10.2 Parabolas

Conic sections

- Are created from the intersection of a plane with a cone
- Parabola – the plane is parallel to the side of the cone
- Ellipses – the plane intersects through the cone
- Hyperbola – The plane is parallel to the axis of the cone

Parabola

- Set of all points in a plane that are equidistant from a fixed line, directrix, and a fixed point, focus.
- Vertex is the maximum or minimum point and is the midpoint between the focus and directrix.
- Axis is the line perpendicular to the directrix and goes through the focus and vertex.

<table>
<thead>
<tr>
<th>Vertical Parabola</th>
<th>Horizontal Parabola</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Parabola Diagram" /></td>
<td><img src="image" alt="Parabola Diagram" /></td>
</tr>
<tr>
<td>d1 = d2</td>
<td>Directrix (x = -p)</td>
</tr>
<tr>
<td>Focus (x, y)</td>
<td>y</td>
</tr>
<tr>
<td>Vertex</td>
<td>(x - h)² = 4p(y - k)</td>
</tr>
<tr>
<td>Directrix y = k - p</td>
<td>(y - k)² = 4p(x - h)</td>
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</table>

Example 1:

Find the vertex, focus, and directrix of the parabola given by \( y = 0.5x^2 \).

Solution

Rearrange the equation to fit the standard form.

\[ x^2 = 2y \]

Since it is \( x^2 \), it is a vertical parabola. Compare it to the standard form equation.

\[ h = 0, k = 0, \text{ and } 4p = 2 \text{ so } p = \frac{1}{2}. \]

The vertex is (0, 0).

The focus is \( \left( \frac{1}{2}, 0 \right) \).

Directrix is \( y = -\frac{1}{2} \).

Example 2:
Find the standard form of the equation of a parabola with vertex at (0, 0) and focus (-2, 0).

**Solution**

Graph the vertex and focus, and notice that the line through the two points is horizontal. This line is the axis. Since the vertex = (0, 0) = (h, k), both h = k = 0.

The distance from the vertex to the focus is 2 to the left, so p = -2. Now fill out the standard form equation for a horizontal parabola.

\[
(y - 0)^2 = 4(-2)(x - 0) \\
y^2 = -8x
\]

**Example 3:**

Find the vertex, focus, and directrix of the parabola given by \(x^2 - 2x - 16y - 31 = 0\).

**Solution**

It is \(x^2\), so this is a vertical parabola. Start arranging the terms to fit the standard form.

\[x^2 - 2x = 16y + 31\]

Complete the square in the x by adding \(\left(\frac{1}{2}b\right)^2\) to each side of the equation.

\[x^2 - 2x + \left(\frac{1}{2}(-2)\right)^2 = 16y + 31 + \left(\frac{1}{2}(-2)\right)^2\]

\[x^2 - 2x + 1 = 16y + 32\]

Factor both sides.

\[(x - 1)^2 = 16(y + 2)\]

Now compare to the standard form equation.

h = 1, k = -2, and 4p = 16 so p = 4.

Vertex (1, -2).

Focus (0, 2) \(\neq (1, 2)\)

Directrix \(y = -6\)

Graph by solving for y and making a table of values.

\[y = \frac{(x - 1)^2}{16} - 2\]

**Example 4:**

Write the standard form of the equation of the parabola with focus at (1, 2) and directrix \(x = 3\).

**Solution**

The directrix is vertical, so the axis of the parabola is horizontal.

The focus is (1, 2) = (p + h, k), so k = 2.

Graph the directrix and focus. The vertex is halfway in between the focus and directrix, (2, 2).

The distance from the vertex to the focus is 1 to the left, so p = -1.

Fill in the standard form equation for a horizontal parabola.

\[(y - 2)^2 = 4(-1)(x - 2)\]

\[(y - 2)^2 = -4(x - 2)\]

**Application**

Find the equation of a tangent line to a parabola.

Skip this section and the problems in the assignment about them because it is complicated and far easier to do with real calculus.