

## CHAPTER 4 PRACTICE EXERCISES (\*OPTIONAL)

### 4-01 ANGLE MEASURES

1. Draw an angle in standard position. Label the vertex, initial side, and terminal side. 12.  $-\frac{11\pi}{4}$

**For each given angle a) draw the angle in standard position, b) convert it to the other angle unit, c) find a positive coterminal angle, d) find a negative coterminal angle, e) find the complementary angle, and f) find the supplementary angle.**

2.  $300^\circ$
3.  $135^\circ$
4.  $-120^\circ$
5.  $30^\circ$
6.  $405^\circ$
7.  $-540^\circ$
8.  $\frac{\pi}{4}$
9.  $\frac{5\pi}{6}$
10.  $\frac{3\pi}{2}$
11.  $-\frac{\pi}{3}$

13.  $\frac{10\pi}{3}$

#### Problem Solving

14. A car with 16 inch diameter tires is traveling at 25 mi/hr. Find the angular speed of the tires in rad/min. How many revolutions per minute do the tires make?

15. An arc has a central angle of  $30^\circ$  and a radius of 128 ft. Find the (a) length of the arc and (b) the area of the sector.

#### Mixed Review

16. (3-05) A substance has a half-life of 3.1 seconds. If the initial amount of the substance was 100 grams, how many half-lives will have passed before the substance decays to 10 grams? What is the total time of decay?

17. (3-04) Solve  $4 \cdot 3^x + 10 = 7$

18. (3-03) Condense  $\log_3 x - 2 \log_3 y + 7 \log_3 z$

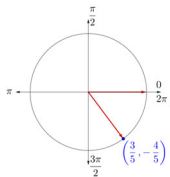
19. (2-06) Find all the zeros of  $f(x) = x^3 - 4x^2 + 6x - 4$

20. (1-07) Identify the parent function and describe the transformations:  $g(x) = -2(x + 3)^2 + 4$

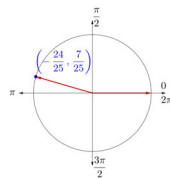
### 4-02 UNIT CIRCLE

1. Draw and label the complete unit circle.

**Evaluate the six trigonometric functions using the point on the unit circle.**



2.



3.

**Evaluate all six trigonometric functions for the given angle using the unit circle.**

4.  $150^\circ$

5.  $\frac{7\pi}{6}$

6. 0

7.  $-\frac{3\pi}{2}$

8.  $480^\circ$

9.  $-\frac{11\pi}{4}$

10. If  $\tan(x) = 1.5$ , what is  $\tan(-x)$ ?

11. If  $-\sec(x) = 2$ , what is  $\sec(-x)$ ?

**Use a calculator to evaluate the expression**

12.  $\cos \frac{5\pi}{12}$

13.  $\sin 100^\circ$

14.  $\csc \frac{\pi}{5}$

**Problem Solving**

15. If a child riding a pink horse starts a ride on a carousel at the point (1, 0) and it rotates in a circle around the origin, what is the coordinates of the child after 45 seconds given the carousel rotates at 1 revolution per minute? 20. (2-08) Identify the asymptotes and graph  $f(x) = \frac{2x-1}{x^2}$ .

16. (4-01) a) draw the angle in standard position, b) convert it to the other angle unit, c) find a positive coterminal angle and d) find a negative coterminal angle of  $\frac{6\pi}{7}$ .

#### Mixed Review

17. (4-01) A race car with an 18-inch diameter wheel is traveling at 180 mi/h. Find the angular speed of the wheels in rad/min. How many revolutions per minute do the wheels make?

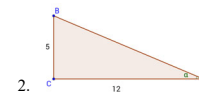
18. (3-02) Evaluate without using a calculator:  $\log_3 81$ .

19. (3-04) Solve  $2 \log_3 (x - 1) = 10$ .

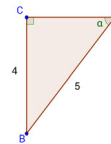
### 4-03 RIGHT TRIANGLE TRIGONOMETRY

1. Draw a right triangle and label one acute angle  $\theta$ . Label the adjacent, opposite, and hypotenuse.

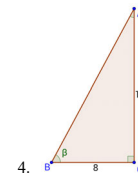
**Evaluate the six trigonometric functions for the indicated angles.**



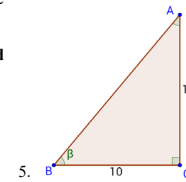
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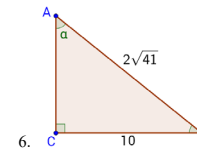
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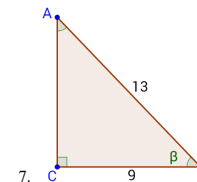
4.



5.



6.



7.

**Use the special right triangles to evaluate the indicated trigonometric function.**

8.  $\sin 30^\circ$

9.  $\csc 45^\circ$

10.  $\cot 60^\circ$

11.  $\sec 30^\circ$
12.  $\cos \frac{\pi}{4}$
13.  $\sec \frac{\pi}{3}$
14.  $\cot \frac{\pi}{4}$
15.  $\csc \frac{\pi}{6}$

**Mixed Review**

16. (4-02) Using the unit circle, evaluate  $\sec \frac{3\pi}{2}$ .
17. (4-02) Using the unit circle, evaluate  $\sin 570^\circ$ .
18. (4-01) Draw the angle,  $\frac{7\pi}{4}$  in standard position, then find a positive and negative coterminal angle.
19. (3-04) Solve  $\log(x) - \log(x+2) = 1$ .
20. (2-01) Divide  $\frac{2-i}{i}$ .

**4-04 RIGHT TRIANGLE TRIGONOMETRY AND IDENTITIES**

1. Explain the cofunction identity.

**Let  $\theta$  be an acute angle. Use the given function value with trigonometric identities to evaluate the given function.**

2. If  $\sin \theta = 0.9$ , find a)  $\cos \theta$  and b)  $\csc \theta$ .
3. If  $\sin \theta = 0.25$ , find a)  $\sin(90^\circ - \theta)$  and b)  $\tan \theta$ .
4. If  $\sec \theta = 1.45$ , find a)  $\cos \theta$  and b)  $\tan \theta$ .
5. If  $\cos \theta = 0.6$ , find a)  $\sin \theta$  and b)  $\cot \theta$ .
6. If  $\csc \theta = 10$ , find a)  $\sin \theta$  and b)  $\csc(90^\circ - \theta)$ .

