Chapter 3 Practice Exercises (*Optional)

3-01 Exponential Functions

Two types of fish have been introduced into a pond. The population of fish A can be modeled by $A(t) = 20(1.05)^t$ and fish B by $B(t) = 40(1.025)^t$ where t is in years. (Round your answers to the nearest whole number.)

1. Which fish population is growing at a faster rate?
2. What type of fish was initially introduced in greater quantity?
3. Assuming the models are still accurate, which type of fish will have the bigger population after 10 years?

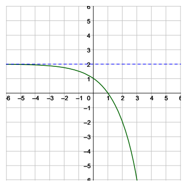
Evaluate the function for the given values.

4. $f(x) = 2\cdot3^x$
   a. $f(0)$
   b. $f(2)$
   c. $f(-1)$
   d. $f(½)$
5. $f(x) = -e^{x+1}$
   a. $f(0)$
   b. $f(2)$
   c. $f(-1)$
   d. $f(½)$
6. What is an asymptote?
7. The graph of $f(x) = 2^x$ is reflected over the x-axis and stretched vertically by a factor of 5. (a) Write the new function $g(x)$, and (b) State its y-intercept, domain, and range.

Write a function that represents the transformation of $f(x) = 3^x$.

8. Shift $f(x)$ 3 units right and 2 units down
9. Reflect $f(x)$ over the y-axis and shift 2 units up

The graph is a transformation of $y = 2^x$. Write an equation describing the transformation.

10. 

Graph the function. Then state the domain, range, asymptote, and whether it is exponential growth, decay, or neither.

11. $f(x) = 3^{x-2}$
12. $g(x) = \left(\frac{1}{2}\right)^x - 3$
13. $h(x) = 2^{1-x} + 1$
14. $j(x) = -e^x + 2$
15. *$k(x) = 2e^x$
16. Sally invests $1500 in an account that pays 7.5% interest. How much is the account worth after 20 years if the interest is compounded
    a. annually?
    b. semiannually?
    c. monthly?
    d. daily?
    e. continuously?

Mixed Review

17. (2-09) Solve $\frac{x+1}{x} \leq 0$.
18. (2-08) Find the asymptotes and graph $f(x) = \frac{x+1}{x}$.
19. (2-05) Find the rational zeros of $x^4 - 2x^3 - 11x^2 + 6x + 24$.
20. (2-06) Find the irrational zeros of $x^4 - 2x^3 - 11x^2 + 6x + 24$.
21. (1-09) Verify that $f(x) = 2x + 3$ and $g(x) = \frac{x-3}{2}$ are inverses by finding their composition.

3-02 Logarithmic Functions

1. What does ln represent?

Rewrite each equation in exponential form.

2. $\log_r(t) = w$
3. $\log_6(m) = n$
4. $\ln(7) = q$

Rewrite each equation in logarithmic form.

5. $a^b = c$
6. $p^3 = 64$
7. $e^t = 8$

Evaluate without using a calculator.

8. $\log_2\left(\sqrt{8}\right)$
9. $4\log_3\left(\frac{1}{9}\right)$
10. $\ln\left(e^{\frac{2}{3}}\right) + 4$

Use properties of logarithms to evaluate.

11. $e^{\ln(6.5)} - 2$
12. $10^{\log(16)}$

Use a calculator to evaluate.

13. log 15
14. ln π

Problem Solving

15. How many decibels is a trumpet played with a sound intensity of 5×10^{-4} W/m²?

Mixed Review

16. (3-01) Evaluate $3 \cdot 4^2$.
17. (3-01) State the domain, range, and asymptote of $f(x) = 2 \cdot e^{x+1} - 3$. Then graph the function.
18. (2-07) Find the asymptotes and intercepts of $g(x) = \frac{2x}{x-1}$.
19. (2-06) Solve $x^3 + 2x^2 - 9x = 18$.
20. (2-04) Divide $(2x^3 + x^2 - 3) \div (x + 1)$.

3-03 Properties of Logarithms

1. Which type of translation affect the domain of a logarithmic function?

Rewrite the expression in terms of $\log_3 4$ and $\log_3 8$.

2. $\log_3 64$
3. $\log_3 2$

Expand the following expressions.

4. $\log_3(3xy^4)$
5. $\log\left(\frac{m}{3n^3}\right)$
6. $\ln\left(\frac{2p^3}{e^t\cdot q}\right)$

Condense the following expressions.

