

Trigonometry

Precalculus
Chapter 4

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- › This Slideshow was developed to accompany the textbook
 - › *Precalculus*
 - › *By Richard Wright*
 - › <https://www.andrews.edu/~rwright/Precalculus-RLW/Text/TOC.html>
- › Some examples and diagrams are taken from the textbook.

Slides created by
Richard Wright, Andrews Academy
rwright@andrews.edu

4-01 Angle Measures

In this section, you will:

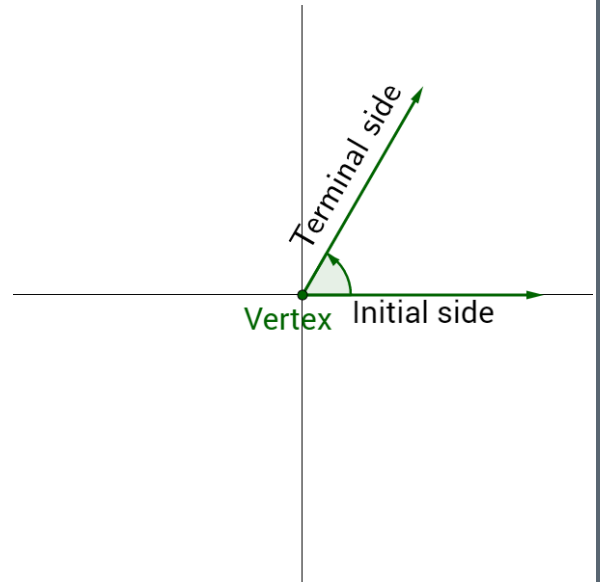
- Draw angles in standard position.
- Convert between degrees and radians.
- Find coterminal angles.
- Use applications of angles.

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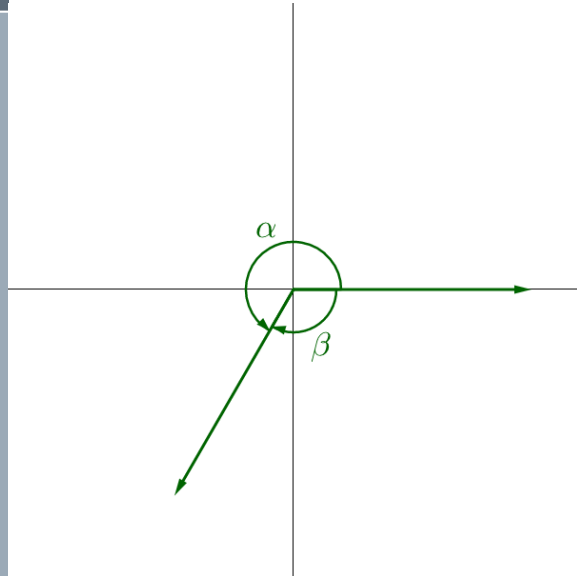
4-01 *Angle Measures*

- › Angles in standard position
 - › Vertex at origin
 - › Initial side on positive x-axis
 - › Terminal side rotates counterclockwise



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4.01 *Angle Measures*



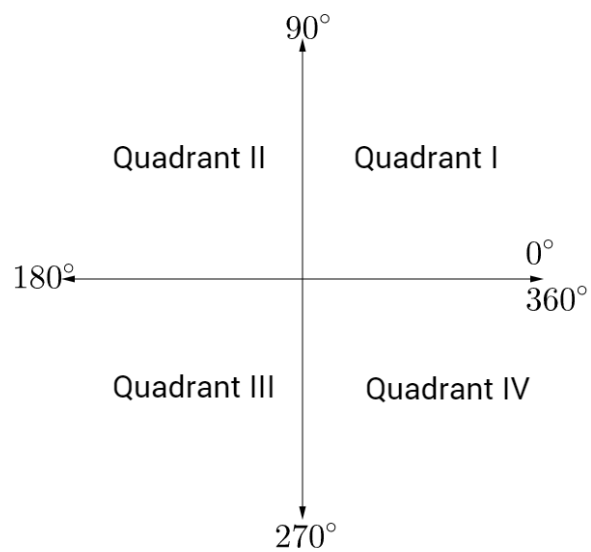
- › Coterminal Angles
- › 2 angles with same sides, but different measures

- › To find coterminal angles
 - › $\theta \pm 360^\circ$

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401 *Angle Measures*

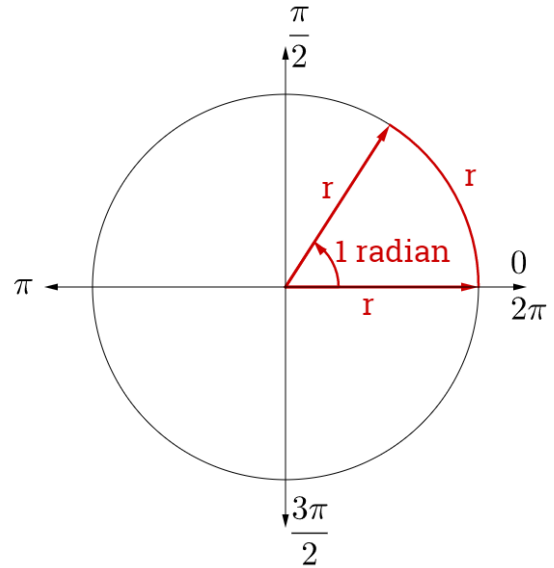
› Degree Measures



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4.01 Angle Measures

- › Radian Measures
 - › Angle where radius = arc length
- › Acute $\rightarrow \theta < 90^\circ, \frac{\pi}{2}$
- › Obtuse $\rightarrow 90^\circ < \theta < 180^\circ$
 - › $\frac{\pi}{2} < \theta < \pi$
- › Complementary $\rightarrow \alpha + \beta = 90^\circ, \frac{\pi}{2}$
- › Supplementary $\rightarrow \alpha + \beta = 180^\circ, \pi$



π **401 Angle Measures**

› Find a coterminal angle
with $\theta = -\frac{\pi}{8}$

› Find the supplement of $\theta = \frac{\pi}{4}$

Coterminal

$$-\frac{\pi}{8} \pm 2\pi = -\frac{\pi}{8} \pm \frac{16\pi}{8} = -\frac{17\pi}{8}, \frac{15\pi}{8}$$

Supplement

$$\begin{aligned} S + \frac{\pi}{4} &= \pi \\ S &= \pi - \frac{\pi}{4} = \frac{3\pi}{4} \end{aligned}$$

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4-01 *Angle Measures*

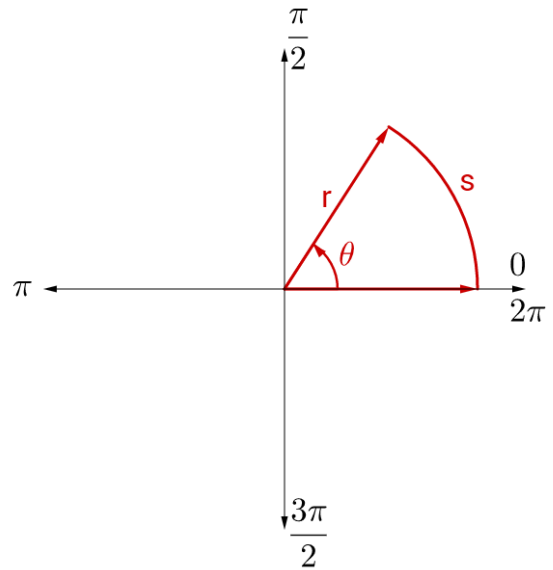
- › Convert radians to degrees
- › Convert 120° to radians
- › $180^\circ = \pi$

$$120^\circ \left(\frac{\pi}{180^\circ} \right) = \frac{120\pi}{180} = \frac{2\pi}{3}$$

