\[ 2.5A \# 17 \]

\[ 2x^3 + 3x^2 - 1 \]

\[ p = \pm 1 \]

\[ q = \pm 1, \pm 2 \]

\[ p = \pm 1, \pm \frac{1}{2} \]

\[ -1 \begin{array}{rrrr}
2 & 3 & 0 & -1 \\
2 & 2 & -2 & 1 \\
\hline & 1 & -1 & 0 \\
\end{array} \]

\[ (2x - 1)(x^2 + 1) \]

\[ 2x - 1 = 0 \quad x + 1 = 0 \]

\[ x = \frac{1}{2} \quad x = -1 \]

\[ x = -1, -1, \frac{1}{2} \]
2.5B Zeros of Polynomials

Write \( h(x) = x^3 - 11x^2 + 41x - 51 \) as a product of linear factors.

Find zeros

\[ p = \pm 1, \pm 3, \pm 17, \pm 51 \]

\[ q = \pm 1 \]

\[ r = \pm 1, \pm 3, \pm 17, \pm 51 \]

\[ r = \pm 1 \]

\[ q = \pm 1 \]

\[ p = \pm 1, \pm 3, \pm 17, \pm 51 \]

\[ x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(17)}}{2(1)} \]

\[ x = 3, 4 \pm i \]

\[ h(x) = (x - 3)(x - (4 + i))(x - (4 - i)) \]

\[ = (x - 3)(x - 4 - i)(x - 4 + i) \]

\[ \begin{array}{c|cccc}
3 & 1 & -11 & 41 & -51 \\
\hline & 3 & -24 & 51 \\
& 1 & -8 & 17 \\
& 1 & 0 & 10 \\
\end{array} \]
Descartes Rule of Signs

Let \( f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 \) be a polynomial with real coefficients and \( a_0 \neq 0 \).

1. The number of positive real zeros is equal to the number of variations in sign of \( f(x) \) or less by even integers.

2. The number of negative real zeros is equal to the number of variations in sign of \( f(-x) \) or less by even integers.
Describe the possible real zeros of
\[ f(x) = -2x^3 + 5x^2 - x + 8 \]

Positive: 3 or 1

Negative: \[ f(-x) = -2(-x)^3 + 5(-x)^2 - (-x) + 8 \]
\[ = 2x^3 + 5x^2 + x + 8 \]
\[ = 0 \]
Upper and Lower Bounds

Let $f(x)$ be a polynomial with real coefficients and positive leading coefficient. Suppose we divide by $x-c$.

1. If $c > 0$ and each number in the last row is $\geq 0$, $c$ is an upper bound for zeros.

2. If $c < 0$ and each number in the last row alternates sign, $c$ is a lower bound for zeros.
Find all real zeros
\[ f(x) = 8x^3 - 4x^2 + 6x - 3 \]

**Rule of Signs**
- Positive: 3 or 1
- Negative: 0

**Possible rational roots**:
- \( P = \pm 1, \pm 3 \)
- \( Q = \pm 1, \pm 2, \pm 4, \pm 8 \)

Dividing by 2 to simplify:
\[
\begin{array}{c|cccc}
& 8 & -4 & 6 & -3 \\
\hline
2 & 4 & -1 & 3 & 0 \\
\hline
& 8 & 0 & 6 & 0 \\
\end{array}
\]

**Coefficients**:
- \( 8x^2 + 6 = 0 \)
- \( x^2 = -\frac{3}{4} \)
- \( x = \pm \frac{\sqrt{3}}{2} i \)

Thus, the real zeros are:
- \( x = \pm \frac{1}{2} \)

No additional zeros found from synthetic division.