7. $\ln x + 2\ln y$
8. $2\log_6 3 + \log_6 w - 4\log_6 t$
9. $\log_2 7 - 2\log_2 m - 4\log_2 n$

Use the change-of-base formula to evaluate each expression as a quotient of natural logs. Use a calculator to approximate each to three decimal places.

10. $\log_7 128$
11. $\log_{½} 7.3$

State the asymptote, domain, and range. Then sketch the graph of the indicated function.

12. $f(x) = \log_2(x - 3)$
13. $g(x) = \log x + 1$
14. $h(x) = -2\ln(x - 3) + 1$

Problem Solving

15. pH is a measure of active hydrogen ion in a solution. The formula pH $= -\log([H^+])$ is used to calculate pH where H^+ is the hydrogen ion concentration in mol/L and pH is the pH level. What is the difference in pH level between battery acid with a hydrogen ion concentration of 1.58×10^{-1} mol/L and orange juice with a hydrogen ion concentration of 5.01×10^{-4} mol/L?

Mixed Review

16. (3-02) Rewrite as an exponential equation: $\ln x = 7$.
17. (3-02) Evaluate without using a calculator: $\log_3(27) - 1$.
18. (3-01) For the following exercises, graph the function. Then state the domain, range, asymptote, and whether it is exponential growth, decay, or neither: $f(x) = ½e^x + 1$
19. (2-09) Solve by graphing: $2x^2 - 5x - 3 \leq 0$.
20. (2-06) Solve $x^3 - 7x^2 + 17x - 15 = 0$.

3-04 Solve Exponential and Logarithmic Equations

1. When can the one-to-one property of logarithms be used to solve an equation?

Solve using the one-to-one property.

2. $4^{3x} = 16$

3. $3^{x-4} = 81^x$

Use logarithms to solve.

4. $3^{x+4} = 8$

5. $-2 \cdot 7^{t+5} - 5 = 10$

6. $5e^{2x-3} - 10 = 14$

Use the one-to-one property of logarithms to solve.

7. $\log_3 (5n + 8) = \log_3 (4 - 3n)$

8. $\ln(x^2 - 8) = \ln(-2x)$

Solve the equation.

9. $3 \log_2 (x) = 21$

10. $\ln(2x + 3) - 5 = 4$

11. $9 + 2 \log_3 (x - 4) = 12$

Solve the equation by graphing.

12. $\log_4 (x - 2) = 1$

13. $\log(2x) - \log(5) = -\frac{1}{2}$

14. $\ln\left(-\frac{x}{2}\right) = \log_3(3x + 4)$

Problem Solving

15. The population of geese in a park can be modeled by $P = 23(1.1)^t$ where t is the number of days since the first geese arrived. How many days until there are 150 geese in the park?

Mixed Review

16. (3-03) Condense $\ln 2 - \ln x - \ln y$.

17. (3-03) State the asymptote, domain, range, and then graph $f(x) = \log_3 (x) + 2$.

18. (3-02) Rewrite in logarithmic form: $3^{x+1} = y$.

19. (3-01) Frank invests $20 in an account that pays 5% interest. How much is the account worth after 15 years if the interest is compounded
   a. monthly?
   b. daily?
   c. continuously?

20. (3-01) State the asymptote, domain, range, and then graph $g(x) = 2e^{x-1}$.

3-05 Exponential and Logarithmic Models

1. Is half-life associated with exponential growth or decay? What about doubling time?

2. Quickly sketch a graph of
   a. exponential growth
   b. exponential decay
   c. logarithmic
   d. logistic
   e. Gaussian

3. The number of people in a certain city infected by a flu virus doubles every 3 days. If the flu started with a household of 4 people, how many people have been infected after 25 days?

4. Suzy has a headache so she takes a regular 325 mg aspirin tablet. The half-life of aspirin in the blood is about 15 minutes. How much aspirin remains in her system after 2 hours?

5. In a certain crate of 250 apples, 20% rot each day. How many good apples remain after one week?

6. A common chemotherapy drug to treat cancer is 5-Fluorouracil. Each hour, 83.9% of it decays. A 75 kg patient is given a 2250 mg dose.
   a. Write an equation to model the amount of 5-Fluorouracil in the body after t hours.
   b. How much of the drug remains after 6 hours?
   c. How long until one quarter of the drug remains?