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4.01 *Angle Measures*

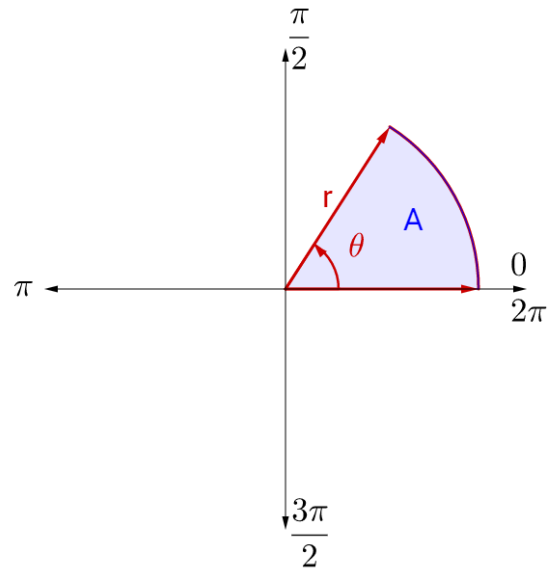
- › Applications
- › Arc Length
 - › $S = r\theta$
 - › Where θ is in radians



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4.01 Angle Measures

- › Area of Sector
- › $A = \text{fraction of circle} \times \pi r^2$
- › $A = \frac{\theta}{2\pi} \times \pi r^2$
- › $A = \frac{1}{2} \theta r^2$
- › Where θ is in radians



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4-01 *Angle Measures*

› Speeds

› Angular speed: $\omega = \frac{\theta}{t}$

› Linear speed (tangential): $v = \frac{s}{t}$

› $v = \frac{r\theta}{t}$

› $v = r\omega$

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401 *Angle Measures*

- › A 6-inch diameter gear makes 2.5 revolutions per second. Find the angular speed in radians per second.
- › How fast is a tooth at the edge of the gear moving in in./s?

$$\frac{2.5 \text{ rev}}{s} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 5\pi \frac{\text{rad}}{s}$$

$$v = (3 \text{ in.}) \left(5\pi \frac{\text{rad}}{s} \right) = 15\pi \text{ in./s}$$

4-02 Unit Circle

In this section, you will:

- Understand the unit circle.
- Use the unit circle to evaluate trigonometric functions.
- Use even and odd trigonometric functions.
- Use a calculator to evaluate trigonometric functions.

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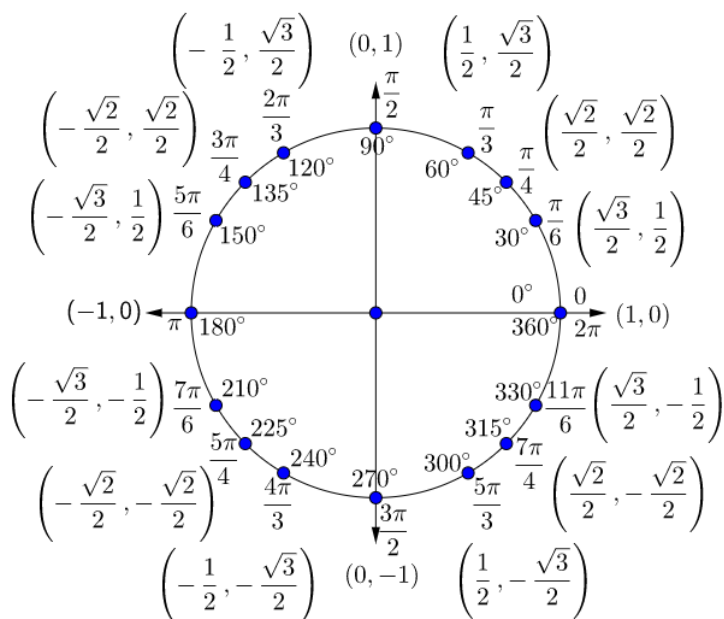
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4-02 Unit Circle

› Unit circle

› $r = 1$

› $x^2 + y^2 = 1$



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4-02 Unit Circle

› Trigonometric Functions
(Unit circle)

› $\sin t = y$

› sine

› $\cos t = x$

› cosine

› $\tan t = \frac{y}{x}$

› tangent

› $\csc t = \frac{1}{y}$

› cosecant

› $\sec t = \frac{1}{x}$

› secant

› $\cot t = \frac{x}{y}$

› cotangent

π **4-02 Unit Circle**

› Evaluate 6 trig functions of $t = \frac{2\pi}{3}$

$$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{2\pi}{3} = -\frac{1}{2}$$

$$\tan \frac{2\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

$$\csc \frac{2\pi}{3} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec \frac{2\pi}{3} = \frac{1}{-\frac{1}{2}} = -2$$

$$\cot \frac{2\pi}{3} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

π **4-02 Unit Circle**

> Evaluate

$$> \csc \frac{11\pi}{6}$$

$$> \sec \frac{4\pi}{3}$$

$$> \cot \frac{3\pi}{4}$$

$$> \sin 2\pi$$

$$> \tan \frac{\pi}{2}$$

$$> \cos 0$$

Draw angles on unit circle for reference

$$\sec \frac{4\pi}{3} = \frac{1}{x} = \frac{1}{-\frac{1}{2}} = -2$$

$$\sin 2\pi = y = 0$$

$$\tan \frac{\pi}{2} = \frac{y}{x} = \frac{1}{0} = \text{undefined}$$

$$\csc \frac{11\pi}{6} = \frac{1}{y} = \frac{1}{-\frac{1}{2}} = -2$$

$$\cot \frac{3\pi}{4} = \frac{x}{y} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$$

$$\cos 0 = x = 1$$

π **4-02 Unit Circle**

› Evaluate

› $\sin\left(-\frac{2\pi}{3}\right)$

› $\sin\left(-\frac{11\pi}{2}\right)$

› $\cos\frac{9\pi}{3}$

Find coterminal angles between 0 and 2π

$$\sin\left(-\frac{2\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = y = -\frac{\sqrt{3}}{2}$$

$$\cos\frac{9\pi}{3} = \cos\pi = x = -1$$

$$\sin\left(-\frac{11\pi}{2}\right) = \sin\frac{\pi}{2} = y = 1$$

4-03 Right Triangle Trigonometry

In this section, you will:

- Use right triangles to evaluate trigonometric functions.
- Use special right triangles to evaluate trigonometric functions of common angles.

