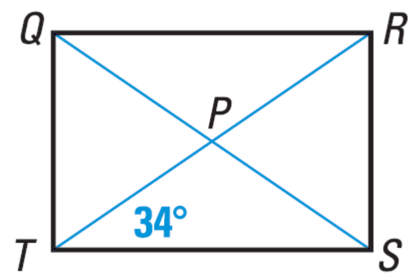
Opposite angles ≅: $m\angle BCE = 53°$

ΔABE is right Δ: $m\angle ABD = 90° - 53° = 37°$

Diagonals are ⊥: $m\angle AED = 90°$

7.4 Properties of Special Parallelograms

- In rectangle $QRST$, $QS = 7x - 15$ and $RT = 2x + 25$. Find the lengths of the diagonals of $QRST$.



- Try #24

30

Diagonals of Rectangle are ≅:

$$7x - 15 = 2x + 25$$
$$5x - 15 = 25$$
$$5x = 40$$
$$x = 8$$
$$QS = RT = 7(8) - 15 = 41$$

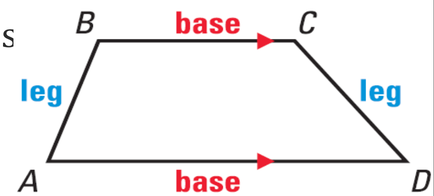
7.5 Properties of Trapezoids and Kites

After this lesson…

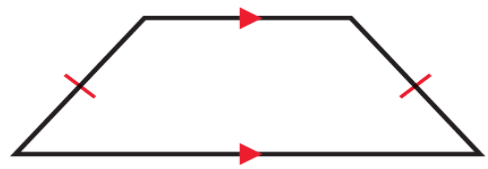
- I can identify trapezoids and kites.
- I can use properties of trapezoids and kites to solve problems.
- I can find the length of the midsegment of a trapezoid.
- I can explain the hierarchy of quadrilaterals.

7.5 Properties of Trapezoids and Kites

- Trapezoid
  - Quadrilateral with exactly one pair of parallel sides

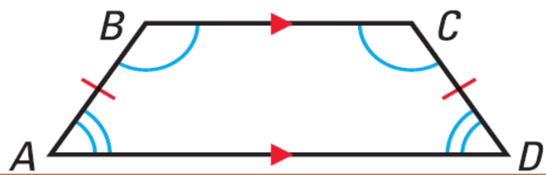


- If the legs are ≅, then the trap is isosceles



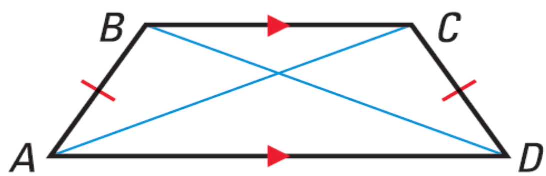
7.5 Properties of Trapezoids and Kites

If isosceles trapezoid, then each pair of base angles is ≅.



If isosceles trapezoid, then diagonals are ≅.

- The converses are also true



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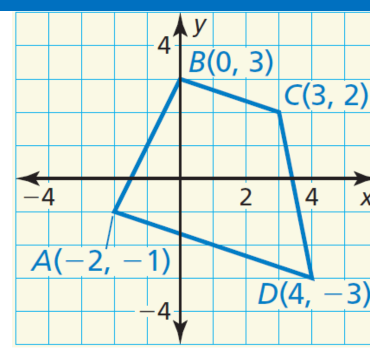
7.5 Properties of Trapezoids and Kites

- Show that *ABCD* is a trapezoid. Then decide whether it is isosceles.



- Try #2

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Slopes: $m_{BC} = \frac{2-3}{3-0} = \frac{1}{3}$; $m_{AD} = \frac{-3-(-1)}{4-(-2)} = -\frac{1}{3}$

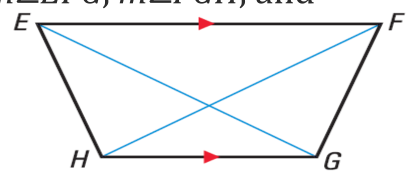
Since only 1 pair of sides is ∥, it is a trapezoid

Check for isosceles: $AB = \sqrt{\left(0-(-2)\right)^2 + \left(3-(-1)\right)^2} = \sqrt{20}$; $CD = \sqrt{(4-3)^2 + (-3-2)^2} = \sqrt{26}$

Not isosceles

7.5 Properties of Trapezoids and Kites

- If the trapezoid is isosceles and $m\angle HEF = 70°$, find $m\angle EFG$, $m\angle FGH$, and $m\angle GHE$.



- Try #6

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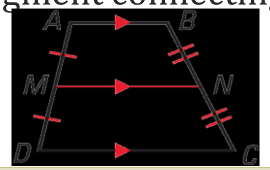
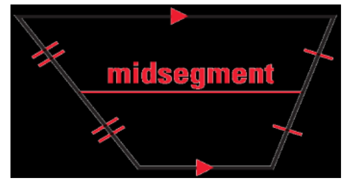
Base angles are ≅; $m\angle EFG = 70°$
Consecutive interior angles are supplementary; $m\angle FGH = m\angle GHE = 110°$

7.5 Properties of Trapezoids and Kites

- Midsegment of a Trapezoid
  - Segment connecting the midpoints of each leg





Midsegment Theorem for Trapezoids

The midsegment of a trapezoid is parallel to the bases and its length is the average of the lengths of the bases.

$$MN = \frac{1}{2}(b_1 + b_2)$$

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7.5 Properties of Trapezoids and Kites

- In trapezoid $JKLM$, $\angle J$ and $\angle M$ are right angles, and JK = 9 cm. The length of the midsegment $\overline{NP}$ of trapezoid $JKLM$ is 12 cm. Find ML.