7. Use the logistic growth model $f(x) = \frac{200}{1+12e^{-0.5x}}$.
   a. Find and interpret $f(0)$.
   b. Find the carrying capacity.

8. The number of sparrows in a backyard after t years can be modeled by $S(t) = \frac{125}{1+17e^{-0.452t}}$.
   a. Graph the function.
   b. What is the initial population of sparrows?
   c. How many sparrows are in the backyard after 5 years?
   d. How long will it take for the population to reach 90 birds?
   e. What is the carrying capacity?

9. The function $R(t) = \frac{230}{1+45e^{-0.6t}}$ models the number of students at a school who have heard a certain rumor after t days.
   a. How many people started the rumor?
   b. How many people have heard the rumor after 4 days?
   c. As t increases, what value does $R(t)$ approach? Interpret your answer.
   d. Read Proverbs 14:15. What does the wise man suggest about believing what you hear?

10. The IQ scores for a sample of a class are approximated by the normal distribution $y = 0.03e^{-\frac{(x-100)^2}{450}}, 70 \le x \le 115$, where x is the IQ score.
   a. Graph the function.
   b. Find the average IQ score.

11. In a small college, the amount of time in hours per week a student uses a math-tutoring center approximates the normal distribution $f(t) = 0.7979e^{-\frac{(t-5.4)^2}{0.5}}$, $4 \le t \le 7$, where t is the number of hours.
   a. Graph the function.
   b. What is the average number of hours per week that a students uses the center?

12. Use the Richter scale $R = \log \frac{I}{I_0}$ and let $I_0 = 1$.
   a. Find the intensity of the 2.5 magnitude earthquake in Alpena, Michigan in 2015.
   b. Find the intensity of the 5.8 magnitude earthquake in Louisa, Virginia that damaged Washington, D.C. in 2011.
   c. Find the magnitude, R, of an earthquake with I = 10,000,000.
   d. Find the magnitude, R, of an earthquake with I = 20,000.

13. The sound level in decibels is given by the formula $\beta = 10 \log\left(\frac{I}{I_0}\right)$ where β is the sound level, I is the sound intensity, and I_0 is 10^{-12} W/m^2 or the softest audible sound.
   a. Find the sound level, β, if $I = 10^{-7}$ W/m^2 (average home).
   b. Find the sound level, β if $I = 10^{-1}$ W/m^2 (chainsaw at 1 m).
   c. Find the intensity, I, if β = 90 dB (diesel truck at 10 m).
   d. Find the intensity, I, if β = 10 dB (rustling leaves).

Enter the data from each table into a graphing calculator and graph the resulting scatter plots. Determine whether the data from the table could represent a function that is linear, exponential, or logarithmic.

14.

x	1	2	3	4	5	6	7	8	9	10
***f*(*x*)**	-0.631	0	0.369	0.631	0.834	1	1.140	1.262	1.369	1.465

15.

x	1	2	3	4	5	6	7	8	9	10
***f*(*x*)**	1.2	1.44	1.728	2.074	2.488	2.986	3.583	4.300	5.160	6.192

Mixed Review

16. (3-04) Solve $2 + 3 \log_3 (x + 1) = 4$.

17. (3-04) Solve $5 \cdot 4^{t+3} = 10$.

18. (3-03) Expand $\log_5 \left(\frac{5r}{t^3}\right)$.

19. (3-02) Solve by rewriting as an exponential equation: $5 = \log_2 x$.

20. (3-01) The graph of $f(x) = 5^x$ is reflected over the x-axis and stretched vertically by a factor of 3. Then it is shifted 2 units left and 4 units up. What is the equation of the new function, $g(x)$? State its y-intercept, domain, and range.

3-Review

Take this test as you would take a test in class. When you are finished, check your work against the answers. On this assignment round your answers to three decimal places unless otherwise directed.

Simplify

1. $4 \log_3 3^{2x}$

2. $4 \cdot 5^{\log_5 3t}$

Identify the asymptote, domain, and range. Then graph the function.