**Problem Solving**

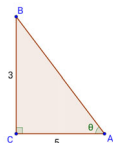
7. A 20-ft ladder leans against a building so that the angle between the ladder and the ground is  $75^\circ$ . How high up the building does the ladder reach?
8. A 30-ft ladder leans against a building so that the angle between the ladder and the ground is  $70^\circ$ . How far from the building is the base of the ladder?
9. The angle of elevation to the top of the Willis Tower is  $33.2^\circ$  when you are a half-mile from the base of the tower. How high is the tower?
10. \*If the Empire State Building is 1250 ft high and the angle of the elevation to the top is  $52^\circ$ , how far from the building are you?
11. A group of civil engineers wants to build a bridge over a canyon, but they do not know how wide the canyon is. They raise different tall objects up beside the canyon until one of them casts a shadow to the other side of the canyon. The height of the object is 80 ft and they estimate the angle of elevation of the sun is  $35^\circ$ . Roughly, how wide is the canyon? (Ben P)
12. A tall pine tree grows vertically. If Sam is 50 feet from the tree and measures the angle of elevation as  $80^\circ$ , how tall is the tree?
13. A large advertising banner hangs on the side of a building. Duane works in a neighboring building 75 feet away and measures to angle of elevation to the top of the banner as  $50^\circ$

and the angle of depression to the bottom as  $20^\circ$ . How long is the banner?

14. Marie is standing on a platform waiting to ride a roller coaster. She measures the angle of depression to the bottom of the long hill as  $13^\circ$  and the angle of elevation to the top of the hill as  $52^\circ$ . If she is 110 feet away, how high is the hill?
15. A steeple is on top of a church. Marco stands 52 ft from the church and measures the angle of elevation to the base of the steeple as  $44^\circ$ . He measures the angle of elevation to the top of the steeple as  $56^\circ$ . How tall is the steeple?
16. Philip is standing on Inspiration Point in Arcadia Scenic Turnout 800 feet above Lake Michigan. He can see two ship, one behind the other. If the angle of depression to the closer ship is  $18^\circ$  and the farther ship is  $15^\circ$ , how far apart are the ships?

**Mixed Review**

17. (4-03) Use a special right triangle to evaluate a)  $\tan 30^\circ$  and b)  $\sec \frac{\pi}{4}$ .
18. (4-03) Evaluate the six trigonometric functions for the given angle.



19. (4-02) Evaluate the six trigonometric functions for  $\frac{4\pi}{3}$  using the unit circle.
20. (4-01) A car with a 30-inch diameter wheels is traveling at 50 mi/h. Find the angular speed of the wheels in rad/min. How many revolutions per minute do the wheels make?
21. (3-02) What is the intensity of a loud stereo blaring music at 95 dB?

**4-05 TRIGONOMETRIC FUNCTIONS OF ANY ANGLE**

**Evaluate the six trigonometric functions based on the given point on the terminal side of an angle in standard position.**

1.  $(3, -5)$
2.  $(-2, -7)$
3.  $\frac{\pi}{2}$
4.  $2\pi$

**Evaluate the function of  $\theta$ .**

5. If  $\sin \theta = \frac{1}{5}$  and  $\theta$  is in quadrant II, find a)  $\cos \theta$  and b)  $\tan \theta$ .
6. If  $\sec \theta = \frac{4}{3}$  and  $\theta$  is in quadrant IV, find a)  $\sin \theta$  and b)  $\csc \theta$ .
7. If  $\tan \theta = -\frac{3}{4}$  and  $\sin \theta > 0$ , find a)  $\cos \theta$  and b)  $\sec \theta$ .
8. If  $\cos \theta = -\frac{8}{17}$  and  $\tan \theta < 0$ , find a)  $\sin \theta$  and b)  $\cot \theta$ .

**Find the reference angle of the given angle.**

9.  $\frac{6\pi}{5}$
10.  $\frac{4\pi}{7}$
11.  $-\frac{8\pi}{9}$

12.  $\frac{15\pi}{4}$

**Evaluate the given trigonometric functions using reference angles.**

13.  $\sin \frac{3\pi}{4}$
14.  $\tan \frac{11\pi}{6}$
15.  $\cos \frac{5\pi}{4}$

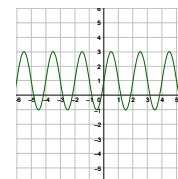
**Mixed Review**

16. (4-04) Let  $\theta$  be an acute angle. Use the given function value with trigonometric identities to evaluate the given function. If  $\csc \theta = 2$ , find a)  $\cot \theta$  and b)  $\sin \theta$ .
17. (4-04) A student is standing on the third floor of a building 30 feet above the ground. There are two kids on a lawn playing catch with a Frisbee. The angles of depression from the student in the building to the kids are  $45^\circ$  and  $55^\circ$ . How far apart are the students?
18. (4-03) Use special right triangles to evaluate a)  $\sin \frac{\pi}{3}$  and b)  $\cot \frac{\pi}{4}$ .
19. (4-02) Use the unit circle to evaluate a)  $\cos \frac{\pi}{6}$  and b)  $\tan \frac{7\pi}{6}$ .
20. (4-01) a) Draw the  $\frac{17\pi}{6}$  in standard position and find a b) positive and c) negative coterminal angle.

**4-06 GRAPHS OF SINE AND COSINE**

1. Why are sine and cosine called periodic functions?

**Graph two full periods of each function and state the amplitude, period, and midline. State the maximum and minimum y-values and their corresponding x-values on one period for  $x > 0$ . State the phase shift and midline. Round answers to two decimal places if necessary.**



2.  $y = 2 \sin x$

3.  $f(x) = \frac{3}{4} \cos x$

4.  $g(x) = \sin(\frac{1}{2}x)$

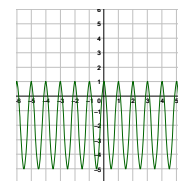
5.  $y = 3 \cos(\pi x)$

6.  $f(x) = -2 \sin(\frac{\pi}{2}x - \pi)$

7.  $g(x) = -\frac{1}{2} \cos(2x) - 3$

8. Determine the amplitude, midline, period, and an equation involving the sine function for the graph.

9. Determine the amplitude, period, midline, and an equation involving cosine for the graph.



10. The Centennial Wheel is a large observational wheel in Chicago with a diameter of 196 ft. Passengers load at the bottom of the wheel from a platform that is 10 ft high. The wheel completes 3 revolutions in 15 minutes. Let  $h(t)$  be a function that gives the

height of a passenger at time  $t$ .