- Try #10

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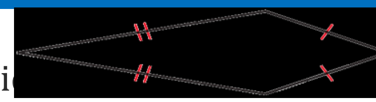
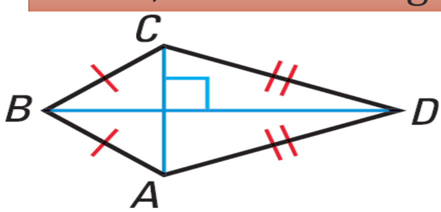
$$midsegment = \frac{1}{2}(b_1 + b_2)$$
$$12 = \frac{1}{2}(ML + 9)$$
$$24 = ML + 9$$
$$ML = 15$$

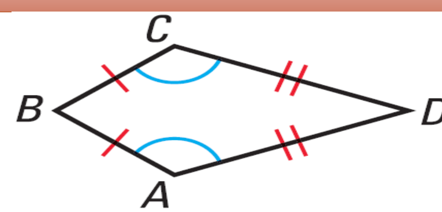
7.5 PROPERTIES OF TRAPEZOIDS AND KITES

- Kites
  - Quadrilateral with 2 pairs of consecutive congruent si



If kite, then the diagonals are perpendicular.

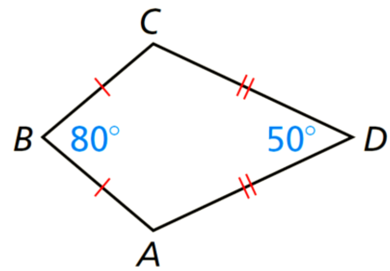




If kite, then exactly one pair of opposite angles are congruent.

7.5 Properties of Trapezoids and Kites

- Find $m\angle C$ in the kite shown.



- Try #16

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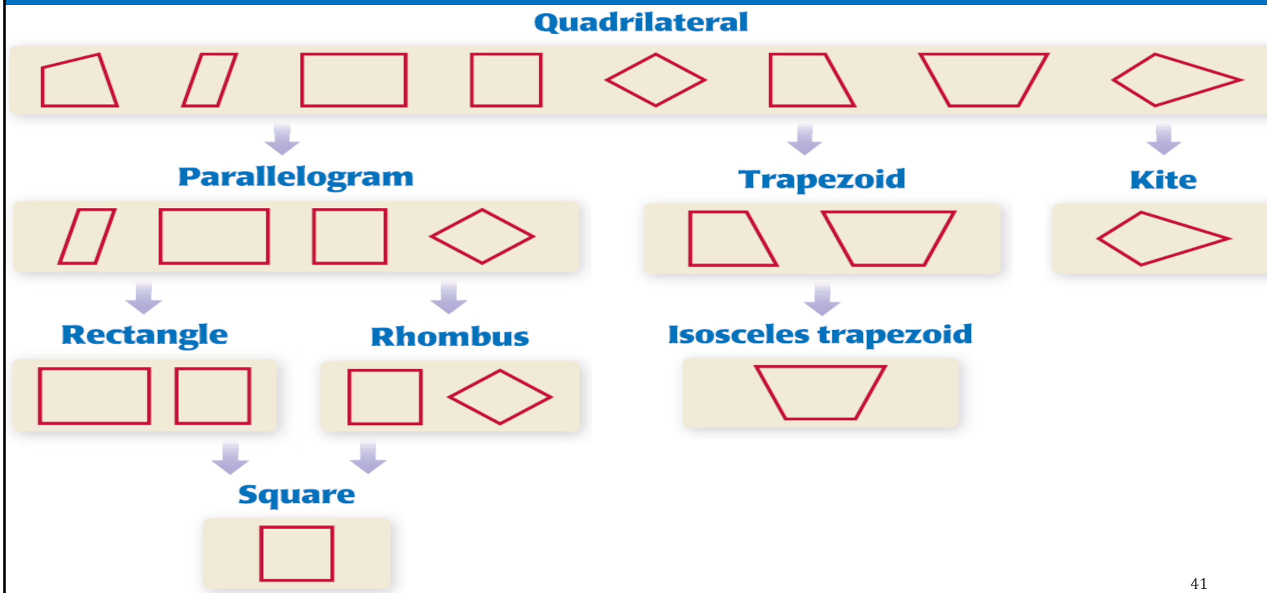
$$x° + x° + 80° + 50° = 360°$$
$$2x° + 130° = 360°$$
$$2x° = 230°$$
$$x = 115°$$

7.5 Properties of Trapezoids and Kites



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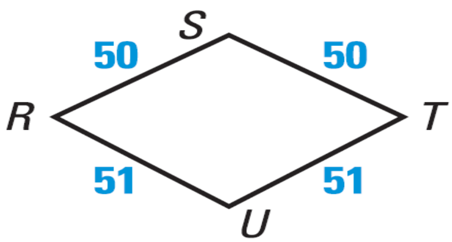
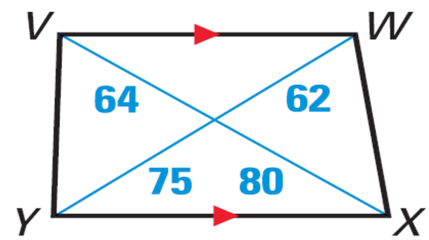
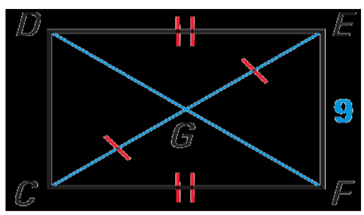
7.5 Properties of Trapezoids and Kites

- Give the most specific name for the quadrilateral.







- Try #22

Kite (≅ consecutive sides)

Trapezoid (exactly one pair of parallel sides, diagonals not ≅)

Quadrilateral (not enough information to be more specific)