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4-03 Right Triangle Trigonometry

$$\triangleright \sin A = \frac{opp}{hyp}$$

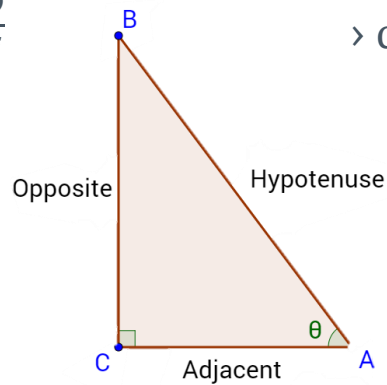
$$\triangleright \csc A = \frac{hyp}{opp}$$

$$\triangleright \cos A = \frac{adj}{hyp}$$

$$\triangleright \sec A = \frac{hyp}{adj}$$

$$\triangleright \tan A = \frac{opp}{adj}$$

$$\triangleright \cot A = \frac{adj}{opp}$$



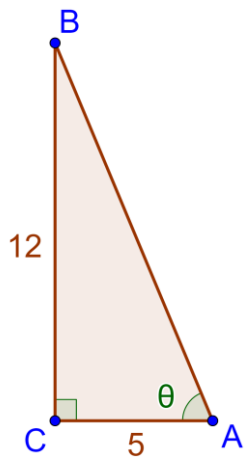
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4-03 *Right Triangle Trigonometry*

› Find the values of the six trig functions



$$\text{hyp} = \sqrt{5^2 + 12^2} = 13$$

$$\sin \theta = \frac{12}{13}$$

$$\cos \theta = \frac{5}{13}$$

$$\tan \theta = \frac{12}{5}$$

$$\csc \theta = \frac{13}{12}$$

$$\sec \theta = \frac{13}{5}$$

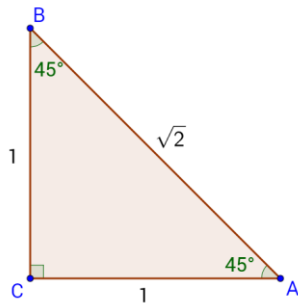
$$\cot \theta = \frac{5}{12}$$

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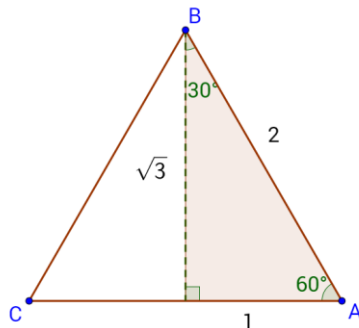
4-03 Right Triangle Trigonometry

> Special right triangles

> $\sin \frac{\pi}{4}$



> $\csc \frac{\pi}{3}$



> $\tan 30^\circ$

$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\csc \frac{\pi}{3} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

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4-03 *Right Triangle Trigonometry*

- › Sketch a triangle and find the other 5 trig functions
- › $\tan \theta = 3$

$$\tan \theta = 3 = \frac{3}{1} = \frac{\text{opp}}{\text{adj}}$$

$$\sin \theta = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\cos \theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\tan \theta = 3$$

$$\csc \theta = \frac{\sqrt{10}}{3}$$

$$\sec \theta = \frac{\sqrt{10}}{1} = \sqrt{10}$$

$$\cot \theta = \frac{1}{3}$$

4-04 Right Triangle Trigonometry and Identities

In this section, you will:

- Use right triangles to evaluate trigonometric functions.
- Use special right triangles to evaluate trigonometric functions of common angles.

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4-04 Right Triangle Trigonometry and Identities

› Basic Identities

› Reciprocal

$$\sin u = \frac{1}{\csc u}$$

$$\cos u = \frac{1}{\sec u}$$

$$\tan u = \frac{1}{\cot u}$$

$$\textcolor{red}{\csc u = \frac{1}{\sin u}}$$

$$\textcolor{red}{\sec u = \frac{1}{\cos u}}$$

$$\textcolor{red}{\cot u = \frac{1}{\tan u}}$$

› Quotient

$$\textcolor{red}{\tan u = \frac{\sin u}{\cos u}}$$

$$\cot u = \frac{\cos u}{\sin u}$$

› Pythagorean

$$\textcolor{red}{\sin^2 u + \cos^2 u = 1}$$

$$1 + \tan^2 u = \sec^2 u$$

$$\cot^2 u + 1 = \csc^2 u$$

› Note: $\sin^2 u = (\sin u)^2$

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4-04 Right Triangle Trigonometry and Identities

› Cofunction Identities

› $\sin(90^\circ - u) = \cos u$

› $\cos(90^\circ - u) = \sin u$

› $\tan(90^\circ - u) = \cot u$

› $\cot(90^\circ - u) = \tan u$

› $\sec(90^\circ - u) = \csc u$

› $\csc(90^\circ - u) = \sec u$

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4-04 *Right Triangle Trigonometry and Identities*

- › Let θ be an acute angle such that $\cos \theta = 0.96$
- › Find $\tan \theta$
- › Find $\sin \theta$

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \sin^2 \theta + 0.96^2 &= 1 \\ \sin^2 \theta + 0.9216 &= 1 \\ \sin^2 \theta &= 0.0784 \\ \sin \theta &= 0.28\end{aligned}$$

$$\begin{aligned}\tan \theta &= \frac{\sin \theta}{\cos \theta} \\ &= \frac{0.28}{0.96} \\ &= 0.2917\end{aligned}$$

These could also have been solved using right triangles

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4-04 *Right Triangle Trigonometry and Identities*

- › Let β be an acute angle such that $\tan \beta = 4$
- › Find $\cot \beta$
- › $\sec \beta$

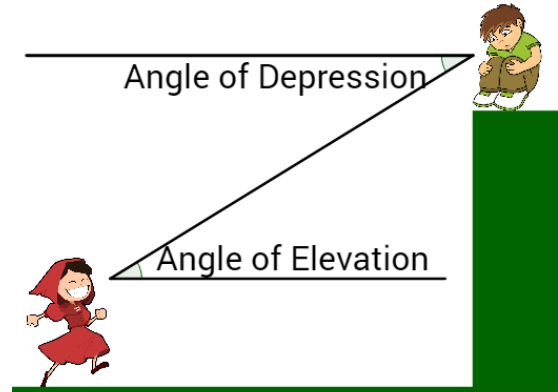
$$\cot \beta = \frac{1}{\tan \beta}$$
$$\cot \beta = \frac{1}{4}$$

$$1 + \tan^2 \beta = \sec^2 \beta$$
$$1 + 4^2 = \sec^2 \beta$$
$$\sqrt{17} = \sec \beta$$

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4-04 *Right Triangle Trigonometry and Identities*

- › Angles of Elevation and Depression
- › Both are measured from the horizontal



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4-04 Right Triangle Trigonometry and Identities

- › A 12-meter flagpole casts a 6-meter shadow. Find the angle of elevation of the sun.

$$\begin{aligned}\tan \theta &= \frac{12}{6} \\ \tan \theta &= 2 \\ \theta &\approx 63.4^\circ\end{aligned}$$

4-05 Trigonometric Functions of Any Angle

In this section, you will:

- Evaluate trigonometric functions of any angle.
- Find reference angles.

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4-05 Trigonometric Functions of Any Angle

$$\triangleright \sin \theta = \frac{y}{r}$$

$$\csc \theta = \frac{r}{y}$$

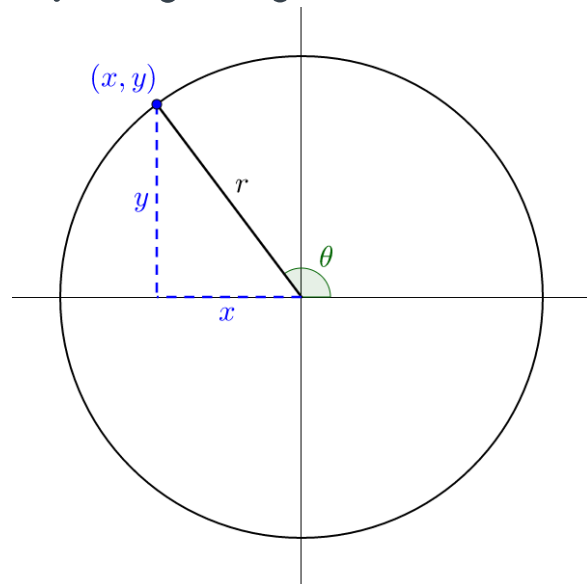
$$\triangleright \cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}$$

$$\triangleright \tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$

$$\triangleright r = \sqrt{x^2 + y^2}$$



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4-05 Trigonometric Functions of Any Angle

› Let $(-2, 3)$ be a point on the terminal side of θ . Find sine, cosine, and tangent of θ .