- Find the amplitude, midline, and period of  $h(t)$ .
- Find a formula for the height function  $h(t)$ .
- How high off the ground is a person after 10 minutes?



Figure 1:  
(pixabay/858265)

quadrant II, find a)  $\cos \theta$  and b)  $\tan \theta$ .

- (4-05) Evaluate the six trigonometric function of  $\theta = \pi$ .
- (4-04) If  $\cos \theta = 0.8$ , find a)  $\sin \theta$  and b)  $\cot \theta$  using identities.
- (4-03) Use special right triangles to evaluate the six trigonometric functions for  $\frac{\pi}{3}$ .
- (4-02) Use the unit circle to evaluate the six trigonometric functions for  $-\frac{\pi}{6}$ .

#### Mixed Review

- (4-05) Evaluate the function of  $\theta$ . If  $\sin \theta = \frac{1}{3}$  and  $\theta$  is in

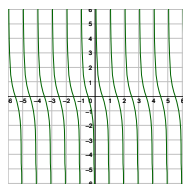
### 4-07 GRAPHS OF OTHER TRIGONOMETRIC FUNCTIONS

- Explain how the graph of cosine can be used to graph secant.

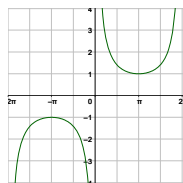
Sketch two periods of the graph of the function. Identify the stretching factor, period, and horizontal shift.

- $n(x) = 2 \sec(\pi x)$
- $p(x) = \tan\left(\frac{\pi}{2}x - \frac{\pi}{4}\right)$
- $r(x) = \frac{1}{2} \cot(x)$
- $t(x) = -3 \csc(2\pi x)$
- $f(x) = 2 \tan\left(x - \frac{\pi}{8}\right)$
- $y = \sec(4x) + 2$

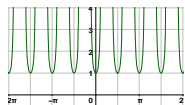
Write an equation for the graph.



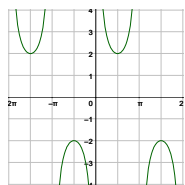
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11.

Use a graphing calculator to graph two periods of the given function. Note: most graphing calculators do not have a cosecant button; therefore, you will need to input  $\csc x$  as  $\frac{1}{\sin x}$ .

- $f(x) = |\sec(x)|$
- $f(x) = \cot(x)\tan(x)$

Graph the damped trigonometric function.

- $y = \frac{1}{2}x \sin(x)$
- $f(x) = |x| \cos(x)$

#### Mixed Review

- (4-06) Graph two full periods of  $f(x) = 2 \sin(\pi x)$ .
- (4-06) Graph two full periods of  $g(x) = \cos\left(2x - \frac{1}{2}\right)$ .
- (4-05) Evaluate  $\cot \frac{17\pi}{6}$  using reference angles.
- (4-03) Evaluate all six trigonometric functions of  $\frac{\pi}{3}$  using special right triangles.

- (4-02) Evaluate all six trigonometric functions of  $\frac{5\pi}{4}$  using the unit circle.

### 4-08 INVERSE TRIGONOMETRIC FUNCTIONS

- Why do  $f(x) = \sin^{-1} x$  and  $g(x) = \cos^{-1} x$  have different ranges?
- Why must the domain of the trigonometric functions be restricted for the inverse trigonometric functions to exist?

#### Mixed Review

- (4-07) Sketch two periods of the graph for each of the following functions. Identify the stretching factor, period, and asymptotes.  
 $y = 2 \sec(\pi x)$

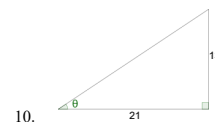
Evaluate the expressions.

- $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
- $\cos^{-1}\left(-\frac{1}{2}\right)$
- $\arctan(-\sqrt{3})$
- $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

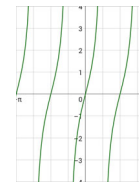
Use a calculator to evaluate each expression. Round to the nearest hundredth.

- $\sin^{-1}(-0.3)$
- $\arccos(0.6)$
- $\tan^{-1}(1.2)$

Find the angle  $\theta$  in the given right triangle. Round to the nearest hundredth.

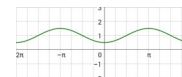


10.



- (4-07) Find an equation for the graph of the function.

- (4-06) Determine the amplitude, midline, period, and an equation involving the sine function for the graph.



- (4-04) Let  $\theta$  be an acute angle. Use the given function value with trigonometric identities to evaluate the given function.  
If  $\cos \theta = \frac{3}{5}$ , find a)  $\sec \theta$  and b)  $\sin \theta$ .

- (4-03) Use the special right triangles to evaluate the indicated trigonometric function.  
 $\csc\left(\frac{\pi}{3}\right)$

### 4-09 COMPOSITIONS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS

Find the exact value, if possible, without a calculator. If it is not possible, explain why.

- $\tan^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right)$
- $\tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$
- $\sin^{-1}(\tan(\pi))$
- $\cos^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$
- $\cos\left(\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right)$
- $\tan\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$

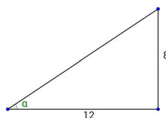
- $\tan(\cos^{-1}(x+1))$
- $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$
- $\sin(\tan^{-1}(x-2))$

- For what value of  $x$  does  $\sin x = \sin^{-1} x$ ? Use a graphing calculator to approximate the answer.

#### Mixed Review

- (4-08) Evaluate  $\arctan\left(\frac{\sqrt{3}}{3}\right)$ .
- (4-08) Evaluate  $\sin^{-1} 1$ .
- (4-05) If  $\sin \theta = \frac{2}{3}$  and  $\tan \theta < 0$ , find a)  $\cos \theta$  and b)  $\cot \theta$ .
- \*(4-03) Evaluate the six trigonometric functions for the

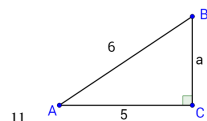
indicated angles.



