

## CHAPTER 12 PRACTICE EXERCISES (\*OPTIONAL)

### 12-01 INTRODUCTION TO LIMITS

Use a table to evaluate the limit.

1.  $\lim_{x \rightarrow -4} \frac{x^2 + 3x - 4}{x + 4}$

2.  $\lim_{x \rightarrow 0} \frac{3x^2 + 4x}{2x}$

3.  $\lim_{x \rightarrow -1} \begin{cases} 2x + 1, & x \leq -1 \\ x^2 - 2, & x > -1 \end{cases}$

Use a graph to determine if the limit exists. If it does not, explain why. If it does, evaluate the limit.

4.  $\lim_{x \rightarrow 0} 3 \cos\left(\frac{1}{2x}\right)$

5.  $\lim_{x \rightarrow 0} \frac{-x^3 + 3x}{x^3}$

6.  $\lim_{x \rightarrow 1} \frac{2x + 1}{3x - 4}$

7.  $\lim_{x \rightarrow 2} \frac{2\sqrt{x} + 2}{x - 2}$

8.  $\lim_{x \rightarrow 0} \begin{cases} x - 3, & x \leq 0 \\ |x + 3|, & x > 0 \end{cases}$

Evaluate the limit by direct substitution. If it does not exist, say so.

9.  $\lim_{x \rightarrow 1} \sin\left(\frac{\pi}{x}\right)$

10.  $\lim_{x \rightarrow 2} 2x^3 - 5x$

11.  $\lim_{x \rightarrow 0} 3\sqrt{x+1}$

12.  $\lim_{x \rightarrow 2} \frac{x^2 + 3x + 1}{x + 1}$

13.  $\lim_{x \rightarrow 0} 2x(x - 1)$

14.  $\lim_{x \rightarrow 2} \begin{cases} -x^2, & x \leq 0 \\ x^2, & x > 0 \end{cases}$

**Problem Solving**

15. The population of some sparrows in the backyard can be modeled by  $P(t) = \frac{75}{1+e^{-0.1t}}$  where  $P$  is the number of sparrows in the backyard after  $t$  years. Find the limit of the population as time approaches 4 years.

**Mixed Review**

16. (11-04) Find the distance from the point  $(3, 2, -3)$  to the plane  $x + 2y + z - 5 = 0$ .

17. (11-04) Find the parametric equations of the line passing through  $(2, 0, -1)$  and  $(4, 2, 2)$ .

18. (11-03) Evaluate  $(2, 2, -1) \times (-3, 0, 1)$ .

19. (10-02) Evaluate the summation using the shortcut formulas.

$$\sum_{i=1}^{15} i^2 - 3i$$

20. (1-04) Simplify  $\frac{f(x+h)-f(x)}{h}$  when  $f(x) = 2x^2 - x$ .

5.  $\lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x}$

6.  $\lim_{x \rightarrow 0} \frac{\sqrt{x+25} - 5}{x}$

7.  $\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x - 1}$

8. a.  $\lim_{x \rightarrow 0^+} \frac{5x}{|x|}$

b.  $\lim_{x \rightarrow 0^-} \frac{5x}{|x|}$

### 12-02 EVALUATING LIMITS

Evaluate the limit.

1.  $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x - 2}$

2.  $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x^2 - 3x}$

3.  $\lim_{x \rightarrow -1} \frac{x^3 + 4x^2 + 3x}{x + 1}$

4.  $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 12x + 20}$

9. a.  $\lim_{x \rightarrow 1^+} \begin{cases} 2x + 3, & x < 1 \\ -x^2 + 1, & x \geq 1 \end{cases}$

b.  $\lim_{x \rightarrow 1^-} \begin{cases} 2x + 3, & x < 1 \\ -x^2 + 1, & x \geq 1 \end{cases}$

10. a.  $\lim_{x \rightarrow -2^+} \begin{cases} \sqrt{x+3}, & x \leq -2 \\ x, & x > -2 \end{cases}$

b.  $\lim_{x \rightarrow -2^-} \begin{cases} \sqrt{x+3}, & x \leq -2 \\ x, & x > -2 \end{cases}$

Evaluate the limit from calculus.