3. $f(x) = 2 \cdot 3^{x-1} - 4$

4. $g(x) = \frac{1}{2} \ln (x + 2) + 1$

Evaluate

5. $\log_2 32$

6. $\log_{1/2} 8$

7. $3^{1/3}$

8. Expand $\log_4 \frac{16x^2y^3}{x^4z^5}$

9. Condense $3 \log x - 2 \log y + \log z$

Solve

10. $3^x = 81$

11. $2^{x+8} = 16$

12. $3 \cdot 5^x - 2 = 10$

13. $4e^{2x-1} = 16$

14. $\log_5 (x + 1) = 3$

15. $2 \log (x + 3) + 4 = 15$

16. $\ln (x) + \ln (x + 3) = 4$

17. An archaeologist finds a mummy whose body contains 68% of the carbon-14 found in living bodies. If the half-life of carbon-14 is 5730 years, how old is the mummy?

In electronics, a capacitor can act as a small battery. If a charged capacitor is used to power a circuit, the function $V = V_0e^{-t/\tau}$ models the voltage of the capacitor. V_0 is the initial voltage, τ is the time constant, and t is the time in seconds.

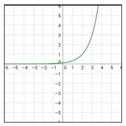
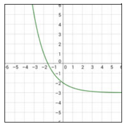
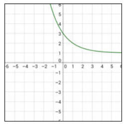
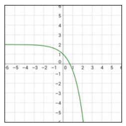
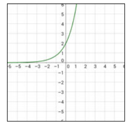
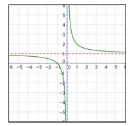
18. If the initial voltage is 12V and the voltage is 4V after 1.5 s, find an equation to model the capacitor.

19. How long for the capacitor to discharge to 2V?

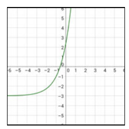
20. What will the voltage be after 1.3 s?

Answers

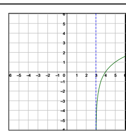
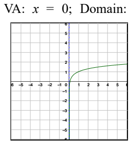
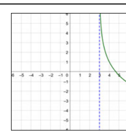
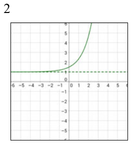
3-01

1. Fish A has a larger base (1.05).
2. Fish B had 20 more than fish A.
3. Fish A will have 33 fish and B will have 51 fish.
4. 2; 18; ⅔; $2\sqrt{3}$
5. −2.718; −20.086; −1; −4.482
6. An asymptote is a line that the graph of a function approaches, as x either increases or decreases without bound or approaches from the left or right. The horizontal asymptote of an exponential function tells us the limit of the function's values as the independent variable gets either extremely large or extremely small.
7. $g(x) = -5(2)^x$; y-int: (0, −5); domain: all real numbers; range: (−∞, 0)
8. $g(x) = 3^{x-3} - 2$
9. $g(x) = 3^{-x} + 2$
10. $y = -2^x + 2$
11.  ; domain: all real numbers; range: (0, ∞); $y = 0$; growth
12.  ; domain: all real numbers; range: (−3, ∞); $y = -3$; decay
13.  ; domain: all real numbers; range: (1, ∞); $y = 1$; decay
14.  ; domain: all real numbers; range: (−∞, 2); $y = 2$; neither
15.  ; domain: all real numbers; range: (0, ∞); $y = 0$; growth
16. \$6371.78; \$6540.57; \$6691.23; \$6721.50; \$6722.53
17. [−1, 0)
18. VA: $x = 0$; HA: $y = 1$; 
19. −2, 4
20. $\pm\sqrt{3}$
21. $f(g(x)) = x$

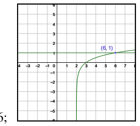
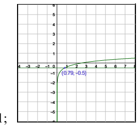
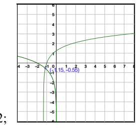
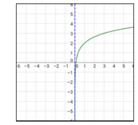
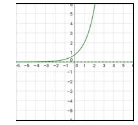
3-02

1. ln is a logarithm with base e called the natural log.
2. $r^w = t$
3. $6^n = m$
4. $e^q = 7$
5. $\log_a c = b$
6. $\log_p 64 = 3$
7. $\ln 8 = t$
8. $\frac{3}{2}$
9. −8
10. $\frac{14}{3}$
11. 4.5
12. 16
13. 1.176
14. 1.145
15. 87.0 dB
16. 48
17. Domain: All Real Numbers; Range: (−3, ∞); HA: $y = -3$; 
18. VA: $x = 1$; HA: $y = 2$; x-int: (0, 0); y-int: (0, 0)
19. −3, −2, 3
20. $2x^2 - x + 1 + \frac{-4}{x+1}$