Use Pythagorean Theorem to find r

$$r = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$$

$$\sin \theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

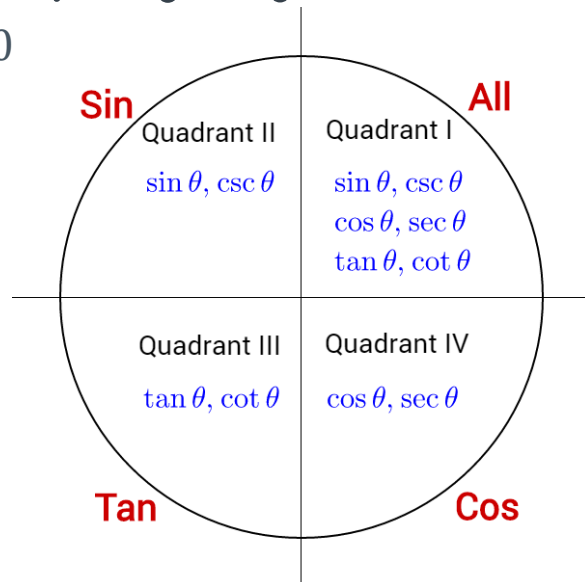
$$\cos \theta = -\frac{2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$$

$$\tan \theta = -\frac{3}{2}$$

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4-05 Trigonometric Functions of Any Angle

› Given $\sin \theta = \frac{4}{5}$ and $\tan \theta < 0$
find $\cos \theta$ and $\csc \theta$.



Quadrant II (sine +, tangent -)

$$\sin \theta = \frac{4}{5} = \frac{y}{r}$$

Use Pythagorean theorem to find $r = -3$

$$\cos \theta = -\frac{3}{5}$$

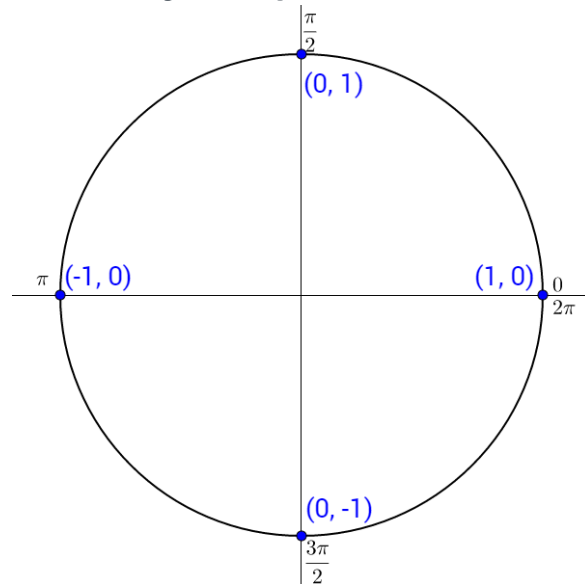
$$\csc \theta = \frac{5}{4}$$

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4-05 Trigonometric Functions of Any Angle

› Evaluate $\sin \pi$

› $\tan \frac{\pi}{2}$



$$\sin \pi = \frac{y}{r} = \frac{0}{1} = 0$$

$$\tan \frac{\pi}{2} = \frac{y}{x} = \frac{1}{0} = \text{undefined}$$

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4-05 *Trigonometric Functions of Any Angle*

- › Reference Angle
 - › Angle between terminal side and nearest x-axis
- › Find the reference angle for $\frac{5\pi}{3}$
- › Find the reference angle for $\frac{5\pi}{4}$

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4-05 Trigonometric Functions of Any Angle

› Use a reference angle to evaluate $\cos \frac{5\pi}{3}$ › $\sin 150^\circ$

Reference angle is $\frac{\pi}{3}$

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

Quadrant IV where cos is +

Reference angle is 30°

$$\sin 30^\circ = \frac{1}{2}$$

Quadrant II where sin is +

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4-05 Trigonometric Functions of Any Angle

› Use a reference angle to evaluate $\tan \frac{11\pi}{6}$

Reference angle is $\frac{\pi}{6}$

$$\tan \frac{\pi}{6} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

Quadrant IV where tan is -

π **4-05 Trigonometric Functions of Any Angle**

› Let θ be an angle in quadrant III such that

$$\sin \theta = -\frac{5}{13}. \text{ Find}$$

› $\tan \theta$

› $\sec \theta$

$$\sin \theta = -\frac{5}{13} = \frac{y}{r}$$

$$y = -5, r = 13$$

Use Pythagorean theorem to find $x = -12$

$$\sec \theta = \frac{r}{x} = -\frac{13}{12}$$

$$\tan \theta = \frac{y}{x} = \frac{-5}{-12} = \frac{5}{12}$$

4-06 Graphs of Sine and Cosine

In this section, you will:

- Graph $y = \sin x$ and $y = \cos x$.
- Graph transformations of sine and cosine graphs.
- Write mathematical models using sine and cosine.

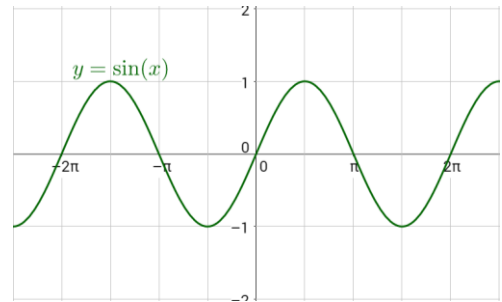
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4.06 Graphs of Sine and Cosine

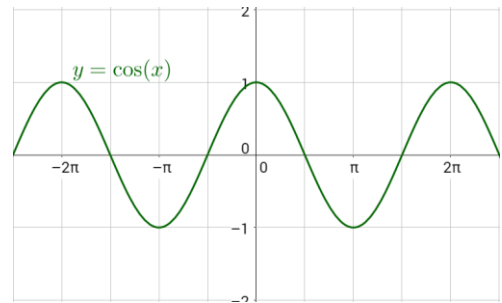
› $y = \sin x$

- › Starts at 0
- › Amplitude = 1
- › Period = 2π



› $y = \cos x$

- › Starts at 1
- › Amplitude = 1
- › Period = 2π



Point out

- Amplitude
- period
- key points

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4-06 *Graphs of Sine and Cosine*

› Transformations

› $y = a \sin(bx - c) + d$

› a = amplitude = vertical stretch

› b = horizontal shrink

› Period $T = \frac{2\pi}{b}$

› c = horizontal shift

› Phase shift $PS = \frac{c}{b}$ (Right if c is positive)