#### 4-10 APPLICATIONS OF RIGHT TRIANGLE TRIGONOMETRY

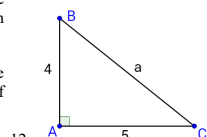
- A 12-foot ladder is leaning against a rain gutter 10 feet above the ground. What angle, in radians, does the ladder make with the ground?
- Two people climb 220 feet up the side of a sand dune so that the change in elevation is 120 feet. What is the angle of elevation of the side of the sand dune?
- The congruent legs of an isosceles triangle are 10 cm, and the base is 6 cm. What is the measure of a base angle of the triangle?
- Without using a calculator, estimate the value of  $\tan^{-1}(1,000,000)$ . Explain your reasoning.
- A guy-wire is a cable that attaches to the top of an electrical pole at an angle to hold it upright. It forms a right triangle with the pole and the ground. If the pole is 13 feet tall and the guy-wire attaches to the ground 5 feet from the pole, what angle does the wire make with the pole?
- What is the angle that the line  $y = \frac{2}{3}x$  makes with the positive  $x$ -axis?
- What is the angle that the line  $y = \frac{5}{2}x$  makes with the positive  $x$ -axis?
- The percent grade of a road is the change in height over a 100-foot horizontal distance. What is the percent grade of a road with a  $3^\circ$  angle of elevation?
- One of the trusses on a railroad bridge is shaped like a right triangle. If the vertical leg is 20 feet and the horizontal leg is 12 feet, what angle does the hypotenuse make with the horizontal leg?
- Frank is building a chicken coop. The frame for the roof will be an isosceles triangle with a base of 4 feet and a height of 1.5 feet. What angle should he cut the wood at the end of the base to get a tight fit?

#### Solve the Right Triangle

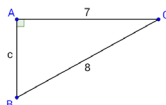


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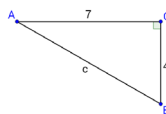
- (3-04) Solve  $2^{x+2} = 64$ .
- (2-09) Solve  $2x^2 + 3x + 1 > 0$ .



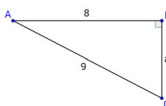
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13.



14.



15.

#### Mixed Review

- (4-09) Evaluate  $\sin(\cos^{-1} \frac{4}{5})$ .
- (4-09) Evaluate  $\tan^{-1}(\tan \frac{\pi}{3})$ .
- (4-08) Evaluate  $\arcsin \frac{\sqrt{3}}{2}$ .
- (4-07) Graph  $y = \frac{\cos x}{\sin x}$  and  $y = \cot x$  on the same graph. What is the relationship between the two functions?
- (4-05) If  $\sec \theta = -3$  and  $\sin \theta < 0$ , find a)  $\tan \theta$  and b)  $\csc \theta$ .

#### 4-11 BEARINGS AND SIMPLE HARMONIC MOTION

##### Bearings

- A plane leaves the airport and flies for 1 hour at 130 mph at  $E 25^\circ N$ . Then it turns and flies for 2 hours at 110 mph at  $E 10^\circ S$ . Finally, it lands. What distance and direction from the airport did the plane land?
- For exercise, Jim leaves his house and runs at 3.5 mph for 30 min at  $W 10^\circ N$ , then he runs at 3.1 mph for 45 min at  $E 80^\circ N$  where he stops at an ice cream shop. How far away and at what direction is the ice cream shop from Jim's house?
- A ship leaves port and travels for 2 hours at 3 knots due north. The it changes course to  $N 10^\circ W$  for 4 hours. Find the distance and bearing from the starting point.
- A naturalist studies wolves. One particular wolf has a GPS tracking collar. The naturalist sees that the wolf ran 3 miles at  $S 20^\circ E$  and then walked 2 miles due north. Finally, the wolf walked 1 mile at  $N 45^\circ E$ . What distance and bearing should the naturalist travel from the wolf's starting point to find the wolf?
- A safari guide leads his group across the savanna at  $W 20^\circ S$  towards a camp 10 km away. Then after traveling 2 km, he discovers he should have been traveling  $S 20^\circ W$ . What bearing and distance should he travel to reach camp?

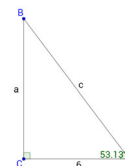
##### Simple Harmonic Motion

- The displacement of a mass hanging from a spring is modeled by  $h(t) = 3 \cos(\frac{8\pi}{3}t)$  where  $t$  is in seconds. Find the amplitude, period, and frequency of the displacement.
- A mass is hanging on a spring and moving with simple harmonic motion. The amplitude is 8 cm, the frequency is 0.5 cycles per second, and it starts at the lowest point. Write a function to model the mass's displacement.
- A mass suspended by a spring is moving up and down with simple harmonic motion. If the mass is at the highest point,  $y = 3$  cm, at  $t = 0$  and returns to the highest point after 0.50 seconds, write an equation to model the motion.

- A pendulum is swinging back and forth 5 cm from the center with simple harmonic motion. If the pendulum is at the center point at  $t = 0$  and completes one full swing in 1.5 seconds, write an equation to model the horizontal position of the pendulum.
- A daredevil bungee jumped off a bridge and is now bouncing up and down with simple harmonic motion. a) If the lowest point at  $t = 0$  is at 10 feet above the water and the highest point at  $t = 5$  s is 50 feet above the water. How high above the water is the equilibrium point? b) What is the amplitude? c) Write an equation modeling the motion from equilibrium. d) Write an equation modeling the motion of the height above the water.

##### Mixed Review

- (4-10) A meter stick is placed vertically on the ground. If its shadow is 1.3 m long, what is the angle of elevation of the sun?
- (4-10) Solve the triangle.



- (4-09) Find the exact value of the expression in terms of  $x$ .  $\sin(\cos^{-1}(\frac{x}{2}))$
- (4-08) What is the domain and range of a)  $\sin^{-1}$ , b)  $\cos^{-1}$ , and c)  $\tan^{-1}$ ?
- (4-06) Graph two full periods of each function and state the amplitude, period, and midline. State the phase shift and vertical translation, if applicable. Round answers to two decimal places if necessary.  
 $y = -2 \sin(\pi x) + 3$

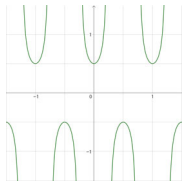
#### 4-REVIEW

**T**ake this test as you would take a test in class. When you are finished, check your work against the answers. On this assignment round your answers to three decimal places unless otherwise directed.