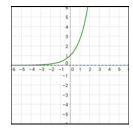
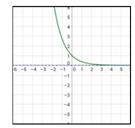
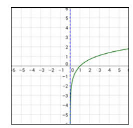
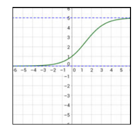
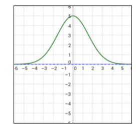
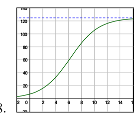
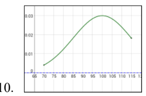
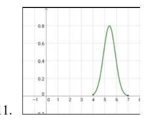
3-03

1. Shifting the function right or left will affect its domain.
2. $3 \log_3 4$
3. $\log_3 8 - \log_3 4$
4. $1 + \log_3(x) + 4\log_3(y)$
5. $\log m - \log 3 - 3\log n$
6. $\ln 2 + 3\ln p - t - \ln q$
7. $\ln xy^2$
8. $\log_6\left(\frac{9w}{t^4}\right)$
9. $\log_2\left(\frac{7}{m^2n^4}\right)$
10. 2.493
11. −2.868
12. VA: $x = 3$; Domain: (3, ∞); Range: (−∞, ∞); 
13. VA: $x = 0$; Domain: (0, ∞); Range: (−∞, ∞); 
14. VA: $x = 3$; Domain: (3, ∞); Range: (−∞, ∞); 
15. 2.499
16. $e^7 = x$
17. 2
18.  ; Domain: (−∞, ∞); Range: (1, ∞); HA: $y = 1$; Exponential growth
19. [−½, 3]
20. 3, 2 + i, 2 − i

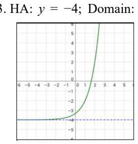
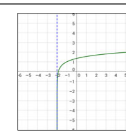
3-04

1. The one-to-one property can be used if both sides of the equation can be rewritten as a single logarithm with the same base.
2. ⅔
3. −⅘
4. −2.107
5. No solution
6. 2.284
7. −½
8. −4
9. 128
10. 4050.042
11. 9.196
12. 6; 
13. 0.791; 
14. −1.152; 
15. About 20 days
16. $\ln\left(\frac{2}{xy}\right)$
17. VA: $x = 0$; Domain: (0, ∞); Range: (−∞, ∞); 
18. $\log_3 y = x + 1$
19. \$42.27; \$42.34; \$42.34
20. HA: $y = 0$; Domain: (−∞, ∞); Range: (0, ∞); 

3-05

1. Exponential decay; Exponential growth
2.  ;  ;  ;  ; 
3. $f(t) = 4e^{0.231t}$; 1290 people
4. $A = 325e^{-0.0462t}$; 1.27 mg
5. $A = 250e^{-0.2231t}$; 52 apples
6. $f(t) = 2250e^{-1.8264t}$; 0.039 mg; 0.759 hours
7. initial amount = 15.4 units; 200 units
8.  ; 7 sparrows; 45 sparrows; 8.36 years; 125 sparrows
9. 5 people; 45 people; 230 people, maximum number of people who will have heard the rumor; look it up
10.  ; 100
11.  ; 5.4 hrs
12. 316; 630,957; 7; 4.3
13. 50 dB; 110 dB; 10^{-3} W/m²; 10^{-11} W/m²
14. logarithmic y = −0.6310 + 0.9102 ln(x)
15. exponential ($y = 1.2^x$)
16. $x \approx 1.080$
17. $t = -2.5$
18. $1 + \log_5 r - 3\log_5 t$
19. $x = 2^5 = 32$
20. $g(x) = -3 \cdot 5^{x+2} + 4$; y-int: (0, −71); Domain: (−∞, ∞); Range: (−∞, 4)

3-Review

1. $8x$
2. $12t$
3. HA: $y = -4$; Domain: (−∞, ∞); Range: (−4, ∞); 
4. VA: $x = -2$; Domain: (−2, ∞); Range: (−∞, ∞); 
5. 5
6. −3
7. 1.442
8. $2 + 3\log_4 y - 2\log_4 x - 5\log_4 z$
9. $\log \frac{x^3 z}{y^2}$
10. 4
11. −4
12. 0.861
13. 1.193
14. 124
15. 316224.766
16. 6.040
17. 3188 years
18. $V = 12e^{-t/1.37}$
19. 2.45 s
20. 4.65 V