› d = vertical shift

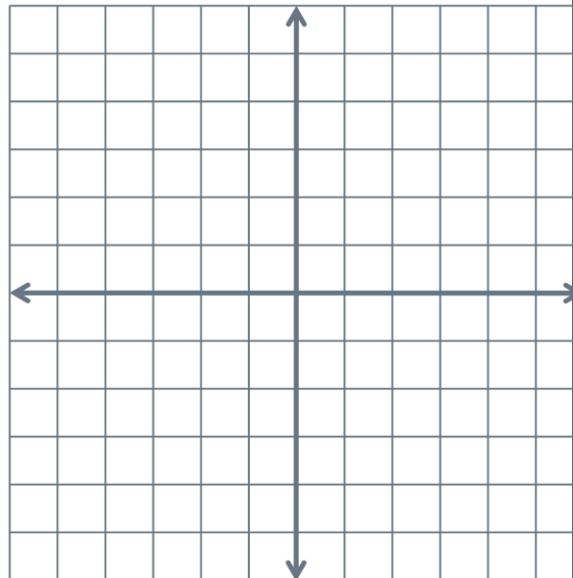
› Midline $y = d$

c is like h
 d is like k

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4-06 Graphs of Sine and Cosine

› Graph $f(x) = 2 \sin x$

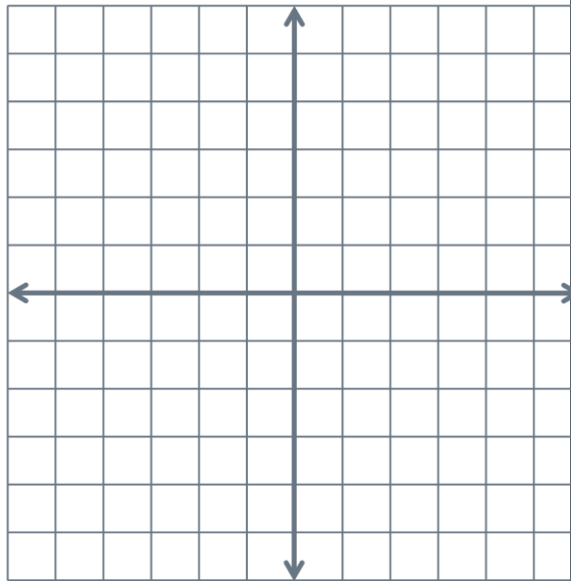


Same as sine, but amp = 2

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406 Graphs of Sine and Cosine

› Graph $y = \cos \frac{x}{2}$



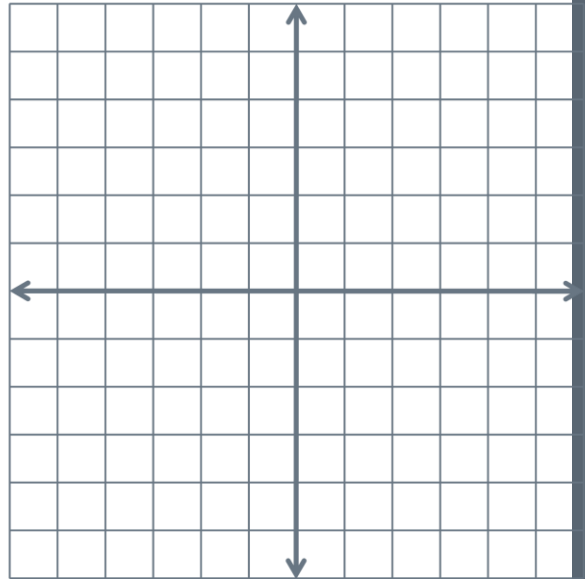
Period $T = \frac{2\pi}{b}$

$$T = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

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4-06 Graphs of Sine and Cosine

› Graph $y = 2 \sin \left(x - \frac{\pi}{2} \right)$



$$a = 2$$

$$b = 1 \rightarrow T = 2\pi$$

$$h = \frac{\pi}{2} \rightarrow PS = \frac{h}{b} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2} \text{ to right}$$

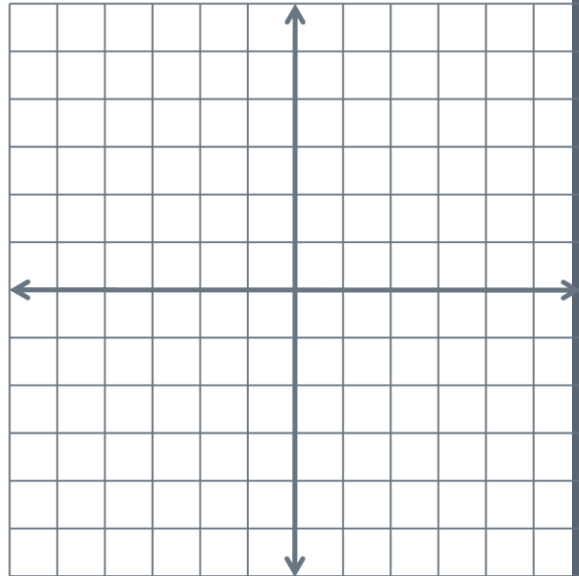
Draw $2 \sin x$ first and then do the phase shift

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4-06 Graphs of Sine and Cosine

› Graph

$$y = -\frac{1}{2}\sin(\pi x + \pi) + 1$$



$$a = -\frac{1}{2} = \text{amp}$$

$$b = \pi \rightarrow T = \frac{2\pi}{b} \rightarrow \frac{2\pi}{\pi} = 2$$

$$h = -\pi \rightarrow PS = \frac{h}{b} \rightarrow -\frac{\pi}{\pi} = -1$$

PS left 1

$$k = 1$$

Shift up 1

Graph $\frac{1}{2}\sin \pi x$ first labeling the key points with a period of 2

Reflect over the x -axis because a is negative

Shift left 1 and up 1

4-07 Graphs of Other Trigonometric Functions

In this section, you will:

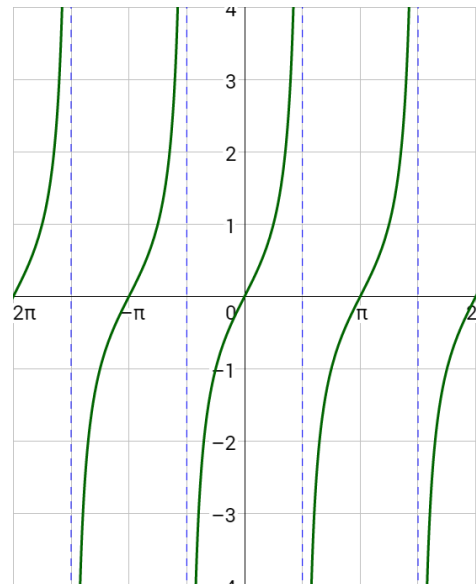
- Graph tangent, secant, cosecant, and cotangent
- Graph a damped trigonometric function

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4.07 Graphs of Other Trigonometric Functions

- › $y = \tan x$
 - › Period = π
 - › $T = \frac{\pi}{b}$
 - › Asymptotes where tangent undefined, $\frac{\pi}{2}, \frac{3\pi}{2}$



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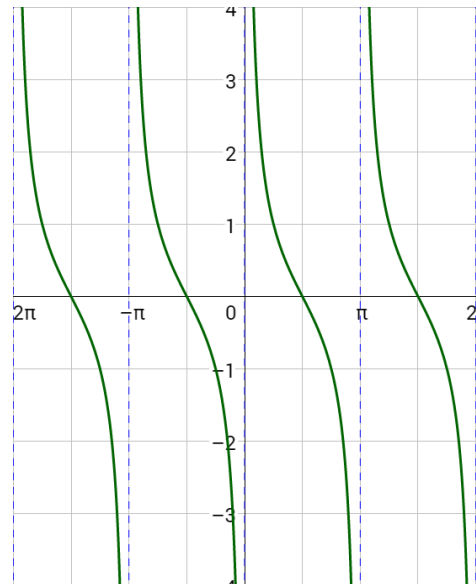
4.07 Graphs of Other Trigonometric Functions