- Sketch the following angles in standard position.
  - $135^\circ$
  - $\frac{11\pi}{6}$
  - $\frac{\pi}{2}$
  - $4.5$
- Find two coterminal angles—one positive and one negative—for a)  $\frac{2\pi}{3}$  and b)  $420^\circ$
- Convert to the other angle unit. a)  $120^\circ$  b)  $15^\circ$  c)  $\frac{4\pi}{3}$  d)  $\frac{\pi}{10}$
- Find the reference angle in radians. a)  $\frac{5\pi}{3}$  b)  $\frac{\pi}{4}$  c)  $\frac{3\pi}{4}$
- Using the unit circle or special right triangles, evaluate the following. a)  $\cos \frac{\pi}{3}$  b)  $\tan \frac{3\pi}{2}$  c)  $\csc \frac{5\pi}{6}$  d)  $\sec \pi$
- A point on a angle  $\alpha$  is  $(3, 7)$ . Evaluate a)  $\sin \alpha$  b)  $\cos \alpha$  c)  $\tan \alpha$  d)  $\sec \alpha$ .
- Given that  $\sin \beta = \frac{4}{5}$  and  $\beta$  is an acute angle, find a)  $\cos \beta$  and b)  $\cot \beta$ .
- Given that  $\tan \theta = -\frac{\sqrt{3}}{3}$  and  $\cos \theta > 0$ ; a) what quadrant does  $\theta$  lie in? b) Evaluate  $\sec \theta$  and c)  $\sin \theta$ .

9. Consider  $y = 2 \sin(\pi x - \frac{\pi}{2})$ . a) Find the amplitude. b) Find the period. c) Find the phase shift.

10. Find a function to model this graph.



11. Find the exact value of  $\cos(\sin^{-1} \frac{3}{4})$ .

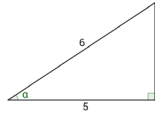
12. Find the exact value of  $\arcsin(\cos \pi)$

13. A ship is 10 miles north and 20 miles east of its destination.

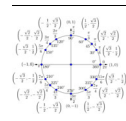
How far and at what bearing should it sail? (Round to 1 decimal place and use degrees.)

14. A mass is bouncing on the end of a spring. If its height at  $t = 0$  is 5 cm above equilibrium and it returns to the highest point after 3 seconds, write a function to model the height from equilibrium.

15. Use the right triangle to evaluate



- $\cot \alpha$
- $\sin \alpha$
- $\sec \alpha$
- $\alpha$  (in degrees)



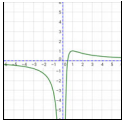
- $\sin \theta = -\frac{4}{5}$     $\csc \theta = -\frac{5}{4}$   
 $\cos \theta = \frac{3}{5}$     $\sec \theta = \frac{5}{3}$   
 $\tan \theta = -\frac{4}{3}$     $\cot \theta = -\frac{3}{4}$
- $\sin \theta = \frac{7}{25}$     $\csc \theta = \frac{25}{7}$   
 $\cos \theta = -\frac{24}{25}$     $\sec \theta = -\frac{25}{24}$   
 $\tan \theta = -\frac{7}{24}$     $\cot \theta = -\frac{24}{7}$
- $\sin \theta = \frac{1}{2}$     $\csc \theta = 2$   
 $\cos \theta = -\frac{\sqrt{3}}{2}$     $\sec \theta = -\frac{2\sqrt{3}}{3}$   
 $\tan \theta = -\frac{\sqrt{3}}{3}$     $\cot \theta = -\sqrt{3}$

- $\sin \theta = -\frac{1}{2}$     $\csc \theta = -2$   
 $\cos \theta = -\frac{\sqrt{3}}{2}$     $\sec \theta = -\frac{2\sqrt{3}}{3}$   
 $\tan \theta = \frac{\sqrt{3}}{3}$     $\cot \theta = \sqrt{3}$
- $\sin \theta = 0$     $\csc \theta = \text{undefined}$   
 $\cos \theta = 1$     $\sec \theta = 1$   
 $\tan \theta = 0$     $\cot \theta = \text{undefined}$
- $\sin \theta = 1$     $\csc \theta = 1$   
 $\cos \theta = 0$     $\sec \theta = \text{undefined}$   
 $\tan \theta = \text{undefined}$     $\cot \theta = 0$
- $\sin \theta = \frac{\sqrt{3}}{2}$     $\csc \theta = \frac{2\sqrt{3}}{3}$   
 $\cos \theta = -\frac{1}{2}$     $\sec \theta = -2$   
 $\tan \theta = -\sqrt{3}$     $\cot \theta = -\frac{\sqrt{3}}{3}$
- $\sin \theta = -\frac{\sqrt{2}}{2}$     $\csc \theta = -\sqrt{2}$   
 $\cos \theta = -\frac{\sqrt{2}}{2}$     $\sec \theta = -\sqrt{2}$   
 $\tan \theta = 1$     $\cot \theta = 1$

- 2
- 0.2588
- 0.9848
- 1.7013
- (0, -1)



- $\frac{1080}{7}$ ;  $(\frac{1080}{7})^\circ$ ;  $\frac{20\pi}{7}$ ;  $-\frac{8\pi}{7}$
- 2.1120 rad/min; 3361 rev/min
- 4
- 244



20. VA:  $x = 0$ ; HA:  $y = 0$ ;

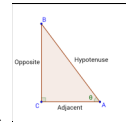
ANSWERS

4-01

<p>1.  ; <math>\frac{5\pi}{3}</math>; <math>660^\circ</math>; <math>-60^\circ</math>; none</p> <p>2.  ; <math>\frac{3\pi}{4}</math>; <math>495^\circ</math>; <math>-225^\circ</math>; none</p> <p>3.  ; <math>\frac{3\pi}{4}</math>; <math>495^\circ</math>; <math>-225^\circ</math>; none</p> <p>4.  ; <math>-\frac{2\pi}{3}</math>; <math>240^\circ</math>; <math>-480^\circ</math>; none</p> <p>5.  ; <math>\frac{\pi}{6}</math>; <math>390^\circ</math>; <math>-330^\circ</math>; <math>60^\circ</math></p>	<p>6.  ; <math>\frac{9\pi}{4}</math>; <math>45^\circ</math>; <math>-315^\circ</math>; none</p> <p>7.  ; <math>-3\pi</math> <math>180^\circ</math>; <math>-180^\circ</math>; none</p> <p>8.  ; <math>45^\circ</math>; <math>\frac{9\pi}{4}</math>; <math>-\frac{7\pi}{4}</math>; <math>\frac{\pi}{4}</math>; <math>\frac{3\pi}{4}</math></p> <p>9.  ; <math>150^\circ</math>; <math>\frac{17\pi}{6}</math>; <math>-\frac{7\pi}{6}</math>; none</p> <p>10.  ; <math>270^\circ</math>; <math>\frac{7\pi}{2}</math>; <math>-\frac{\pi}{2}</math>; none</p>	<p>11. none</p> <p>12. none</p> <p>13. none</p> <p>14. 3300 rad/min; 525.2 rev/min</p> <p>15. <math>\frac{64\pi}{3}</math> ft.; 4289.3 ft<sup>2</sup></p> <p>16. 3.32 half-lives; 10.298 s</p> <p>17. -9.491</p> <p>18. <math>\log_3 \frac{x+7}{y^2}</math></p> <p>19. <math>2, 1+i, 1-i</math></p> <p>20. quadratic function; reflected over the x-axis, vertical stretch by factor of 2, translated left 3 and up 4</p>
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4-02