11. For the function  $f(x) = 2x^2$  evaluate  $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$ .

12. For the function  $f(x) = x^2 + 3$  evaluate  $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$ .

13. For the function  $f(x) = x^2 - 2x$  evaluate  $\lim_{h \rightarrow 0} \frac{f(-1+h) - f(-1)}{h}$ .

14. For the function  $f(x) = 2x^3 + x$  evaluate  $\lim_{h \rightarrow 0} \frac{f(-2+h) - f(-2)}{h}$ .

15. For the function  $f(x) = \frac{1}{x}$  evaluate  $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$ .

**Mixed Review**

16. (12-01) Evaluate  $\lim_{x \rightarrow 3} (2x^3 - 18x)$ .

17. (12-01) Evaluate  $\lim_{x \rightarrow -2} \frac{x^2 - 9}{x + 2}$ .

18. (11-04) Find the distance from  $(2, 0, 1)$  to  $2x + y + 3z - 2 = 0$ .

19. (10-02) Evaluate the sum:  $\sum_{i=1}^{15} (i^2 + 3i)$

20. (10-02) Evaluate the sum:  $\sum_{r=1}^{20} \left(\frac{4}{3}\pi r^3\right)$

### 12-03 DERIVATIVES

1. A function that gives the slope of another function is called a   ?

**Problem Solving**

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Find the derivative of the function.

2.  $f(x) = 3x^2$

3.  $f(x) = (x - 2)^2$

4.  $f(x) = x^3 + 2x$

5.  $f(x) = -4x^2 - 2x + 3$

6.  $f(x) = \sqrt{x - 3}$

7.  $f(x) = \sqrt{x + 2} - 4$

8.  $f(x) = \frac{1}{x}$

Find the slope of the function at the given point.

9.  $f(x) = 3x^2 - 4$  at  $(2, 8)$

10.  $f(x) = 2x^3 + x$  at  $(-1, -3)$

11.  $f(x) = \sqrt{x}$  at  $(4, 2)$

12.  $f(x) = \frac{2}{x^2}$  at  $(1, 2)$

13.  $f(x) = x^2 - 3x + 2$  at  $(0, 2)$

14. Velocity is the derivative of position with respect to time. A falling object's position can be modeled by  $x(t) = -4.9t^2 + 100$  where  $t$  is time in seconds and  $x(t)$  is position in meters. Find the velocity at  $t = 2$  seconds.

15. Acceleration is the derivative of velocity with respect to time. A falling object's velocity can be modeled by  $v(t) = -9.8t$  where  $t$  is time in seconds and  $v(t)$  is velocity in meters per second. Find the acceleration at  $t = 2$  seconds.

**Mixed Review**

16. (12-02) Evaluate  $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x - 3}$ .

17. (12-02) Evaluate  $\lim_{x \rightarrow 0^+} \frac{4|x|}{x}$ .

18. (12-01) Evaluate  $\lim_{x \rightarrow 2} \cos \pi x$ .

19. (10-02) Use formulas to evaluate  $\sum_{i=1}^{10} (x^2 - 3x)$ .

20. (10-02) Use formulas to evaluate  $\sum_{i=1}^{100} (5x^2 - x^3)$ .