› $y = \cot x$

› Period = π

› $T = \frac{\pi}{b}$

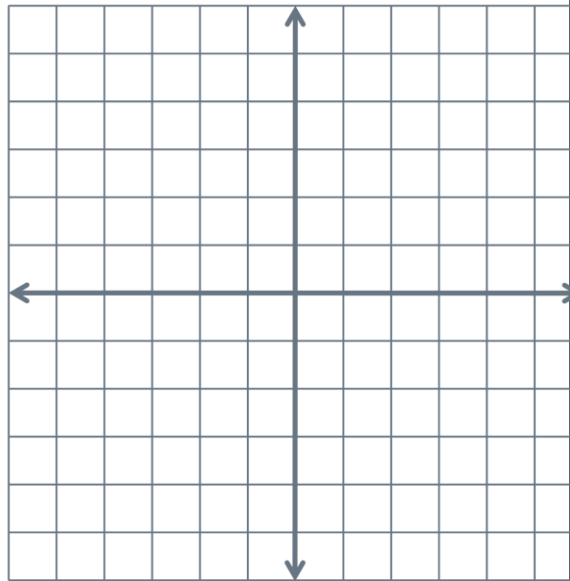
› Asymptotes at $0, \pi, 2\pi$



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4-07 Graphs of Other Trigonometric Functions

› Graph $y = \tan \frac{x}{4}$

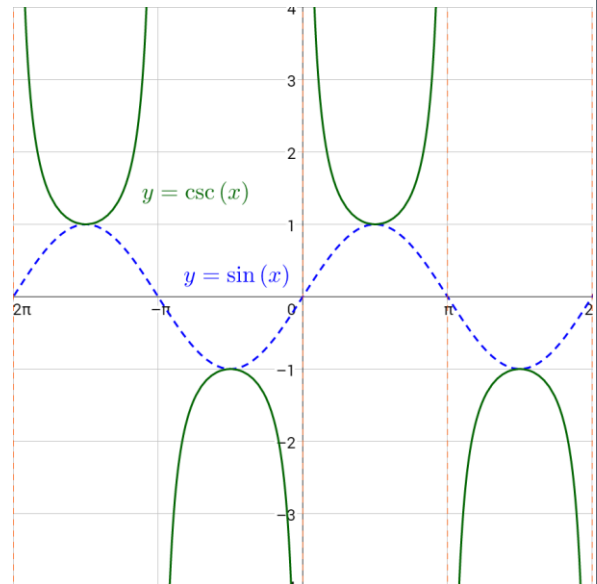


$$b = \frac{1}{4}$$
$$T = \frac{\pi}{b} = \frac{\pi}{\frac{1}{4}} = 4\pi$$
$$a = 1$$

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4.07 Graphs of Other Trigonometric Functions

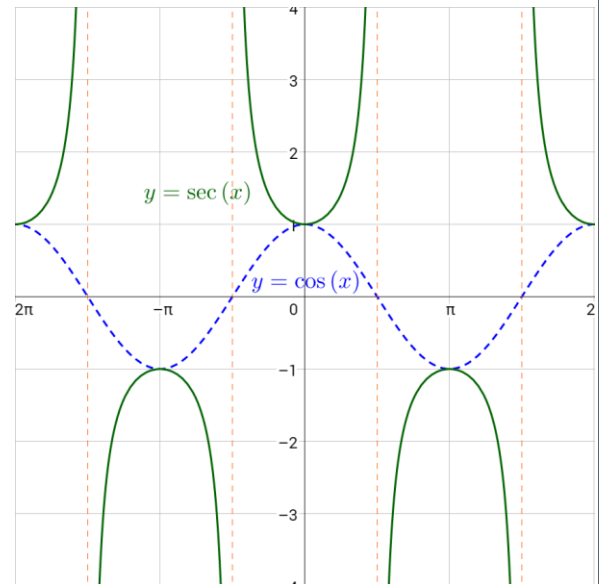
- › $y = \csc x$
- › Period = 2π
- › Asymptotes where sine = 0
 - › $0, \pi, 2\pi$



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4.07 Graphs of Other Trigonometric Functions

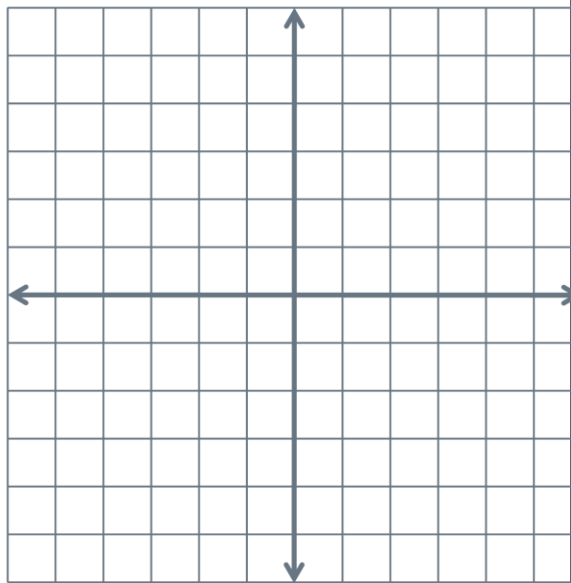
- › $y = \sec x$
- › Period = 2π
- › Asymptotes where cosine = 0
 - › $\frac{\pi}{2}, \frac{3\pi}{2}$



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4-07 Graphs of Other Trigonometric Functions

› Graph $y = 2 \csc\left(x + \frac{\pi}{2}\right)$



$$\begin{aligned}a &= 2 \\b &= 1 \\T &= \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi \\c &= -\frac{\pi}{2} \\PS &= \frac{c}{b} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2} \\k &= 0\end{aligned}$$

Start by graphing $2 \sin x$

Then shift left $\frac{\pi}{2}$

Then draw asymptotes at the x-intercepts

Then draw csc graph

π

4.07 Graphs of Other Trigonometric Functions

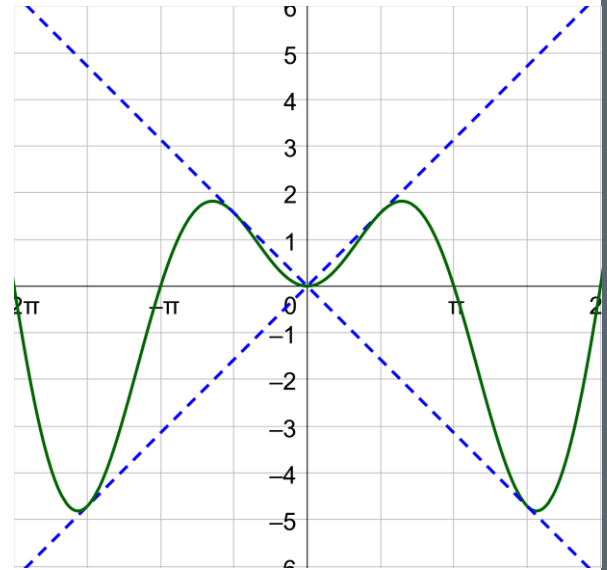
› Damped Trig Functions

› $y = \boxed{x} \sin x$

› The x is the damping function

› Graph the damping function and its reflection over x -axis

› Graph the trig between



4-08 Inverse Trigonometric Functions

In this section, you will:

- Use the inverse sine, cosine, and tangent functions
- Evaluate inverse trigonometric functions

π

π

4-08 *Inverse Trigonometric Functions*

- › Inverses switch x and y
 - › *Reflects graph over $y = x$*
- › $y = \sin x \leftrightarrow x = \sin^{-1} y$
- › Inverse trig functions give the angle

π

4-08 Inverse Trigonometric

› Inverse Sine

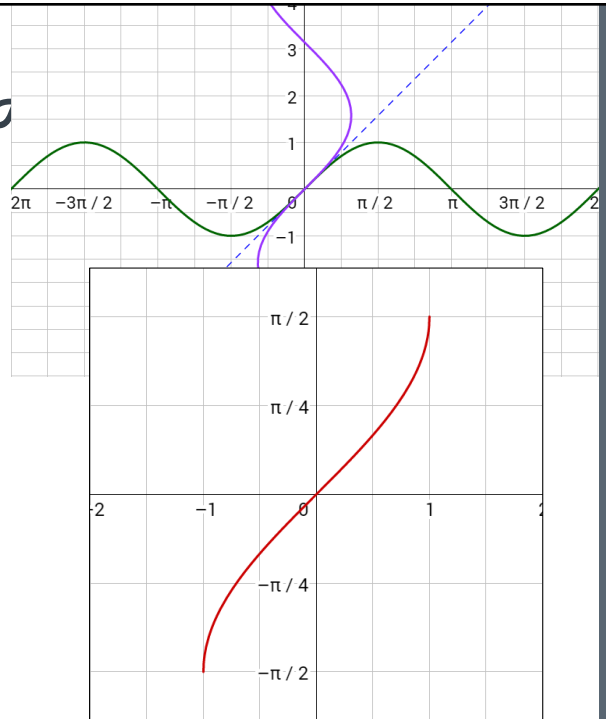
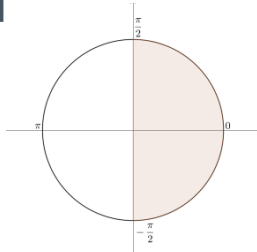
$$y = \sin^{-1} x$$

$$y = \arcsin x$$

› Domain: $[-1, 1]$

› Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

› $\arcsin(-1)$



$$\arcsin(-1) = -\frac{\pi}{2}$$

$$\sin \theta = y = -1$$

π

4-08 Inverse Trigonometric Functions

› Inverse Cosine

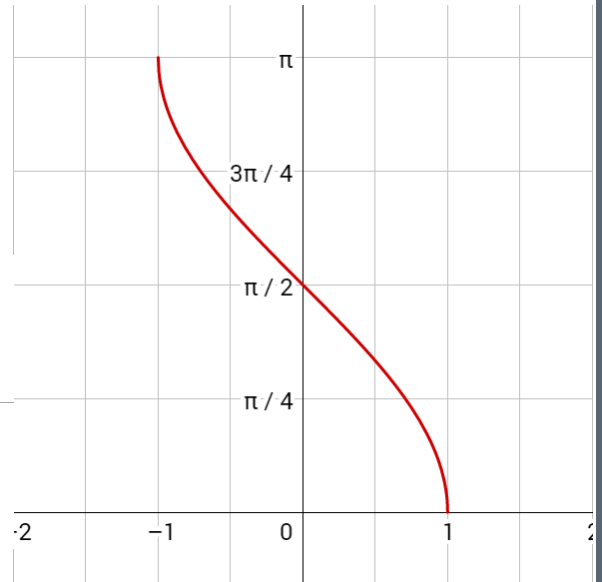
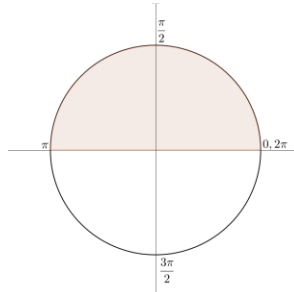
› $y = \cos^{-1} x$

› $y = \arccos x$

› Domain: $[-1, 1]$

› Range: $[0, \pi]$

› $\arccos \frac{1}{2}$



Think $\cos \theta = \frac{1}{2}$

$$\arccos \frac{1}{2} = \frac{\pi}{3}$$

π

4-08 Inverse Trigonometric Functions

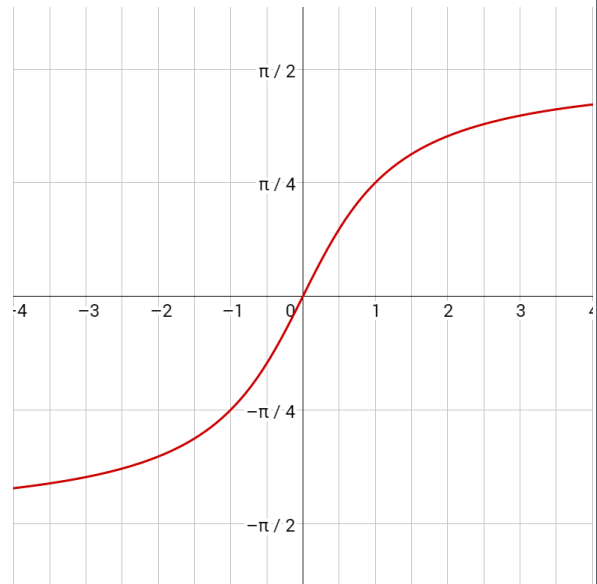
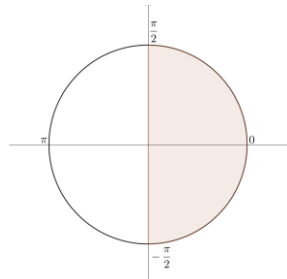
› Inverse Tangent

› $y = \tan^{-1} x$

› $y = \arctan x$

› Domain: $(-\infty, \infty)$

› Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



π

4-08 Inverse Trigonometric Functions

› Evaluate

› $\arcsin \sqrt{3}$

› $\sin^{-1} \left(\frac{1}{2} \right)$

Think $\sin \theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{6}$

Think $\sin \theta = \sqrt{3} \rightarrow$ Not possible

π

408 Inverse Trigonometric Functions

› Evaluate

$$\text{› } \arctan \frac{\sqrt{3}}{3}$$

$$\text{› } \cos^{-1} \frac{\sqrt{3}}{2}$$

$$\text{Think } \cos \theta = \frac{\sqrt{3}}{2} \rightarrow \theta = \frac{\pi}{6}$$

$$\text{Think } \tan \theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \rightarrow \theta = \frac{\pi}{6}$$

4-09 Compositions involving Inverse Trigonometric Functions

In this section, you will:

- Evaluate compositions of inverse functions

π

409 Compositions involving Inverse Trigonometric Functions

π

› If $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, then $\sin(\arcsin x) = x$ and $\arcsin(\sin y) = y$

› $\tan(\arctan(-14))$

$$\sin(\arcsin \pi)$$

Check domain of inner: $\arctan$ domain $x \neq \frac{\pi}{2}n$ so -14 is in domain.