4-03



- $\sin \alpha = \frac{5}{13}$ ,  $\cos \alpha = \frac{12}{13}$ ,  $\tan \alpha = \frac{5}{12}$ ,  $\csc \alpha = \frac{13}{5}$ ,  $\sec \alpha = \frac{13}{12}$ ,  $\cot \alpha = \frac{12}{5}$
- $\sin \alpha = \frac{4}{5}$ ,  $\cos \alpha = \frac{3}{5}$ ,  $\tan \alpha = \frac{4}{3}$ ,  $\csc \alpha = \frac{5}{4}$ ,  $\sec \alpha = \frac{5}{3}$ ,  $\cot \alpha = \frac{3}{4}$
- $\sin \beta = \frac{15}{17}$ ,  $\cos \beta = \frac{8}{17}$ ,  $\tan \beta = \frac{15}{8}$ ,  $\csc \beta = \frac{17}{15}$ ,  $\sec \beta = \frac{17}{8}$ ,  $\cot \beta = \frac{8}{15}$

- $\sin \beta = \frac{6\sqrt{61}}{61}$ ,  $\cos \beta = \frac{5\sqrt{61}}{61}$ ,  $\tan \beta = \frac{6}{5}$ ,  $\csc \beta = \frac{61}{6\sqrt{61}}$ ,  $\sec \beta = \frac{61}{5\sqrt{61}}$ ,  $\cot \beta = \frac{5}{6}$
- $\sin \alpha = \frac{5\sqrt{41}}{41}$ ,  $\cos \alpha = \frac{4\sqrt{41}}{41}$ ,  $\tan \alpha = \frac{5}{4}$ ,  $\csc \alpha = \frac{41}{5\sqrt{41}}$ ,  $\sec \alpha = \frac{41}{4\sqrt{41}}$ ,  $\cot \alpha = \frac{4}{5}$
- $\sin \beta = \frac{2\sqrt{22}}{13}$ ,  $\cos \beta = \frac{9}{13}$ ,  $\tan \beta = \frac{2\sqrt{22}}{9}$ ,  $\csc \beta = \frac{13}{2\sqrt{22}}$ ,  $\sec \beta = \frac{13}{9}$ ,  $\cot \beta = \frac{9\sqrt{22}}{41}$
- $\frac{8}{5}$   
 $\frac{9}{2}$   
 $\frac{\sqrt{3}}{3}$
- ;  $\frac{15\pi}{4}$ ;  $-\frac{\pi}{4}$

4-04

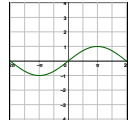
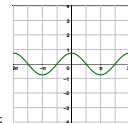
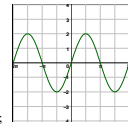
- For example, the sine of an angle is equal to the cosine of its complement; the cosine of an angle is equal to the sine of its complement.
- 0.4359; 19%
- 0.9682; 0.2582
- 0.6897; 1.05
- 0.8; 0.75
- 0.1; 1.0050
7. 19.3 ft
8. 10.3 ft
9. 1728 ft
10. 977 ft
11. 114 ft
12. 284 ft
13. 116.7 ft
14. 166.2 ft
15. 26.9 ft
16. 523 ft
17.  $\frac{\sqrt{3}}{2}$ ;  $\sqrt{2}$
18.  $\sin \theta = \frac{3\sqrt{34}}{34}$ ,  $\cos \theta = \frac{5\sqrt{34}}{34}$ ,  $\tan \theta = \frac{3}{5}$ ,  $\csc \theta = \frac{\sqrt{3}}{3}$
19.  $\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$ ,  $\cos \frac{4\pi}{3} = -\frac{1}{2}$ ,  $\tan \frac{4\pi}{3} = \sqrt{3}$ ,  $\csc \frac{4\pi}{3} = \frac{2\sqrt{3}}{3}$
20. 3520 rad/min; 560.2 rev/min
21. 0.00316 W/m<sup>2</sup>

4-05

- $\sin \theta = \frac{5\sqrt{31}}{34}$ ,  $\cos \theta = \frac{3\sqrt{31}}{34}$ ,  $\tan \theta = \frac{5}{3}$ ,  $\csc \theta = \frac{34}{5\sqrt{31}}$ ,  $\sec \theta = \frac{34}{3\sqrt{31}}$ ,  $\cot \theta = -\frac{3}{5}$
- $\sin \theta = \frac{7\sqrt{53}}{53}$ ,  $\cos \theta = \frac{2\sqrt{53}}{53}$ ,  $\tan \theta = \frac{7}{2}$ ,  $\csc \theta = \frac{53}{7\sqrt{53}}$ ,  $\sec \theta = -\frac{\sqrt{53}}{2}$ ,  $\cot \theta = \frac{2}{7}$
- $\sin \theta = 1$ ,  $\cos \theta = 0$ ,  $\tan \theta = \text{undefined}$ ,  $\csc \theta = 1$ ,  $\sec \theta = \text{undefined}$ ,  $\cot \theta = 0$
- $\sin \theta = 0$ ,  $\cos \theta = 1$ ,  $\tan \theta = 0$ ,  $\csc \theta = \text{undefined}$ ,  $\sec \theta = 1$ ,  $\cot \theta = \text{undefined}$
- $\frac{2\sqrt{6}}{5}$ ,  $\frac{\sqrt{6}}{12}$   
 $-\frac{\sqrt{4}}{4}$ ,  $-\frac{4\sqrt{7}}{7}$
- $-\frac{4}{4}$ ,  $-\frac{5}{7}$   
 $-\frac{4}{4}$ ,  $-\frac{5}{7}$
- $\frac{15}{5}$ ,  $-\frac{8}{15}$   
 $\frac{15}{5}$ ,  $-\frac{8}{15}$
- $\frac{\sqrt{3}}{2}$ ; 1  
 $\frac{\sqrt{3}}{2}$ ;  $\sqrt{3}$
- ;  $\frac{5\pi}{6}$ ;  $-\frac{7\pi}{6}$