### 12-04 LIMITS AT INFINITY AND LIMITS OF SEQUENCES

1. What does it mean to find the limit of a sequence?

2.  $\lim_{x \rightarrow \infty} \frac{6 - 2x + x^2 - 4x^3}{x^3}$

Evaluate the limit if it exists.

$$3. \lim_{x \rightarrow \infty} \frac{x^4 + 2x^2 - 1}{2 + 5x + 3x^3}$$

$$4. \lim_{x \rightarrow \infty} \frac{17 - 3x^3 + x^2}{6x^3}$$

$$5. \lim_{x \rightarrow \infty} \frac{(x+4)(x-9)}{(2x+1)(3x-4)}$$

Find the limit of the sequence and state whether it converges.

$$6. a_n = \frac{(3n-1)(n^2+4)}{2-4n-5n^2}$$

$$7. a_n = \frac{(n^2+2n-1)(3n+2)}{5-2n-9n^3}$$

$$8. a_n = \frac{10}{n^4} \cdot \left[ \frac{n^2(n+1)^2}{4} \right]$$

$$9. a_n = \frac{20}{n^3} \cdot \left[ \frac{n(n+1)(2n+1)}{6} \right]$$

$$10. a_n = \frac{5}{n^2} \cdot \left[ \frac{n(n+1)}{2} \right]$$

### Mixed Review

11. (12-03) Find the derivative of  $f(x) = 2x^3 - 3x$ .

12. (12-03) Find the derivative of  $f(x) = 4\sqrt{x} + 5$ .

13. (12-02) Evaluate  $\lim_{x \rightarrow -3} \frac{x+3}{x^2+2x-3}$ .

14. (12-01) Evaluate  $\lim_{x \rightarrow 2} \frac{x+3}{x^2+2x-3}$ .

15. (10-02) Evaluate  $\sum_{n=1}^{20} (n^3 - n)$ .

## 12-05 INTEGRALS

1. What is an integral?

Evaluate the limit of the sum.

$$2. \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^2}{n^3}$$

$$3. \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i-4}{n^2}$$

$$4. \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^2+5}{n^3}$$

$$5. \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2i^2+i}{n^3}$$

Find the area between  $f(x)$  and the  $x$ -axis between the given  $x$ -values.

6.  $f(x) = 2x + 1$ ; between  $x = 0$  and  $x = 2$

7.  $f(x) = -3x^2 + 3$ ; between  $x = 0$  and  $x = 1$

Evaluate the integral.

8.  $\int_{-1}^2 -3x \, dx$

$$9. \int_2^3 2x^2 \, dx$$

$$10. \int_1^4 (-x^2 + 3x + 4) \, dx$$

### Problem Solving

11. In physics, the integral of a velocity function from  $t = 0$  to  $t = b$  gives the displacement at  $b$  seconds. If the velocity of an object thrown up at 4.5 m/s is modeled by  $v = -9.8t + 4.5$ , what is the displacement of the object after 2 seconds?

### Mixed Review

12. (12-04) Evaluate  $\lim_{x \rightarrow \infty} \frac{3-x+5x^2}{x^2}$ .

13. (12-04) Find the limit of  $a_n = \frac{(3n^2+1)(2n-3)}{1-2n-10n^2}$ .

14. (12-03) Find the slope of  $f(x) = 3x^2 - 4$  at  $x = -2$ .

15. (12-02) Evaluate  $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$ .

16. (12-01) Evaluate  $\lim_{x \rightarrow 0} \sin\left(\frac{2}{x}\right)$ .

## 12-REVIEW

Take this test as you would take a test in class. When you are finished, check your work against the answers. On this assignment round your answers to three decimal places unless otherwise directed.

Evaluate each limit.

$$1. \lim_{x \rightarrow 2} \frac{x-2}{x^2+3x-10}$$

$$2. \lim_{x \rightarrow 1} \frac{x^2+2}{x-4}$$

$$3. \lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4}$$

Use a table or graph to find the limit to 4 decimal places. Draw the table or graph.

$$4. \lim_{x \rightarrow \pi} \frac{3}{\sin x}$$

$$5. \lim_{x \rightarrow 2} \frac{x^3-8}{x-2}$$

Find the derivative.