Check range of outer: $\tan$ range $(-\infty, \infty)$ so -14 is in range

Ans: -14

Check domain of inner: $\arcsin$ domain $[-1, 1]$

π is not in domain, so not possible

409 Compositions involving Inverse Trigonometric Functions

π

$$> \arcsin\left(\sin \frac{5\pi}{3}\right)$$

$$> \arccos\left(\cos \frac{7\pi}{6}\right)$$

Check domain of inner: $\sin$ domain $(-\infty, \infty)$ so $\frac{5\pi}{3}$ is included

Check range of outer: $\arcsin$ range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ so use coterminal angle

$$\text{Ans } -\frac{\pi}{3}$$

Check domain of inner: $\cos$ domain $(-\infty, \infty)$ so $\frac{7\pi}{6}$ is included

Check range of outer: $\arccos$ range $[0, \pi]$ so use reference angle to find another angle with same sign and reference angle

$$\text{Ans } \frac{5\pi}{6}$$

409 Compositions involving Inverse Trigonometric Functions

π

$$> \tan^{-1}(\cos \pi)$$

$$> \cos^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right)$$

Check domain of inner: $\cos$ domain $(-\infty, \infty)$ so π is in domain.

Use the unit circle to find inner: $\cos \pi = -1$

$$\text{Evaluate outer } \tan^{-1}(-1) = -\frac{\pi}{4}$$

Check domain of inner: $\sin$ domain $(-\infty, \infty)$ so $\frac{\pi}{6}$ is in domain.

$$\text{Use the unit circle to find inner: } \sin \frac{\pi}{6} = \frac{1}{2}$$

$$\text{Evaluate outer } \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

4.09 Compositions involving Inverse Trigonometric Functions

π

$$> \cos \left(\tan^{-1} \left(-\frac{3}{4} \right) \right)$$

$$> \sin \left(\cos^{-1} \left(\frac{2}{3} \right) \right)$$

The input is in arctan so they are ratio of sides. Use those to make a triangle.

Use Pythagorean theorem to find r

Evaluate cos of that angle in the triangle

Ans: $\frac{4}{5}$

The input is in arccos so they are ratio of sides. Use those to make a triangle.

Use Pythagorean theorem to find y

Evaluate sin of that angle in the triangle

Ans: $\frac{\sqrt{5}}{3}$

409 Compositions involving Inverse Trigonometric Functions

π

› $\sec(\arctan x)$

The input is in arctan so they are ratio of sides. Use those to make a triangle.

Use Pythagorean theorem to find $r = \sqrt{x^2 + 1}$

Evaluate sec of that angle in the triangle

Ans: $\frac{\sqrt{x^2+1}}{1}$

4.10 Applications of Right Triangle Trigonometry

In this section, you will:

- Solve problems with right triangles and trigonometry

π

π

4-10 Applications of Right Triangle Trigonometry

- › Right triangle trigonometry
- › Draw a triangle and label it
- › Solve

π

4-10 *Applications of Right Triangle Trigonometry*

- › A ladder leaning against a house reaches 24 ft up the side of the house. The ladder makes a 60° angle with the ground. How far is the base of the ladder from the house?

Draw picture

$$\tan 60^\circ = \frac{24}{x}$$

$$\sqrt{3} = \frac{24}{x}$$

$$x = \frac{24}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3} \approx 13.86 \text{ ft}$$

4.11 Bearings and Simple Harmonic Motion

In this section, you will:

- Solve problems involving bearings
- Solve problems involving simple harmonic motion

π

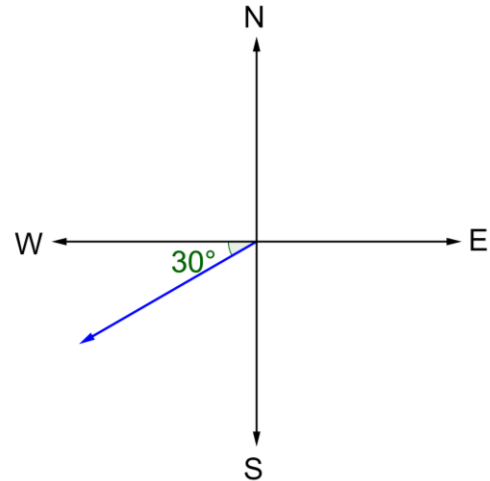
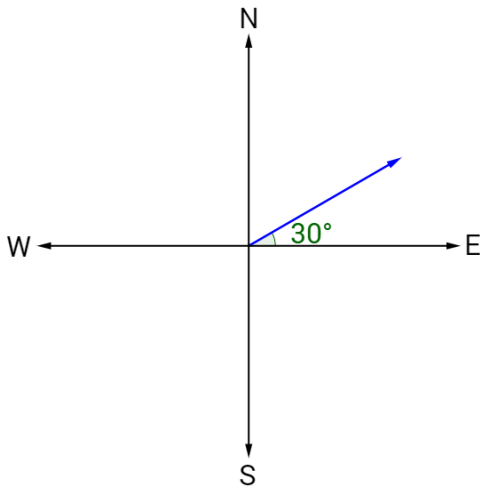
π

4.11 Bearings and Simple Harmonic Motion

› Bearings show direction

› W 30° S

› E 30° N



π

4.11 Bearings and Simple Harmonic Motion

- › A sailboat leave a pier and heads due west at 8 knots. After 15 minutes the sailboat tacks, changing course to N 16° W at 10 knots. Find the sailboat's bearing and distance from the pier after 12 minutes on this course.

Draw a diagram and find all components of the N 16° W
Add the x components
Add the y components
Draw a new triangle with those sums
Use Pythagorean theorem to find the hypotenuse
Use inverse tangent to find the angle

3.19 mi at W 37.0° N

π

4.11 Bearings and Simple Harmonic Motion

› Simple Harmonic Motion (SHM)

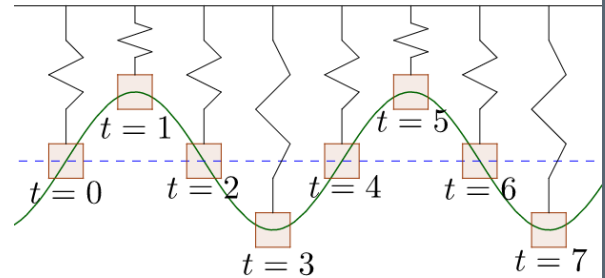
› $y = a \sin \omega x$

› $y = a \cos \omega x$

› Period $T = \frac{2\pi}{\omega}$

› Frequency (cycles per second) $f = \frac{\omega}{2\pi}$

› Equilibrium is the centerline



π

4.11 Bearings and Simple Harmonic Motion

- › Find a model for simple harmonic motion with displacement at $t = 0$ is 0, amplitude of 4 cm, and period of 6 sec.

$$a = 4 \text{ cm}$$
$$T = \frac{2\pi}{\omega} \rightarrow 6 = \frac{2\pi}{\omega} \rightarrow \omega = \frac{\pi}{3}$$

Starts at 0 so use sine

$$y = a \sin \omega t$$
$$y = 4 \sin \left(\frac{\pi}{3} t \right)$$

π

4.11 Bearings and Simple Harmonic Motion

› Given the equation for simple harmonic motion

$$d = 4 \cos 6\pi t$$

› Find maximum displacement

› Find frequency

› Find value of d when $t = 4$

› Find the least positive value of t for which $d = 0$

4 (amplitude)

$$f = \frac{\omega}{2\pi} = \frac{6\pi}{2\pi} = 3$$

$$d = 4 \cos 6\pi 4 = 4$$

$$0 = 4 \cos 6\pi t \rightarrow 0 = \cos 6\pi t \rightarrow \frac{\pi}{2} = 6\pi t \rightarrow \frac{1}{12} = t$$