4-06

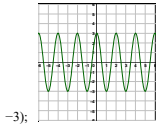
- The shape of the graph repeats regularly.
- Amp = 2; T = 2π; y = 0; PS = 0; Max (π/2, 2); Min (3π/2, -2)
- Amp = 3/4; T = 2π; y = 0; PS = 0; Max (2π, 3/4)
- Amp = π; T = 4π; y = 0; PS = 0; Max (π, 1)



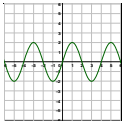
(2π, -2)

(3π, -1)

5. Amp = 3; T = 2; y = 0; PS = 0; Max (2, 3); Min (1,

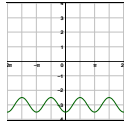


6. Amp = 2; T = 4; y = 0; PS = 2 right; Max (1, 2);



Min (3, -2);

7. Amp =  $\frac{1}{2}$ ; T =  $\pi$ ; y = -3; PS = 0; Max  $(\frac{\pi}{2}, -\frac{5}{2})$ ;



Min  $(\pi, -\frac{7}{2})$ ;

8. Amp = 2; T = 2; y = 1; PS = 0; y = 2 sin( $\pi x$ ) + 1  
 9. Amp = 3; T = 1; y = -2; PS = 0; y = 3 cos(2 $\pi x$ ) - 2  
 10. (a) Amp = 98 ft; midline y = 108 ft; T = 5 min (b)  
 $h(t) = -98 \cos(\frac{2\pi}{5}t) + 108$  (c) 10 ft  
 11.  $-\frac{2\sqrt{2}}{3}$ ;  $-\frac{\sqrt{2}}{4}$   
 12. sin  $\pi = 0$ ; cos  $\pi = -1$ ; tan  $\pi = 0$ ; csc  $\pi = \text{und}$ ; sec  $\pi = \text{und}$

13. 0.6; 4/3  
 $\sin \theta = \frac{\sqrt{3}}{2}$ ;  $\cos \theta = \frac{1}{2}$ ;  $\tan \theta = \sqrt{3}$ ;  
 $\csc \theta = \frac{2\sqrt{3}}{3}$ ;  $\sec \theta = 2$ ;  $\cot \theta = \frac{\sqrt{3}}{3}$ ;  
 $\sin \theta = -\frac{1}{2}$ ;  $\cos \theta = \frac{\sqrt{3}}{2}$ ;  $\tan \theta = -\frac{\sqrt{3}}{3}$ ;  
 $\csc \theta = -2$ ;  $\sec \theta = \frac{2\sqrt{3}}{3}$ ;  $\cot \theta = -\sqrt{3}$

14. 1.667; 0.8

15.  $\frac{2\sqrt{3}}{3}$

#### 4-09

1. 0.46  
 2.  $\frac{\pi}{4}$   
 3. 0  
 4.  $\frac{\pi}{6}$   
 5.  $\frac{\sqrt{3}}{2}$   
 6.  $\frac{\sqrt{3}}{3}$

7.  $\frac{\sqrt{-x^2-2x}}{x+1}$   
 8.  $\frac{\sqrt{x^2-1}}{x}$   
 9.  $\frac{x-2}{\sqrt{x^2-4x+5}}$   
 10. x = 0  
 11.  $\frac{\pi}{6}$   
 12.  $\frac{\pi}{2}$

13.  $-\frac{\sqrt{5}}{3}$ ;  $-\frac{\sqrt{5}}{2}$   
 14. 

$\sin \alpha = \frac{2\sqrt{13}}{13}$	$\cos \alpha = \frac{3\sqrt{13}}{13}$	$\tan \alpha = \frac{2}{3}$
$\csc \alpha = \frac{13}{2\sqrt{13}}$	$\sec \alpha = \frac{13}{3\sqrt{13}}$	$\cot \alpha = \frac{3}{2}$

  
 15. 4  
 16.  $(-\infty, -1) \cup (-\frac{1}{2}, \infty)$

#### 4-10

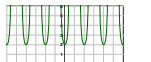
1. 0.985  
 2. 33.1°  
 3. 72.5°  
 4.  $\frac{\pi}{2}$   
 5. 21.0°  
 6. 33.7°  
 7. 68.2°

8. 59°  
 9. 59.0°  
 10. 36.9°  
 11. a =  $\sqrt{11}$ , A  $\approx 33.6^\circ$ , B  $\approx 56.4^\circ$   
 12. a =  $\sqrt{41}$ , B  $\approx 51.3^\circ$ , C  $\approx 38.7^\circ$   
 13. c =  $\sqrt{15}$ , B  $\approx 61.0^\circ$ , C  $\approx 29.0^\circ$   
 14. c =  $\sqrt{65}$ , A  $\approx 29.7^\circ$ , B  $\approx 60.3^\circ$

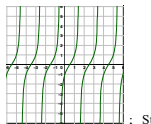
15. a =  $\sqrt{17}$ , A  $\approx 27.3^\circ$ , C  $\approx 62.7^\circ$   
 16.  $\frac{3}{5}$   
 17.  $\frac{1}{3}$   
 18.  $\frac{1}{3}$   
 19. They are the same graph.  
 20.  $2\sqrt{2}$ ;  $-\frac{3\sqrt{2}}{4}$

#### 4-07

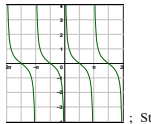
1. Since  $y = \sec x$  is the reciprocal function of  $y = \cos x$ , plot the reciprocal of the coordinates on the graph of  $y = \cos x$  to obtain the y-coordinates of  $y = \sec x$ . The x-intercepts of the graph  $y = \cos x$  are the vertical asymptotes for the graph of  $y = \sec x$ .