6.  $f(x) = 4x + 3$

7.  $f(x) = -3x^2$

8.  $f(x) = -\frac{2}{x^2}$

9. Find the slope of  $f(x) = 2\sqrt{x}$  at  $(9, 6)$ .

Find the limit at infinity.

$$10. \lim_{x \rightarrow \infty} \frac{x(2x+3)}{5x^2-7x+1}$$

$$11. \lim_{x \rightarrow -\infty} \frac{(2x+1)(3x-1)}{2x^3+5x-1}$$

## ANSWERS

### 12-01

1. -5	9. 0	16. $\frac{\sqrt{6}}{7}$
2. 2	10. 6	17. $\begin{cases} x = 2t + 2 \\ y = 2t \\ z = 3t - 1 \end{cases}$
3. -1	11. 3	
4. DNE, oscillates	12. $\frac{11}{3}$	18. (2, 1, 6)
5. DNE, decreases without bound	13. 0	19. 880
6. -3	14. 4	20. $4x + 2h - 1$
7. DNE, increases without bound	15. 66 sparrows	
8. DNE, approaches different values from either side		

### 12-02

1. 7	7. $\frac{1}{4}$	14. 25
2. $-\frac{1}{3}$	8. 5; -5	15. -1
3. -2	9. 0; 5	16. 0
4. $-\frac{1}{8}$	10. -2; 1	17. Does not exist
5. $\frac{1}{6}$	11. 8	18. $\frac{5\sqrt{14}}{14}$
6. $\frac{1}{10}$	12. 2	19. 1600
	13. -4	20. 58800π

### 12-03

1. derivative	7. $f'(x) = \frac{1}{2\sqrt{x+2}}$	13. -3
2. $f'(x) = 6x$	8. $f'(x) = -\frac{1}{x^2}$	14. $v = -19.6$ m/s
3. $f'(x) = 2x - 4$	9. 12	15. $a = -9.8$ m/s <sup>2</sup>
4. $f'(x) = 3x^2 + 2$	10. 7	16. 2
5. $f'(x) = -8x - 2$	11. $\frac{1}{4}$	17. -4
6. $f'(x) = \frac{1}{2\sqrt{x-3}}$	12. -4	18. 1
		19. 220
		20. -23810750

### 12-04

1. Find the value that the terms tend towards as $n$ approaches $\infty$ .	6. does not exist; diverges	11. $f'(x) = 6x^2 - 3$
2. -4	7. $-\frac{1}{3}$ ; converges	12. $f'(x) = \frac{2}{\sqrt{x}}$
3. does not exist	8. $\frac{5}{2}$ ; converges	13. $-\frac{1}{4}$
4. $-\frac{1}{2}$	9. $\frac{20}{3}$ ; converges	14. 1
5. $\frac{1}{6}$	10. $\frac{5}{2}$ ; converges	15. 43890

### 12-05

1. The area bounded by $f(x)$ , the $x$ -axis, and the lines $x = a$ and $x = b$	4. $\frac{1}{3}$	8. $-\frac{9}{2}$
2. $\frac{1}{3}$	5. $\frac{2}{3}$	9. $\frac{38}{3}$
3. $\frac{1}{2}$	6. 6	10. $\frac{27}{2}$
	7. 2	11. -10.6 m

12. 5  
13. Does not exist

14. -12  
15.  $\frac{1}{2}$

16. Does not exist

12-REVIEW

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1.  $\frac{1}{7}$   
2. -1  
3.  $\frac{1}{4}$   
4. Does not exist  
5. 12  
6.  $f^{-1}(x) = 4$

7.  $f^{-1}(x) = -6x$   
8.  $f^{-1}(x) = \frac{4}{x^2}$   
9.  $\frac{1}{3}$   
10.  $\frac{2}{5}$   
11. 0  
12. Does not exist

13.  $\frac{7}{6}$   
14. 0  
15.  $\frac{118}{3}$   
16. 8  
17.  $-9.8 \text{ m/s}^2$   
18. -14.1 m