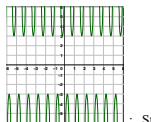
2. ; Stretching factor = 2; Period = 2; Horizontal shift = 0



3. ; Stretching factor = 1; Period = 2; Horizontal shift = \frac{1}{2} to the right.

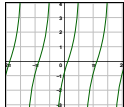


4. ; Stretching factor = 1/2; Period = pi; Horizontal shift = 0.

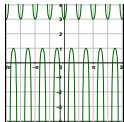


5. ; Stretching factor = 3;

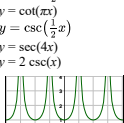
Period = 1; Horizontal shift = 0.



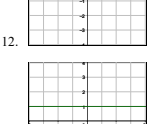
6. ; Stretching factor = 2; Period = pi; Horizontal shift = pi/8 to the right.



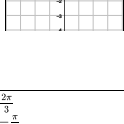
7. ; Stretching factor = 1; Period = pi/2; Horizontal shift = 0.



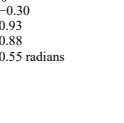
8. y = cot(pi x)



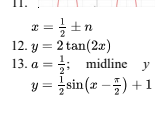
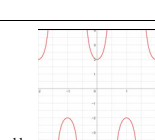
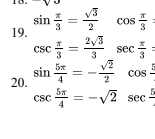
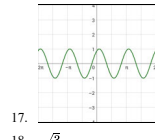
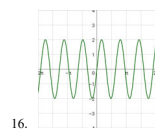
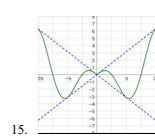
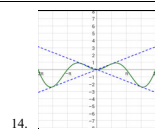
9. y = csc(pi/3 x)



10. y = sec(4x)



11. y = 2 csc(x)

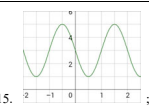


17.  $-\sqrt{3}$   
 $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ ;  $\cos \frac{\pi}{3} = \frac{1}{2}$ ;  $\tan \frac{\pi}{3} = \sqrt{3}$   
 19.  $\csc \frac{\pi}{3} = \frac{2\sqrt{3}}{3}$ ;  $\sec \frac{\pi}{3} = 2$ ;  $\cot \frac{\pi}{3} = \frac{\sqrt{3}}{3}$   
 20.  $\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$ ;  $\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$ ;  $\tan \frac{5\pi}{4} = 1$   
 $\csc \frac{5\pi}{4} = -\sqrt{2}$ ;  $\sec \frac{5\pi}{4} = -\sqrt{2}$ ;  $\cot \frac{5\pi}{4} = 1$

#### 4-11

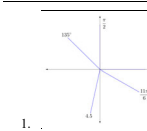
1. 334.9 miles, at E 2.9° N  
 2. 2.9 miles at W 63.0° N  
 3. 17.9 miles at N 6.7° W  
 4. 1.7 miles at E 3.7° S  
 5. 8.85 km at W 80.0° S  
 6. amplitude: 3; period: 3/4; frequency: 4/3  
 7. y = -8 cos(pi t)  
 8. y = 3 cos(4pi t)  
 9. y = 5 sin(pi/8 t)

10. 30 ft; 20 ft;  $y = -20 \cos(\frac{\pi}{5}t)$   
 $y = -20 \cos(\frac{\pi}{5}t) + 30$   
 11. 37.6°  
 12. B = 36.87°; a = 8; c = 10  
 13.  $\frac{\sqrt{4-x^2}}{2}$   
 14. D: [-1, 1] R:  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ ; D: [-1, 1] R: [0, pi]; D:  $(-\infty, \infty)$ ; R:  $(-\frac{\pi}{2}, \frac{\pi}{2})$



15. ; amplitude: 2; period: 2; midline: y = 3; phase shift: 0; vertical translation: 3

#### 4-REVIEW



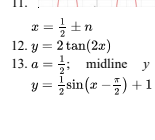
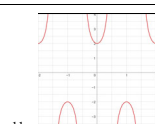
2.  $\frac{8\pi}{3}$ ;  $-\frac{4\pi}{3}$ ; 60°; -300°  
 3.  $\frac{22\pi}{3}$ ;  $\frac{\pi}{12}$ ; 240°; 18°  
 4.  $\frac{\pi}{3}$ ;  $\frac{\pi}{4}$   
 5. 1/2; undefined; 2; -1  
 6.  $\frac{7\sqrt{58}}{58}$ ;  $\frac{3\sqrt{58}}{58}$ ;  $\frac{7}{3}$ ;  $\frac{\sqrt{58}}{3}$   
 7.  $\frac{3}{5}$ ;  $\frac{4}{5}$   
 8. IV;  $\frac{2\sqrt{3}}{3}$ ;  $-\frac{1}{2}$

9. 2; 2;  $\frac{1}{2}$   
 10.  $y = \frac{1}{2} \sec(2\pi x)$   
 11.  $\frac{\sqrt{7}}{4}$   
 12.  $-\frac{\pi}{2}$   
 13. 22.4 miles at W 26.6° S  
 14.  $y = 5 \cos(\frac{2\pi}{3}t)$   
 15.  $\frac{5\sqrt{11}}{11}$ ;  $\frac{\sqrt{11}}{6}$ ;  $\frac{6}{5}$ ; 33.6°

#### 4-08

1. The function  $y = \sin x$  is one-to-one on  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ ; thus, this interval is the range of the inverse function of  $y = \sin x$ ,  $f(x) = \sin^{-1} x$ . The function  $y = \cos x$  is one-to-one on  $[0, \pi]$ ; thus, this interval is the range of the inverse function of  $y = \cos x$ ,  $f(x) = \cos^{-1} x$ . This also gives each function one positive and one negative quadrant.  
 2. In order for any function to have an inverse, the function must be one-to-one and must pass the horizontal line test. The regular sine function is not one-to-one unless its domain is restricted in some way.  
 3.  $-\frac{\pi}{3}$

4.  $\frac{2\pi}{3}$   
 5.  $-\frac{\pi}{3}$   
 6.  $\frac{\pi}{6}$   
 7. -0.30  
 8. 0.93  
 9. 0.88  
 10. 0.55 radians



11. ; a = 2; T = 2; VA:  $x = \frac{1}{2} \pm n$   
 12. y = 2 tan(2x)  
 13. a = 1/2; midline y = 1; period = 2pi;  $y = \frac{1}{2} \sin(x - \frac{\pi}{2}) + 1$